

An Investigation of Wellbore Storage and Skin Effect in Unsteady Liquid Flow: II. Finite Difference Treatment

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ABSTRACT

An investigation of the effect of wellbore storage and skin effect on transient flow was conducted using a finite-difference solution to the basic partial differential equation. The concept of skin effect was generalized to include a damaged annular region adjacent to the wellbore (a composite reservoir). The numerical solutions were compared with analytical solutions for cases with the usual steady-state skin effect. It was found that the solutions for a finite-capacity skin effect compared closely with analytical solutions at short times (wellbore storage controlled) and at long times after the usual straight line was reached. For intermediate times, presence of a finite-capacity skin effect caused significant departures from the infinitesimal skin solutions. Two straight lines occurred on the drawdown plot for cases of large radius of damage. The first had a slope characteristic of the flow capacity of the damaged region; the second straight line had a slope characteristic of the flow capacity of the undamaged region. Results are presented both in tabular form and as log-log plots of dimensionless pressures vs dimensionless times. The log-log plot may be used in a type-curve matching procedure to analyze short-time (before normal straight line) well-test data.

INTRODUCTION

Skin effect was defined by van Everdingen¹ and Hurst² as being an impediment to flow that is caused by an infinitesimally thin damaged region around the wellbore. The additional pressure drop through this skin is proportional to the wellbore flow rate and behaves as though flow through the skin were steady-state.

Wellbore storage is caused by having a moving liquid level in a wellbore, or by simply having a volume of compressible fluid stored in the wellbore.³ When surface flow rates change abruptly, wellbore storage causes a time lag in formation flow rates and a corresponding damped pressure response.

A recent study⁴ was made to determine the combined effects of infinitesimally thin skin and wellbore storage. Analytical methods were used along with numerical integration of a Laplace transformation inversion integral. Tabular and graphical results were presented for various cases. It was recognized during the study that this representation of skin was oversimplified; that skin effect should be thought of as a result of formation damage or improvement to a finite region adjacent to the wellbore.

It was suggested that a skin effect could arise physically in a number of ways. One simple example would be to assume that an annular volume adjacent to the wellbore is reduced uniformly to a lower permeability than the original value. This would be similar to the composite reservoir problem. Perhaps a better example would be to assume that the permeability increases continuously from a low value at the wellbore to a constant value in the undamaged reservoir. In either case, the damaged region would have a finite storage capacity and would lead to transient behavior within the skin region.

A negative skin effect could arise from an increase in permeability within an annular region adjacent to the wellbore. This might physically result from acidizing. But it is believed that cases of more practical importance are those in which negative skin effects are caused by hydraulic fracturing. A high-permeability fracture communicating with the wellbore gives the appearance of a negative skin effect.

For the purposes of this study, it was decided to represent a skin effect, either positive or negative, as an annular region adjacent to the wellbore with either decreased or increased permeability. This then is the composite reservoir problem wherein a permeability k_1 exists from the

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¹References given at end of paper.

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well radius to a radius of the damaged region, r_1 . For composite reservoirs, there are an infinite number of pairs of values, r_1 and k_1 , which correspond to a value of skin effect, s . The main purpose of this study was to investigate the behavior of a well during initial transient flow in the presence of a finite-capacity skin effect with the presence of wellbore storage. The goal was to provide information which might be useful in interpretation of short-time well-test data (either buildup or drawdown).

The initial-value problem is a special case of a composite reservoir problem wherein porosity, viscosity and compressibility are the same for both damaged and undamaged formation regions, but with the permeability changing at the boundary of the two regions; the complication of wellbore storage is also added. The skin effect, s , does not appear explicitly in the formulation of the problem, but may be inferred from the steady-state pressure drop through the damaged annular region adjacent to the wellbore.

The initial-value problem may be stated as follows. For the "damaged" or skin region,

$$\frac{\partial^2 p_{1D}}{\partial r_D^2} + \frac{1}{r_D} \frac{\partial p_{1D}}{\partial r_D} = \frac{k}{k_1} \frac{\partial p_{1D}}{\partial t_D};$$

$$1 \leq r_D < r_{1D} \dots \dots \dots (1)$$

For the undamaged formation,

$$\frac{\partial^2 p_{2D}}{\partial r_D^2} + \frac{1}{r_D} \frac{\partial p_{2D}}{\partial r_D} = \frac{\partial p_{2D}}{\partial t_D};$$

$$r_{1D} < r_D \rightarrow \infty \dots \dots \dots (2)$$

Inner boundary condition,

$$\bar{c} \frac{\partial p_{1D}}{\partial t_D} - \frac{k_1}{k} \left(\frac{\partial p_{1D}}{\partial r_D} \right)_{r_D=1} = 1$$

$$\dots \dots \dots (3)$$

$$p_{wD}(t_D) = p_{1D}(1, t_D)$$

$$\text{for all } t_D \dots \dots \dots (4)$$

Interface conditions between skin region and formation,

$$p_{1D} = p_{2D}; \quad (r_{1D}, t_D) \dots \dots \dots (5)$$

$$k_1 \frac{\partial p_{1D}}{\partial r_D} = k \frac{\partial p_{2D}}{\partial r_D}; \quad (r_{1D}, t_D) \dots \dots \dots (6)$$

Outer boundary condition,

$$\lim_{r_D \rightarrow \infty} p_{2D} = 0 \dots \dots \dots (7)$$

And the initial condition is,

$$p_{1D}(r_D, 0) = p_{2D}(r_D, 0) = 0 \dots \dots \dots (8)$$

The dimensionless variables have usual definitions,

$$r_D = r/r_w \dots \dots \dots (9)$$

$$t_D = \frac{kt}{\phi\mu cr_w^2} \dots \dots \dots (10)$$

$$p_{1D}(r_D, t_D) = \frac{2\pi kh}{q\mu} (p_i - p_{r,t});$$

$$r_w \leq r < r_1 \dots \dots \dots (11)$$

$$p_{2D}(r_D, t_D) = \frac{2\pi kh}{q\mu} (p_i - p_{r,t});$$

$$r_1 < r \rightarrow \infty \dots \dots \dots (12)$$

$$p_{wD}(t_D) = \frac{2\pi kh}{q\mu} (p_i - p_w) \dots \dots \dots (13)$$

$$\bar{c} = \frac{C}{2\pi h \phi c r_w^2} \dots \dots \dots (14)$$

where C represents the fluid storage capacity in the wellbore, cc/atm.

The equivalent steady-state skin effect, s , can be expressed as a function of k , k_1 , r_1 and r_w . That is,

$$s = \left(\frac{k}{k_1} - 1 \right) \ln \left(\frac{r_1}{r_w} \right)$$

$$= \left(\frac{k}{k_1} - 1 \right) \ln r_{1D} \dots \dots \dots (15)$$

Thus, it is possible to select appropriate values of r_{1D} and (k/k_1) to provide a specific value of the skin effect. The relationship between the skin effect and the pressure drop attributable to the skin is

$$s = \frac{2\pi kh}{q\mu} \Delta p_{\text{skin}} \dots \dots \dots (16)$$

Strictly speaking, the above problem may be solved to provide the pressure at any time and radial location within either region. But since the

main objective was information for well-test analysis, only pressures at the well were sought. All results reported in the study are for dimensionless pressures at the well which are denoted as $p_{wD}(t_D)$.

Although the subject problem may be handled analytically with ease, the analytical result would require numerical integration to provide final tabulations of the dimensionless variables. Fortunately, a finite-difference computer program was available that could readily handle the problem. The program was a one-dimensional radial model prepared to solve real gas flow with damage and wellbore storage. The details of this program have been described previously.^{5,6} Solutions for liquid flow were obtained by employing constant-fluid physical properties. The finite-difference solution represented a finite-radius reservoir. But the reservoir radius selected was large enough so that pressure at the outer boundary was not affected for producing times of $t_D \times 10^8$. A check of the outer boundary pressure was made for each run. In addition, all solutions were found to agree closely with the infinite-reservoir analytical solution for the longest producing times run. Since solutions to the diffusivity equation were obtained in terms of pressure, the usual assumption of small pressure gradients throughout the flow system was made.

Given a value of equivalent skin effect, corresponding values of r_{1D} and (k_1/k) were used in the model. Values of $r_{1D} = 1, 10, 100$ and $1,000$ were used to give a range of conditions. The case of $r_{1D} = 1$ is equivalent to the van Everdingen-Hurst infinitesimally thin damaged region. When no wellbore storage is present, the solution to this case can be obtained by adding the skin effect to the $s = 0$ solution. When wellbore storage is present, the solutions to this case are equivalent to those solutions given by Agarwal *et al.*⁴

A negative skin effect in a composite reservoir implies that the region around the wellbore has improved permeability, $k_1 > k$. It should be noted that for a given radius of permeability improvement, r_{1D} , there is an upper limit to the magnitude of negative skin effect. This limit is reached as the permeability around the wellbore approaches infinity. From Eq. 15, it can be seen that the limiting negative skin effect (as $k_1 \rightarrow \infty$) is

$$s = -\ln r_{1D} \dots \dots \dots (17)$$

Table 1 shows the values of (k_1/k) , which correspond to the various values of r_{1D} and s in the study. The first two entries for $s = -5$ are blank because this negative skin effect is not obtainable for $r_{1D} = 10$ or 100 , according to Eq. 15.

RESULTS

Finite-difference solutions were generated for a total of 60 cases. Tables 2 through 11 give the results of these computer runs. Each table gives

TABLE 1 — VALUES OF r_{1D} AND (k_1/k) USED FOR VARIOUS SKIN EFFECTS

Skin-Zone Radius, r_{1D}	(k_1/k) for Skin Effects, s , of		
	-5	+5	+20
10		0.3153	0.1032
100		0.4794	0.1872
1,000	3.6209	0.5801	0.2567

TABLE 2* — $p_{wD}(t_D)$ VS t_D FOR $s = 0$

t_D	$\bar{c} = 0$	10	100	1,000	10,000	100,000
1	0.77935	0.090324	0.0098795	0.00099719	0.000099812	0.0000099822
2	1.0046	0.17234	0.019659	0.0019934	0.00019961	0.000019964
5	1.3493	0.38688	0.048554	0.0049773	0.00049877	0.000049910
1×10^1	1.6411	0.67709	0.09558	0.0099588	0.00099778	0.000099818
2×10^1	1.9513	1.1046	0.18599	0.019821	0.0019950	0.00019963
5×10^1	2.3816	1.9307	0.43514	0.049206	0.0049840	0.00049904
1×10^2	2.7164	2.3889	0.79441	0.097432	0.0099579	0.00099798
2×10^2	3.0560	2.8794	1.3659	0.19146	0.019880	0.0019956
5×10^2	3.5088	3.4328	2.4226	0.45722	0.049467	0.0049866
1×10^3	3.8534	3.8129	3.2538	0.85575	0.098243	0.0099663
2×10^3	4.1984	4.1767	3.9149	1.5237	0.19399	0.019907
5×10^3	4.6549	4.6453	4.5480	2.8668	0.46850	0.049592
1×10^4	5.0008	4.9958	4.9468	4.0120	0.88987	0.089654
2×10^4	5.3465	5.3438	5.3184	4.9150	1.6209	0.19534
5×10^4	5.8033	5.8021	5.7913	5.6617	3.1904	0.47504
1×10^5	6.1501	6.1495	6.1439	6.0825	4.6539	0.91122
2×10^5	6.4959	6.4956	6.4926	6.4615	5.8619	1.6865
5×10^5	6.9526	6.9525	6.9512	6.9382	6.7714	3.4251
1×10^6	7.2993	7.2993	7.2986	7.2919	7.2169	5.1869
2×10^6	7.6451	7.6451	7.6447	7.6412	7.6042	6.7373
5×10^6	8.1024	8.1024	8.1022	8.1007	8.0856	7.8746
1×10^7	8.4484	8.4484	8.4462	8.4476	8.4398	8.3504
2×10^7	8.7947	8.7947	8.7946	8.7942	8.7902	8.7472
5×10^7	9.2517	9.2517	9.2517	9.2515	9.2498	9.2325
1×10^8	9.5982	9.5982	9.5982	9.5981	9.5972	9.5883

*Five-place precision was taken from the computer output but does not imply that the solution have this degree of accuracy.

TABLE 3 — $p_D(t_D)$ VS t_D FOR $s = -5, r_{1D} = 1,000$

t_D	$\bar{c} = 0$	10	100	1,000	10,000	100,000
1	0.33474	0.080373	0.0097524	0.00099588	0.000099799	0.0000099821
2	0.41377	0.14467	0.019266	0.0019893	0.00019957	0.000019964
5	0.52578	0.28523	0.046705	0.0049575	0.00049877	0.000049908
1×10^1	0.61522	0.43189	0.089571	0.0099715	0.00099711	0.000099812
2×10^1	0.70669	0.77706	0.16576	0.019521	0.0019927	0.00019961
5×10^1	0.83044	0.77706	0.35069	0.048019	0.0049717	0.00049892
1×10^2	0.92491	0.89669	0.55918	0.093311	0.0099141	0.00099754
2×10^2	1.0199	1.0049	0.79531	0.17733	0.019723	0.0019940
5×10^2	1.1458	1.1392	1.0581	0.38886	0.048609	0.0049779
1×10^3	1.2412	1.2377	1.2002	0.64782	0.095162	0.0099341
2×10^3	1.3367	1.3348	1.3163	0.96432	0.18302	0.019788
5×10^3	1.4628	1.4620	1.4543	1.3274	0.41214	0.048926
1×10^4	1.5584	1.5580	1.5540	1.5021	0.70857	0.096206
2×10^4	1.6539	1.6537	1.6516	1.6277	1.0970	0.18641
5×10^4	1.7803	1.7802	1.7793	1.7697	1.5772	0.42734
1×10^5	1.8811	1.8810	1.8805	1.8751	1.8023	0.75245
2×10^5	2.0053	2.0052	2.0049	2.0012	1.9616	1.2053
5×10^5	2.2358	2.2357	2.2355	2.2332	2.2093	1.8477
1×10^6	2.4659	2.4659	2.4668	2.4652	2.4493	2.2541
2×10^6	2.7402	2.7402	2.7401	2.7391	2.7289	2.6154
5×10^6	3.1472	3.1472	3.1471	3.1466	3.1414	3.0861
1×10^7	3.4754	3.4754	3.4753	3.4750	3.4720	3.4408
2×10^7	3.8125	3.8125	3.8125	3.8123	3.8106	3.7934
5×10^7	4.2641	4.2641	4.2641	4.2640	4.2633	4.2555
1×10^8	4.6088	4.6088	4.6088	4.6087	4.6083	4.6041

TABLE 4 — $p_D(t_D)$ VS t_D FOR $s = +5, r_{1D} = 1$

t_D	$\bar{c} = 0$	10	100	1,000	10,000	100,000
1	5.7705	0.098841	0.0099724	0.00099812	0.000099818	0.0000099804
2	5.9958	0.19603	0.019928	0.0019961	0.00019963	0.000019956
5	6.3405	0.47808	0.049695	0.0049889	0.00049907	0.000049899
1×10^1	6.6323	0.92007	0.099000	0.0099739	0.00099811	0.000099800
2×10^1	6.9425	1.7112	0.19650	0.019933	0.0019961	0.00019962
5×10^1	7.3728	3.5239	0.48083	0.049724	0.0049892	0.00049906
1×10^2	7.7076	5.3657	0.92990	0.099108	0.0099750	0.00099812
2×10^2	8.0471	7.0361	1.7463	0.19692	0.019937	0.0019961
5×10^2	8.5001	8.2556	3.6826	0.48326	0.049749	0.0049895
1×10^3	8.8446	8.7416	5.7702	0.93875	0.09920	0.0099760
2×10^3	9.1896	9.1399	7.8011	1.7767	0.19728	0.019941
5×10^3	9.6460	9.6260	9.3341	3.8169	0.48521	0.049770
1×10^4	9.9920	9.9818	9.8714	6.1196	0.94572	0.099279
2×10^4	10.338	10.332	10.281	8.5000	1.8005	0.19755
5×10^4	10.794	10.792	10.772	10.396	3.9259	0.48672
1×10^5	11.141	11.140	11.130	11.002	6.4242	0.95138
2×10^5	11.487	11.487	11.481	11.423	9.1463	1.8204
5×10^5	11.944	11.944	11.941	11.919	11.441	4.0193
1×10^6	12.291	12.290	12.289	12.278	12.131	6.6834
2×10^6	12.636	12.636	12.636	12.630	12.565	9.7270
5×10^6	13.094	13.094	13.093	13.091	13.066	12.467
1×10^7	13.440	13.440	13.439	13.438	13.426	13.256
2×10^7	13.786	13.786	13.786	13.785	13.779	13.708
5×10^7	14.243	14.243	14.243	14.243	14.240	14.213
1×10^8	14.589	14.589	14.589	14.589	14.588	14.574

TABLE 5 — $p_D(t_D)$ VS t_D FOR $s = +5$, $r_{1D} = 10$

t_D	$\bar{C}=0$	10	100	1,000	10,000	100,000
1	1.5806	0.094963	0.0099317	0.00099772	0.000099817	0.0000099823
2	2.0907	0.18546	0.019814	0.0019949	0.00019963	0.000019965
5	2.9418	0.43984	0.049259	0.0049845	0.00049905	0.000049911
1 × 10 ¹	3.7167	0.82450	0.097818	0.0099619	0.00099802	0.000099821
2 × 10 ¹	4.5849	1.4903	0.19336	0.019900	0.0019958	0.00019964
5 × 10 ¹	5.8438	2.9887	0.47009	0.049608	0.0049881	0.00049908
1 × 10 ²	6.8136	4.5972	0.90481	0.098820	0.0099721	0.00099812
2 × 10 ²	7.6356	6.3326	1.6942	0.19625	0.019930	0.0019961
5 × 10 ²	8.3728	8.0045	3.5776	0.48142	0.049730	0.0049893
1 × 10 ³	8.7878	8.6617	5.6401	0.93506	0.099163	0.0099756
2 × 10 ³	9.1627	9.1087	7.6997	1.7698	0.19719	0.019940
5 × 10 ³	9.6356	9.6149	9.3069	3.8040	0.48500	0.049767
1 × 10 ⁴	9.9869	9.9764	9.8637	6.1036	0.94531	0.099275
2 × 10 ⁴	10.335	10.330	10.278	8.4872	1.7998	0.19754
5 × 10 ⁴	10.793	10.791	10.771	10.392	3.9245	0.48570
1 × 10 ⁵	11.141	11.140	11.129	11.002	6.4224	0.95134
2 × 10 ⁵	11.487	11.486	11.481	11.423	9.1448	1.8203
5 × 10 ⁵	11.944	11.943	11.941	11.919	11.440	4.0191
1 × 10 ⁶	12.290	12.290	12.289	12.278	12.131	6.6832
2 × 10 ⁶	12.636	12.636	12.636	12.630	12.565	9.7268
5 × 10 ⁶	13.094	13.094	13.093	13.091	13.066	12.467
1 × 10 ⁷	13.440	13.440	13.439	13.438	13.426	13.256
2 × 10 ⁷	13.786	13.786	13.786	13.785	13.779	13.708
5 × 10 ⁷	14.243	14.243	14.243	14.243	14.240	14.213
1 × 10 ⁸	14.589	14.589	14.589	14.589	14.588	14.574

TABLE 6 — $p_D(t_D)$ VS t_D FOR $s = +5$, $r_{1D} = 100$

t_D	$\bar{C}=0$	10	100	1,000	10,000	100,000
1	1.2271	0.093595	0.0099168	0.00099757	0.000099816	0.0000099823
2	1.6117	0.18161	0.019771	0.0019945	0.00019963	0.000019965
5	2.2328	0.42415	0.049654	0.0049825	0.00049903	0.000049911
1 × 10 ¹	2.7824	0.77973	0.097198	0.0099555	0.00099821	0.000099821
2 × 10 ¹	3.3855	1.3675	0.19135	0.019879	0.0019956	0.00019964
5 × 10 ¹	4.2446	2.5813	0.46047	0.049502	0.0049870	0.00049907
1 × 10 ²	4.9255	3.7453	0.87386	0.098456	0.0099684	0.00099809
2 × 10 ²	5.6231	4.9155	1.5999	0.19502	0.019917	0.0019960
5 × 10 ²	6.5601	6.2483	3.2380	0.47553	0.049667	0.0049887
1 × 10 ³	7.2757	7.1110	4.9458	0.91724	0.098965	0.0099736
2 × 10 ³	7.9935	7.9057	6.6889	1.7203	0.19659	0.019934
5 × 10 ³	8.9392	8.9013	8.4575	3.6501	0.48260	0.049742
1 × 10 ⁴	9.6076	9.5894	9.3961	5.8329	0.93915	0.099207
2 × 10 ⁴	10.460	10.152	10.075	8.1808	1.7857	0.19738
5 × 10 ⁴	10.733	10.730	10.706	10.257	3.8917	0.48620
1 × 10 ⁵	11.122	11.111	11.100	10.959	6.3749	0.95029
2 × 10 ⁵	11.473	11.473	11.467	11.407	9.0989	1.8182
5 × 10 ⁵	11.938	11.938	11.936	11.913	11.424	4.0148
1 × 10 ⁶	12.288	12.288	12.287	12.275	12.127	6.6771
2 × 10 ⁶	12.635	12.635	12.634	12.628	12.564	9.7210
5 × 10 ⁶	13.093	13.093	13.093	13.090	13.065	12.464
1 × 10 ⁷	13.439	13.439	13.439	13.438	13.425	13.256
2 × 10 ⁷	13.786	13.786	13.786	13.785	13.779	13.708
5 × 10 ⁷	14.243	14.243	14.243	14.243	14.240	14.213
1 × 10 ⁸	14.589	14.589	14.589	14.589	14.588	14.574

TABLE 7 — $p_D(t_D)$ VS t_D FOR $s = +5$, $r_{1D} = 1,000$

t_D	$\bar{C}=0$	10	100	1,000	10,000	100,000
1	1.0925	0.02866	0.0099086	0.00099748	0.000099815	0.0000099823
2	1.4289	0.17955	0.019747	0.0019943	0.00019962	0.000019965
5	1.9643	0.41578	0.049855	0.0049814	0.00049902	0.000049911
1 × 10 ¹	2.4324	0.75618	0.096850	0.0099518	0.00099792	0.000099820
2 × 10 ¹	2.9415	1.3048	0.19022	0.019867	0.0019955	0.00019964
5 × 10 ¹	3.6610	2.3874	0.45507	0.049441	0.0049864	0.00049907
1 × 10 ²	4.2283	3.3680	0.85662	0.098245	0.0099663	0.00099807
2 × 10 ²	4.8076	4.3105	1.5474	0.19428	0.019910	0.0019959
5 × 10 ²	5.5840	5.3678	3.0399	0.47174	0.049627	0.0099883
1 × 10 ³	6.1762	6.0618	4.4972	0.90432	0.098818	0.0099721
2 × 10 ³	6.7700	6.7088	5.8823	1.6778	0.19506	0.019928
5 × 10 ³	7.5562	7.5293	7.2305	3.4684	0.49773	0.049712
1 × 10 ⁴	8.1523	8.1380	7.9939	5.3747	0.92903	0.099097
2 × 10 ⁴	8.7480	8.7403	8.6671	7.2963	1.7517	0.19698
5 × 10 ⁴	9.5354	9.5321	9.5012	9.0704	3.7500	0.48416
1 × 10 ⁵	10.133	10.131	10.115	9.9275	6.0415	0.94376
2 × 10 ⁵	10.729	10.728	10.720	10.628	8.5293	1.7989
5 × 10 ⁵	11.509	11.509	11.506	11.469	10.867	3.9482
1 × 10 ⁶	12.058	12.058	12.056	12.039	11.822	6.5463
2 × 10 ⁶	12.526	12.526	12.526	12.518	12.435	9.5482
5 × 10 ⁶	13.054	13.054	13.053	13.051	13.023	12.368
1 × 10 ⁷	13.421	13.421	13.420	13.419	13.406	13.225
2 × 10 ⁷	13.777	13.777	13.777	13.776	13.769	13.696
5 × 10 ⁷	14.239	14.239	14.239	14.239	14.236	14.209
1 × 10 ⁸	14.588	14.588	14.588	14.587	14.586	14.572

TABLE 8 — $p_D(t_D)$ VS t_D FOR $s = +20$, $r_{1D} = 1$

t_D	$\bar{C}=0$	10	100	1,000	10,000	100,000
1	20.744	0.099558	0.0099796	0.00099819	0.000099815	0.0000099765
2	20.969	0.19865	0.019954	0.0019963	0.00019961	0.000019962
5	21.314	0.49306	0.049850	0.0049904	0.00049905	0.000049905
1 × 10 ¹	21.606	0.97467	0.099583	0.0099797	0.00099808	0.000099736
2 × 10 ¹	21.916	1.9047	0.19870	0.019955	0.0019962	0.00019954
5 × 10 ¹	22.346	4.4528	0.49334	0.049853	0.0049904	0.00049909
1 × 10 ²	22.681	8.0074	0.97561	0.099593	0.0099798	0.00099807
2 × 10 ²	23.021	13.163	1.9086	0.19874	0.019955	0.0019962
5 × 10 ²	23.474	20.273	4.4757	0.49360	0.049856	0.0049904
1 × 10 ³	23.818	23.120	8.0899	0.97669	0.099604	0.0099799
2 × 10 ³	24.163	24.000	13.408	1.9127	0.19879	0.019956
5 × 10 ³	24.620	24.565	20.966	4.4979	0.49385	0.049858
1 × 10 ⁴	24.966	24.939	24.149	8.1655	0.97767	0.099614
2 × 10 ⁴	25.311	25.298	25.133	13.630	1.9164	0.19882
5 × 10 ⁴	25.768	25.763	25.711	21.621	4.5176	0.49408
1 × 10 ⁵	26.115	26.112	26.087	25.178	8.2396	0.97862
2 × 10 ⁵	26.461	26.459	26.447	26.269	13.854	1.9200
5 × 10 ⁵	26.917	26.917	26.912	26.857	22.279	4.5374
1 × 10 ⁶	27.264	27.264	27.261	27.235	26.185	8.3055
2 × 10 ⁶	27.610	27.610	27.608	27.596	27.401	14.055
5 × 10 ⁶	28.067	28.067	28.057	28.004	28.004	22.953
1 × 10 ⁷	28.413	28.413	28.413	28.410	28.383	27.171
2 × 10 ⁷	28.759	28.759	28.759	28.758	28.745	28.534
5 × 10 ⁷	29.216	29.216	29.216	29.216	29.210	29.151
1 × 10 ⁸	29.563	29.563	29.563	29.563	29.560	29.532

the results for given values of s and r_{1D} . In each table the various wellbore storage cases are listed, ranging from $\bar{C} = 0$ to $\bar{C} = 100,000$.

Only the solutions at selected values of t_D are shown. Smaller time steps were taken between the printed results to reduce the time truncation error. A total of 816 time steps were taken for each case.

The cases of infinitesimally thin skin ($r_{1D} = 1$) were compared to the "analytical" solutions of Ref. 4. Solutions in Ref. 4 were obtained by the numerical integration of an inversion integral. The agreement between the analytical solutions and finite-difference solutions was excellent. The maximum differences between the two were about

TABLE 9 — $p_D(t_D)$ VS t_D FOR $s = +20$, $r_{1D} = 10$

t_D	$\bar{C}=0$	10	100	1,000	10,000	100,000
1	3.1149	0.097455	0.0099581	0.00099798	0.000099820	0.0000099823
2	4.1058	0.19238	0.019980	0.0019957	0.00019960	0.000019965
5	5.9241	0.46780	0.049582	0.0049878	0.00049908	0.000049911
1 × 10 ¹	7.2732	0.90550	0.098823	0.0099722	0.00099812	0.000099821
2 × 10 ¹	8.7622	1.7219	0.19658	0.019933	0.0019961	0.00019964
5 × 10 ¹	13.212	3.8540	0.48534	0.049771	0.0049897	0.00049910
1 × 10 ²	16.034	6.7178	0.95462	0.099372	0.0099777	0.00099818
2 × 10 ²	18.994	10.915	1.8568	0.19917	0.019950	0.0019963
5 × 10 ²	22.256	17.755	4.3309	0.49182	0.049874	0.0049904
1 × 10 ³	25.472	21.878	7.9278	0.97281	0.099564	0.0099796
2 × 10 ³	24.040	23.726	13.033	1.8203	0.19870	0.019950
5 × 10 ³	24.576	24.514	20.652	4.4793	0.49633	0.049856
1 × 10 ⁴	24.945	24.917	24.033	8.1342	0.97722	0.099609
2 × 10 ⁴	25.301	25.287	25.112	13.587	1.9155	0.19881
5 × 10 ⁴	25.764	25.759	25.706	21.586	4.5156	0.49405
1 × 10 ⁵	26.113	26.110	26.085	25.165	8.2362	0.97857
2 × 10 ⁵	26.460	26.458	26.446	26.266	13.849	1.9199
5 × 10 ⁵	26.917	26.916	26.911	26.857	22.275	4.5372
1 × 10 ⁶	27.264	27.264	27.261	27.235	26.183	8.3052
2 × 10 ⁶	27.610	27.610	27.608	27.595	27.401	14.054
5 × 10 ⁶	28.067	28.067	28.067	28.061	28.004	22.952
1 × 10 ⁷						

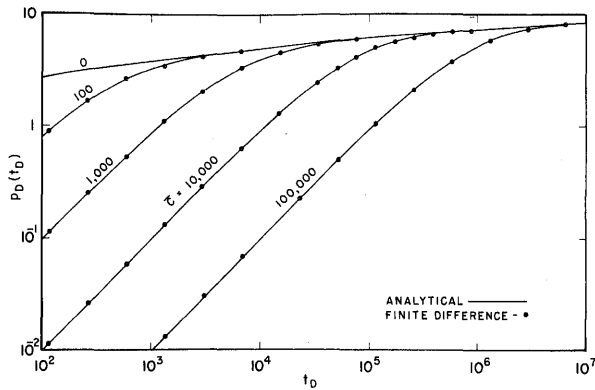


FIG. 1 — $p_D(t_D)$ VS t_D FOR ANALYTICAL AND FINITE-DIFFERENCE SOLUTIONS ($s=0$).

0.5 percent, with the analytical solutions being higher than the finite-difference solutions. This maximum difference occurred for the larger values of \bar{C} at the transition between the wellbore storage-controlled period and the period in which wellbore storage was not important. On a log-log plot of $p_D(t_D)$ vs t_D , this is the region of greatest curvature. At other points on the curve the solutions are almost identical.

Fig. 1 shows a comparison of the analytical solutions from Ref. 4 (solid lines) and finite-difference solutions (points). The differences between the two are not noticeable on a graph. All of the curves in Fig. 1 are for no skin effect, $s = 0$.

Fig. 2 presents results for skin effects of -5 , 0 and $+20$. The skin effect of $+5$ was not plotted to make Fig. 2 more readable. Dimensionless pressures at the well are plotted with skin effect, dimensionless radius of the damaged zone, and the dimensionless wellbore storage constant as parameters. The results for the skin effect of -5 are only for a dimensionless "damaged" zone radius of $1,000$. It was impossible to achieve a negative skin effect as large as -5 with r_{1D} values less than 150 . Thus, the data for a skin effect of $+20$ on Fig. 2 are best for studying the effect of

the radius of the damaged region upon short-time well-test data. Results for a skin effect of $+5$ were similar.

DISCUSSION

As shown in Fig. 2, by comparison of results for different r_{1D} values, representation of a skin effect as an annular region of altered permeability can lead to significant changes in the early pressure-time history, with or without wellbore storage. (All lines on Fig. 2 for $r_{1D} = 1$ represent infinitesimally thin skin cases, or previous analytical cases.) The lines for the finite-radius, damaged regions fall successively below the infinitesimally thin skin case, but finally join the infinitesimally thin skin case at times which increase as the damage radius squared, as would be expected. If there is significant wellbore storage, a separation is noticed within the transition region from storage control to outer formation control, as damaged-zone radius increases. The separation diminishes with increasing wellbore storage constant and is essentially negligible for storage constant of $100,000$. This results because the finite-storage constant cases must join the zero storage case. Thus previous criteria for the duration of the wellbore storage effect⁴ requires some modification. If the damaged-zone radius is great, transients caused by the large volume of the damaged region may last longer than those caused by wellbore storage. In extreme cases, this could result in an early period caused by wellbore storage, followed by two straight lines on a conventional semilog well-test plot — the first having a slope indicative of the permeability of the damaged region; the second having the correct slope indicative of the undamaged formation. This is shown on Fig. 3.

Fig. 3 presents results in a conventional semilog plot of the pressure-time data for a skin effect of $+5$, a storage constant of $1,000$, and two different damaged-zone radii: r_{1D} of 1 and $1,000$. Pressure data to a time of about 2.5×10^3 are almost

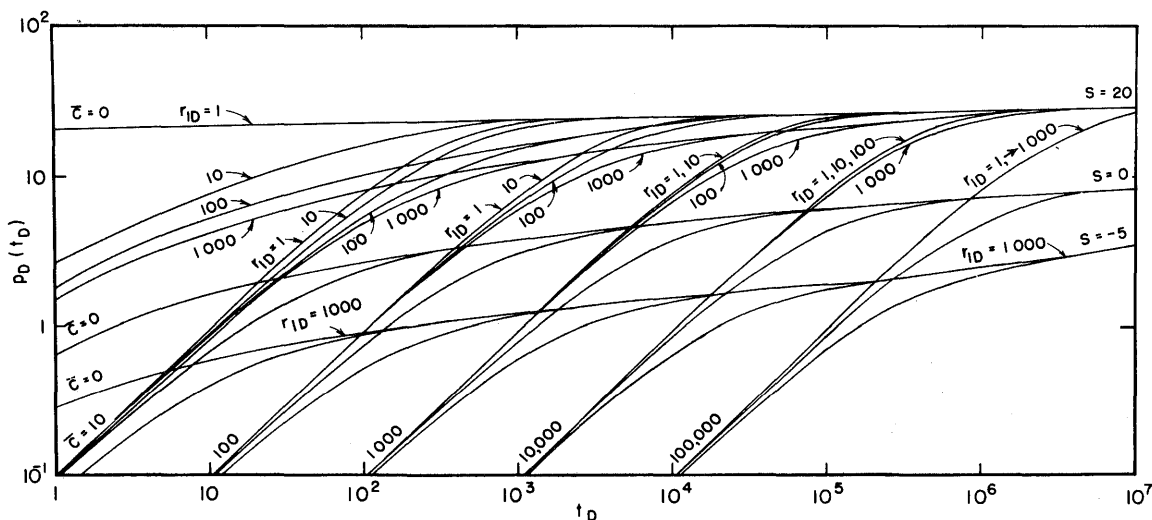


FIG. 2 — SHORT-TIME SOLUTIONS WITH WELLBORE STORAGE AND FORMATION DAMAGE.

completely dominated by the wellbore storage effect. This can be seen better on a log-log plot; the data form near perfect straight lines with a unit slope on a log-log plot. At times of about 2×10^5 , both cases reach a straight line. For the infinitesimally thin skin case, $r_{1D} = 1$, the slope is the correct value of 1.151 and is indicative of the formation permeability. For the large-radius damaged region case, the slope is 1.984, which is indicative of the damaged zone permeability (see Table 1). At a time of about 4×10^6 , the large-radius damaged region case finally reaches the proper straight line. As can be seen on Fig. 3, the proper straight line for the $r_{1D} = 1,000$ case could be easily misinterpreted. It is far easier to interpret this case with a log-log plot and type-curve matching procedures,⁴ and misinterpretation is far less likely. Type-curve interpretation procedures are discussed in detail in Ref. 7.

Several comments regarding well-test interpretation may be made utilizing Fig. 2. First, storage constants for oil well tests often are of the order of 1,000 or greater. Inspection of Fig. 2 indicates that the radius of the damaged region would have to be greater than $100r_w$ to cause a significant effect on the pressure-time history. Thus, previous interpretation methods (such as described in Refs. 4 and 7) should be valid. In other words, the infinitesimal skin concept is valid under these conditions. Second, storage constants for gas well tests may be much less than 1,000. Inspection of Fig. 2 indicates that the radius of the damaged region may play an important role in pressure-time data for these conditions. Thus Fig. 2 represents an additional type-curve which may be particularly useful for interpretation of well-test data when storage constants are small. In our opinion, type-curve matching procedures are extremely useful and will find increasing application in well-test analysis. Finally, it is apparent that Fig. 2 should also be useful in forecasting results of production from a composite reservoir by proper interpretation of the skin-effect parameter.

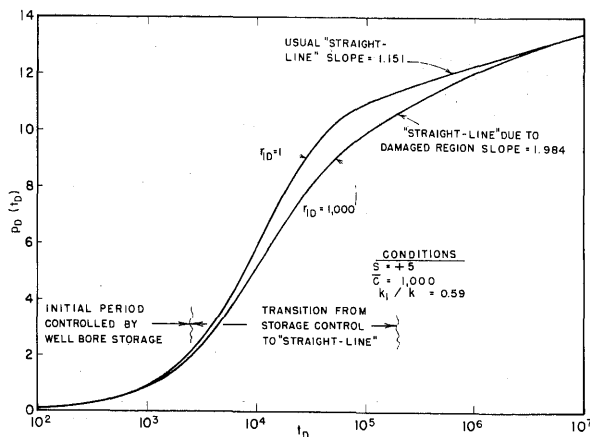


FIG. 3 — EXAMPLE OF BUILDUP DISTORTION WITH WELLBORE STORAGE AND FORMATION DAMAGE.

CONCLUSION

The infinitesimally thin skin concept of van Everdingen and Hurst is applicable if wellbore storage is significant for damaged-zone dimensionless radii from 1 to about 100. If the damaged-zone radius is as large as $1,000r_w$, two straight lines will be evident on a semi-logarithmic plot. If the damaged-zone radius is small, it is not possible to detect the value of the radius from well-test data.

If the damaged-zone radius is greater than $100r_w$, short-time well-test data should be interpreted by means of solutions for a skin region of finite storage capacity. One way to accomplish this end is a type-curve matching procedure employing plots such as Fig. 2.

If wellbore storage is not significant, a damaged- or skin-region radius of $10r_w$ or greater may distort the early pressure data significantly. In this event, two straight lines may appear upon semi-logarithmic plots.

NOMENCLATURE

- c = total system compressibility, atm⁻¹
- C = wellbore storage capacity, cc/atm
- \bar{C} = dimensionless wellbore storage constant, Eq. 14
- b = formation thickness, cm
- k = undamaged formation permeability, darcies
- k_1 = permeability of "damaged" region, darcies
- p = pressure, atm
- p_i = initial pressure, atm
- p_{1D} = dimensionless pressure in "damaged" region, Eq. 11
- p_{2D} = dimensionless pressure in "undamaged" reservoir, Eq. 12
- $p_{wD}(t_D)$ = dimensionless pressure in wellbore
- q = surface flow rate, cc/second
- r = radial distance, cm
- r_1 = radius of "damaged" region, cm
- r_w = wellbore radius, cm
- r_D = dimensionless radius, Eq. 9
- r_{1D} = dimensionless radius of "damaged" region: r_1/r_w
- s = dimensionless skin effect
- t = time, seconds
- t_D = dimensionless time, Eq. 10
- ϕ = porosity, fraction
- μ = viscosity, cp

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