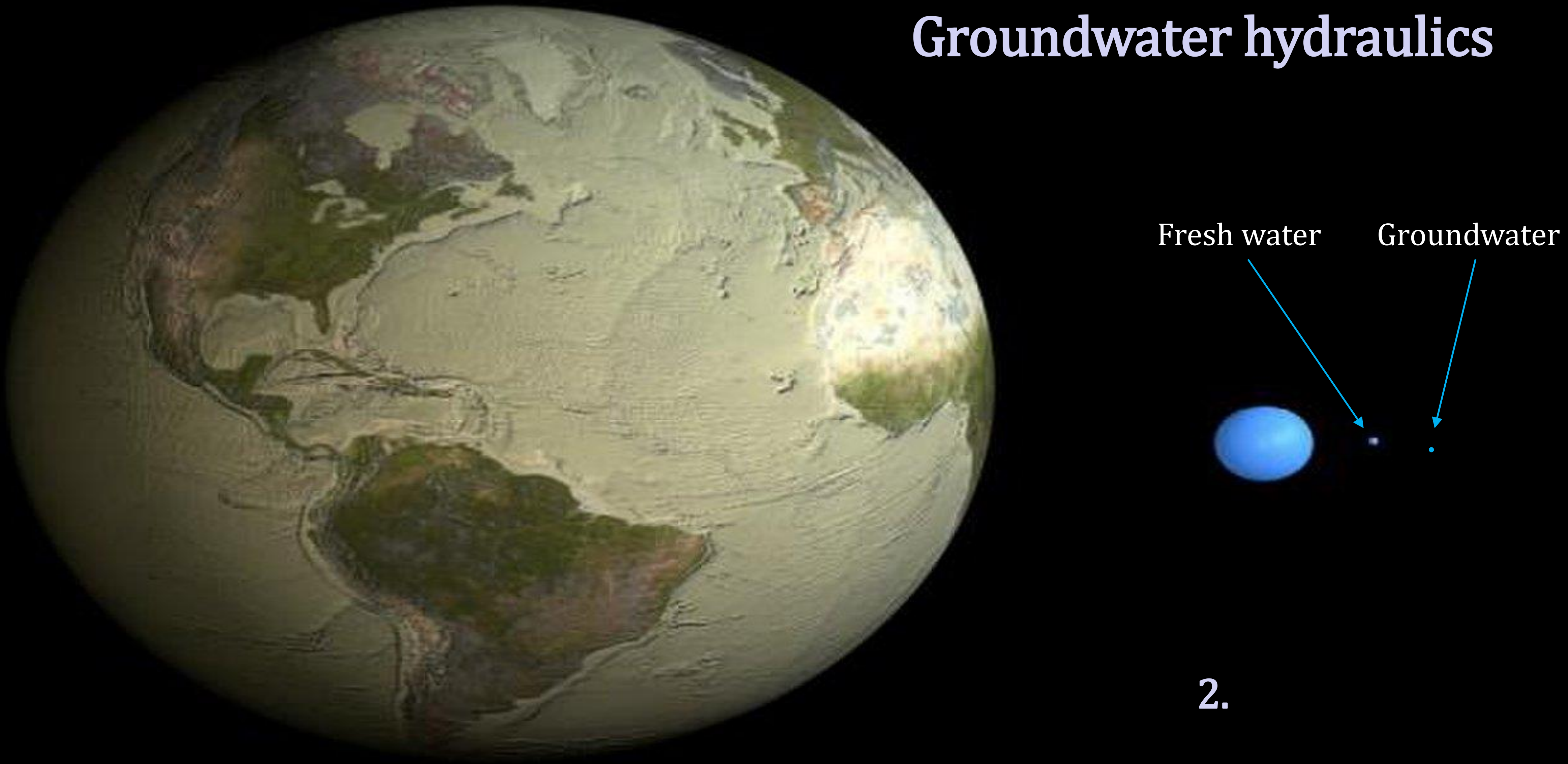
Groundwater hydraulics



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2020_2021

GROUNDWATER HYDRAULICS

LITERATURE

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GROUNDWATER

- **ground water**: the water that lies beneath the ground surface (and groundwater level) , filling the pore space between grains in bodies of sediment and clastic sedimentary rock, and filling cracks and crevices in all types of rock
- **ground water** is a major economic resource,
- source of **ground water** is rain and snow that falls to the ground a portion of which percolates down into the ground to become **ground water**

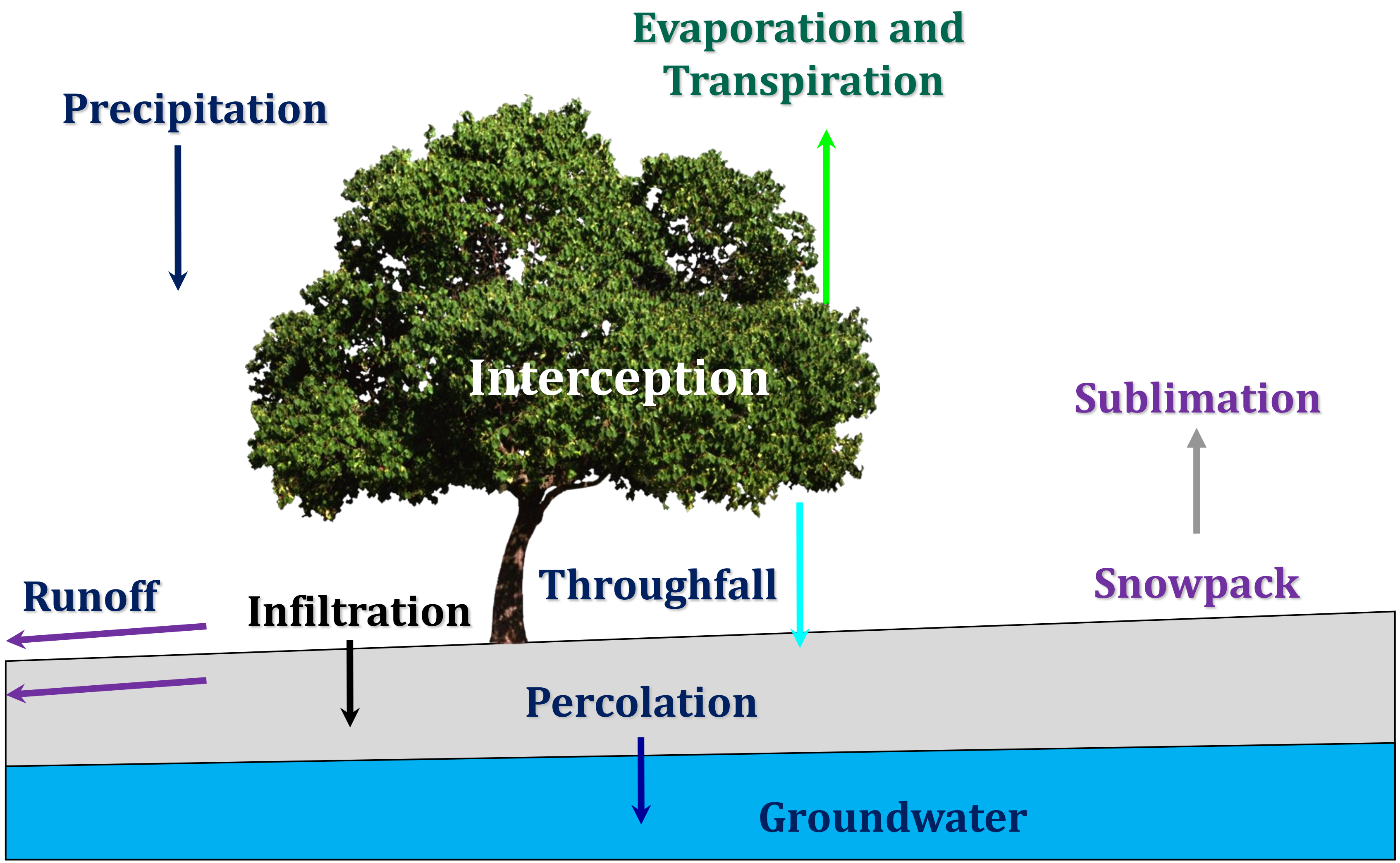
–WHY GROUNDWATER IS GOOD?

- **much less subject to seasonal variations** in availability than surface water
- **slow movement** leads to **high biological purity**
- **Temperature is remarkably constant**
- **Available virtually everywhere if you go deep enough**

Estimate of the World Water Balance

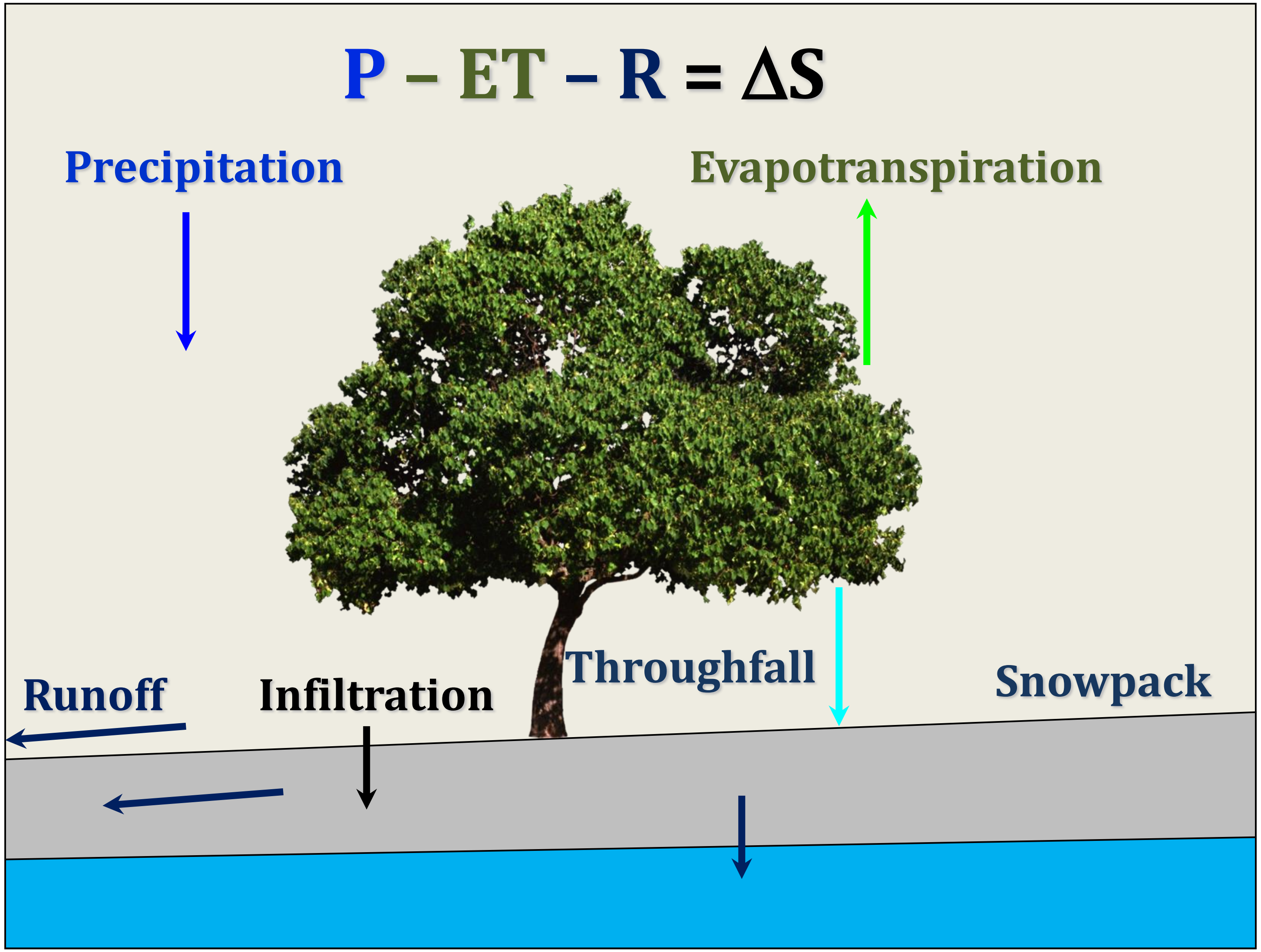
Parameter	Surface area (km^2) X 10^6	Volume (km3) X 10^6	Volume %	Equivalent depth (m)	Residence Time
Oceans and seas	361	1370	94	2500	~4000 years
Lakes and reservoirs	1.55	0.13	<0.01	0.25	~10 years
Swamps	<0.1	<0.01	<0.01	0.007	1-10 years
River channels	<0.1	<0.01	<0.01	0.003	~2 weeks
Soil moisture	130	0.07	<0.01	0.13	2 weeks – 1 year
Groundwater	130	60	4	120	2 weeks – 10,000 years
Icecaps and glaciers	17.8	30	2	60	10-1000 years
Atmospheric water	504	0.01	<0.01	0.025	~10 days
Biospheric water	<0.1	<0.01	<0.01	0.001	~1 week

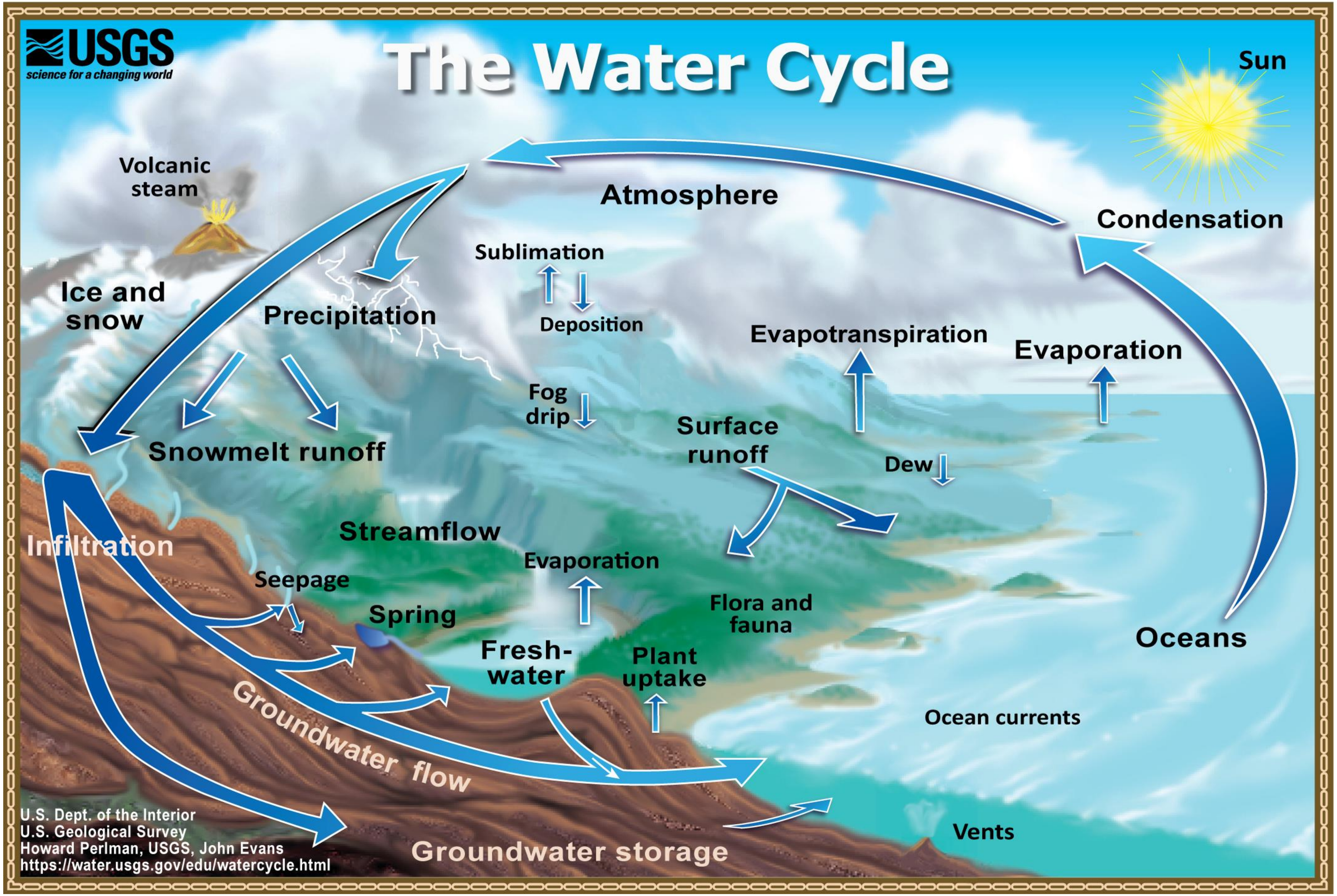
HYDROLOGIC CYCLE - TERMINOLOGY



THE WATER BUDGET: LAW OF MASS CONSERVATION

CLOSED SYSTEM

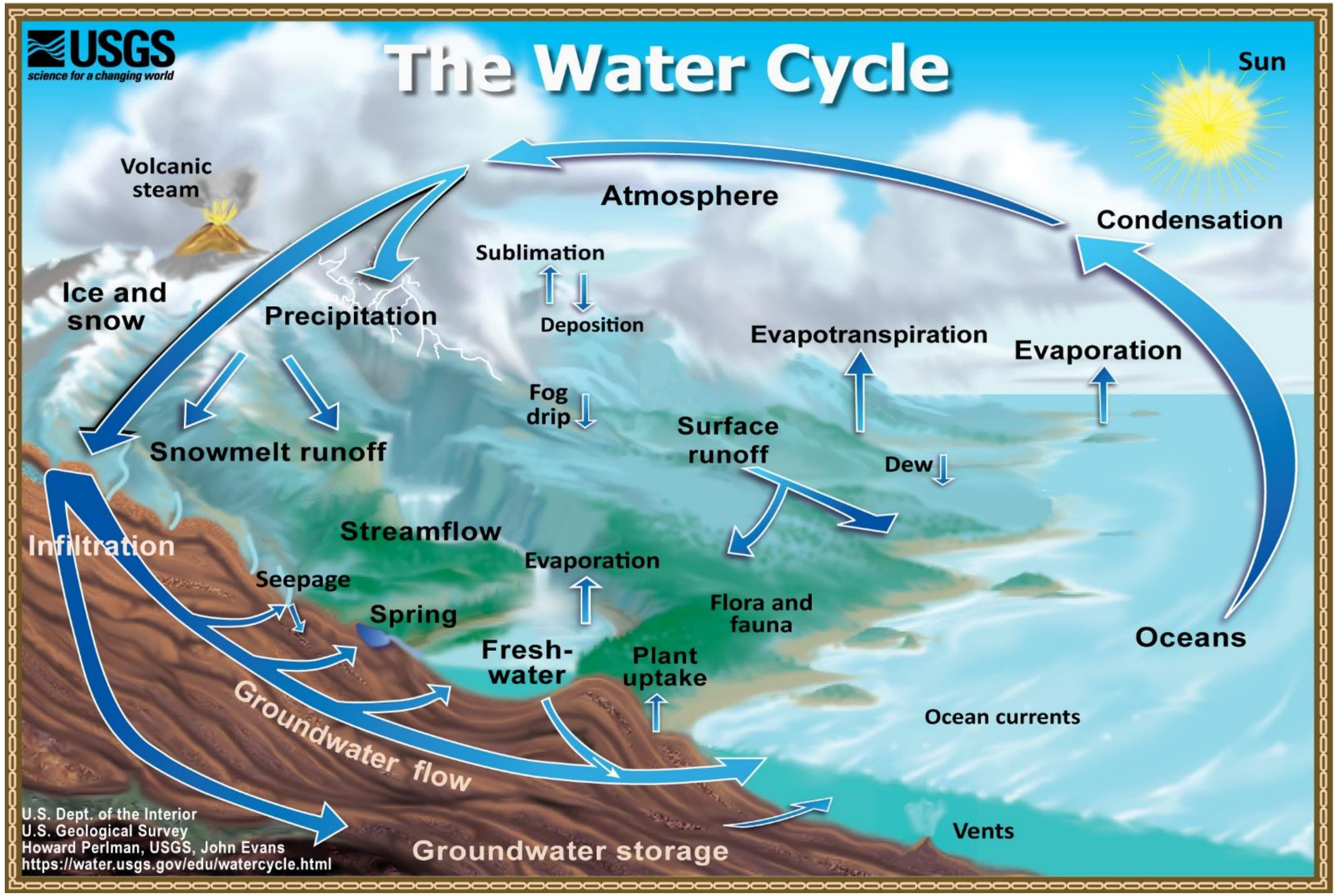



USGS
science for a changing world
The Water Cycle
Sun
Volcanic steam
Atmosphere
Condensation
Sublimation
Deposition
Ice and snow
Precipitation
Evapotranspiration
Evaporation
Fog drip
Surface runoff
Snowmelt runoff
Dew
Streamflow
Infiltration
Seepage
Evaporation
Spring
Flora and fauna
Fresh-water
Plant uptake
Oceans
Groundwater flow
Ocean currents
Vents
Groundwater storage
U.S. Dept. of the Interior
U.S. Geological Survey
Howard Perlman, USGS, John Evans
https://water.usgs.gov/edu/watercycle.html

IMPORTANCE OF GROUNDWATER (GW)

1. It provides water for rivers, streams and wetlands
2. Part of the Hydrologic Cycle
3. It provides a water resources for humans, stock and plants (irrigation)
4. 25% of all the fresh water on Earth
5. Important Environmental Issues
6. **Groundwater** is an important source of **drinking water** for **European** countries, with even 75% of **EU** inhabitants depending on **groundwater** for their **water** supply.

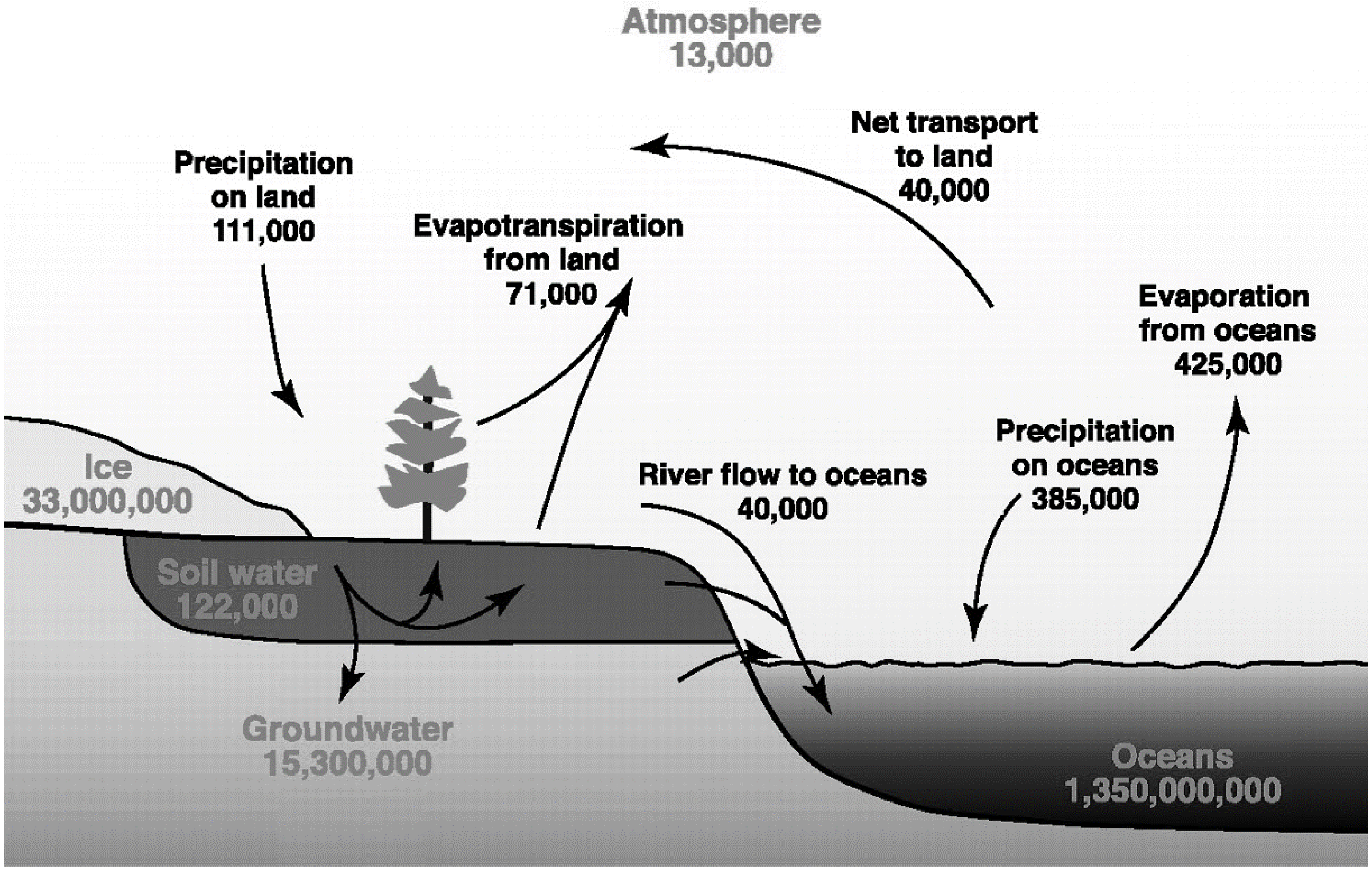
6. It helps maintain lake water levels
7. It can provide a pathway to filter, chemically sequester or remove contaminants (but not always)



Characteristic of GW:
- stable … reliable source of water
- slow … once contaminated, it is very difficult to clean up

World: Groundwater (GW) represents 97 % of all unfrozen fresh water

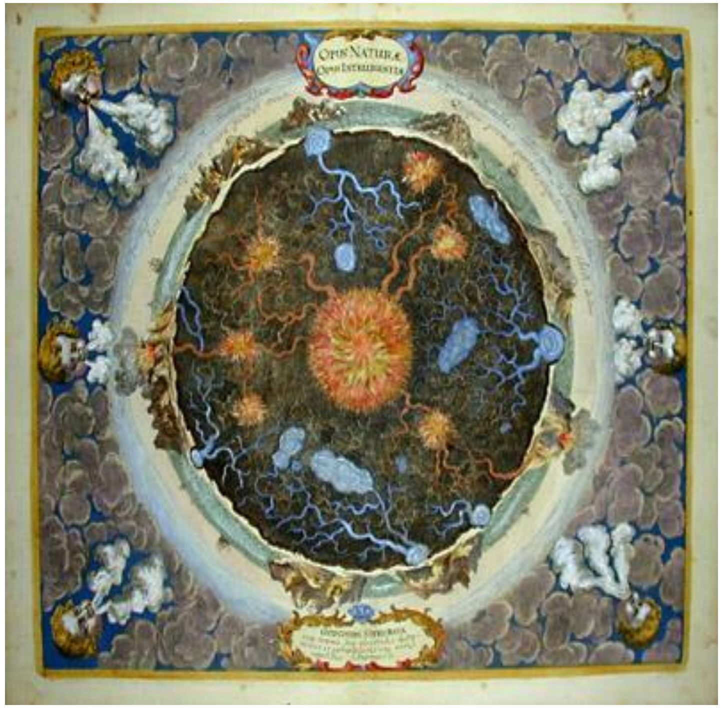
THE GLOBAL WATER CYCLE - GLOBAL WATER VOLUMES & FLUXES (KM^3)



HISTORY

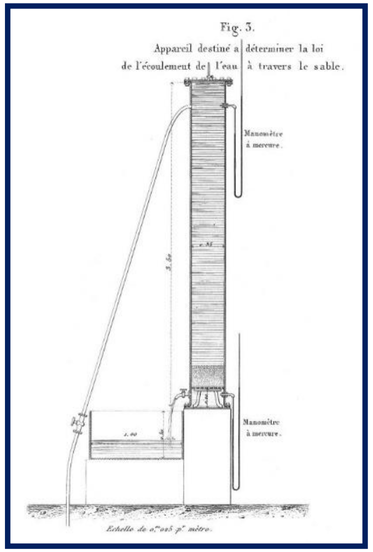
OLD THEORIES

- Homer (~1000 BC):
  - “from whom all rivers are and the entire sea and all springs and all deep wells have their waters”
- Seneca (3 BC -65 AD)
  - “You may be quite sure that it not mere rainwater that is carried down into our greatest rivers.”
- Da Vinci (1452-1519)
  - accurate representation of the hydrologic cycle
- Kircher (1615-1680):
  - Water from the ocean is vaporized by the hot earth, rises, and condenses inside mountains.

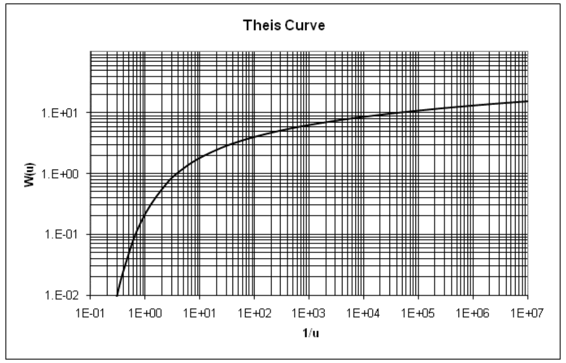


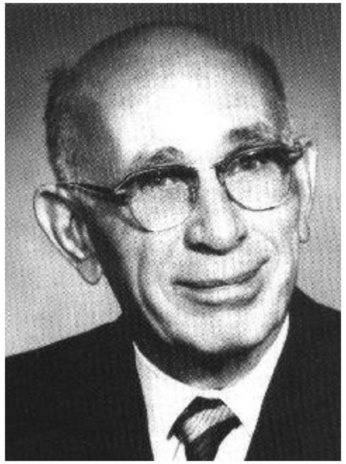
- **PERCOLATION THEORY**
  - **Vitruvius** (~80-20 BC) 8th Book on Water and Aqueducts. Rain and snow on land reappears as springs and rivers
  - **Perrault, Mariotte** (1670): Water balance on the Seine. River flow explained by rainfall..

- **MODERN THEORY**
  - **Henri Darcy** (1856): Relationship for the flow through sand filters. Resistance of flow through aquifers. Solution for unsteady flow.
  - **King** (1899): Water table maps, groundwater flow, cross-section
  - **C.V. Theis** (1930s): Well Hydraulics
  - **C. E. Jacob** (1940) Partial differential equation of transient groundwater flow

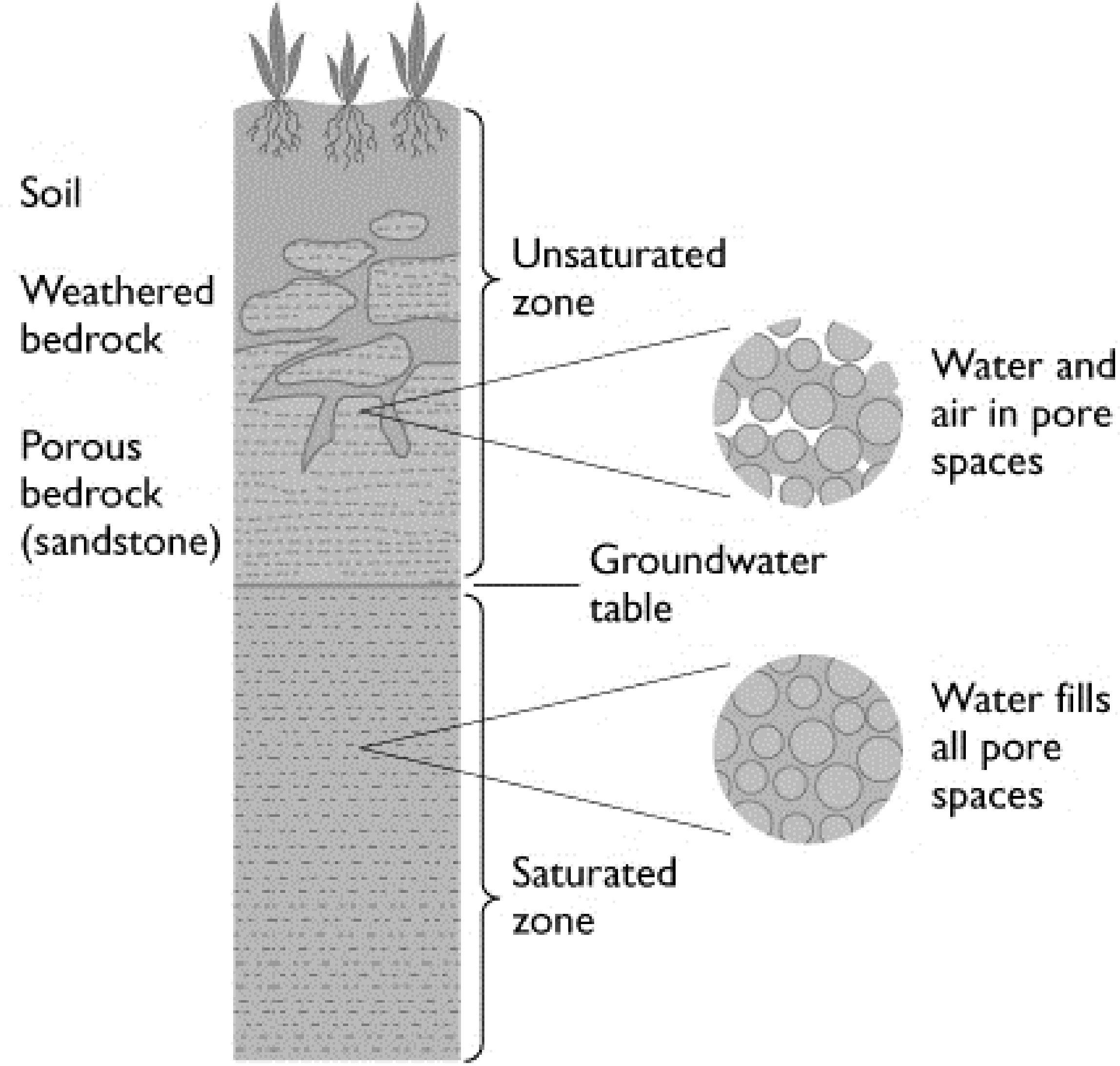


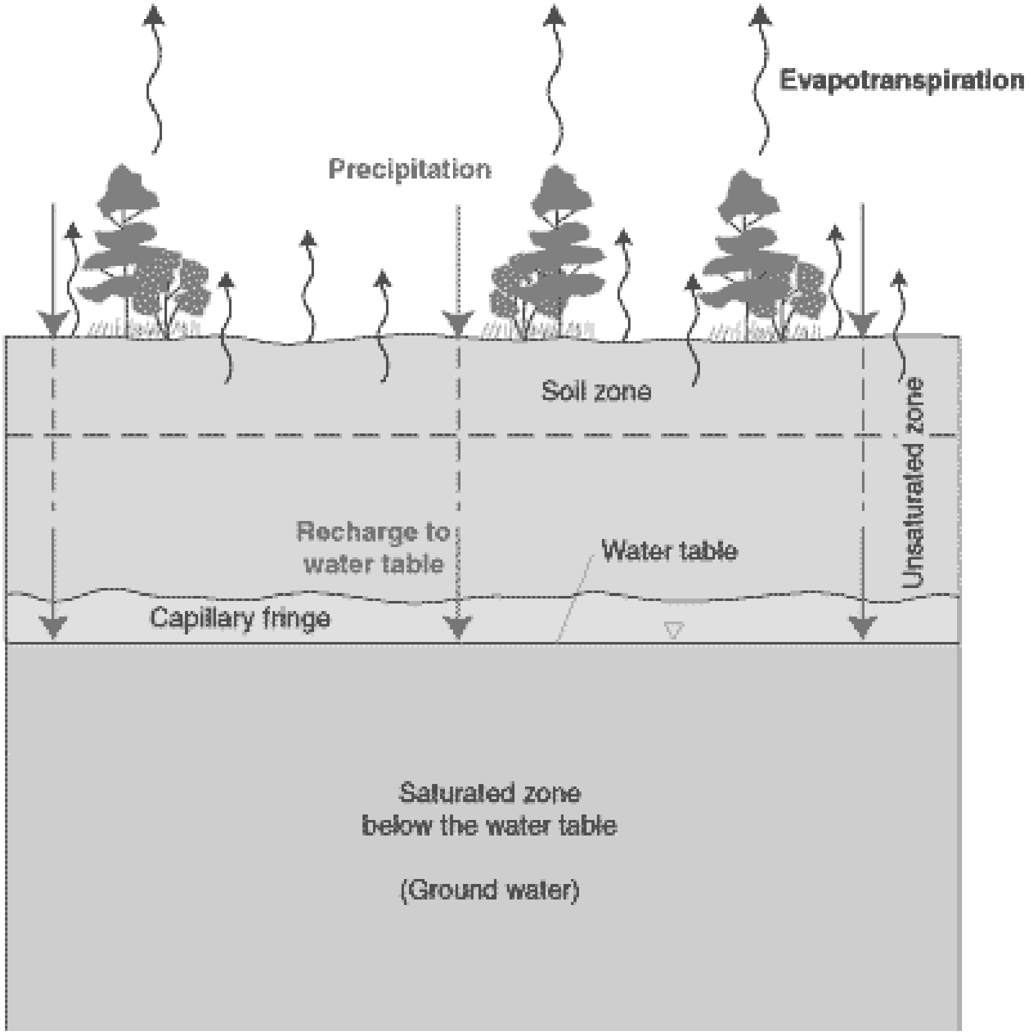

Henri Darcy

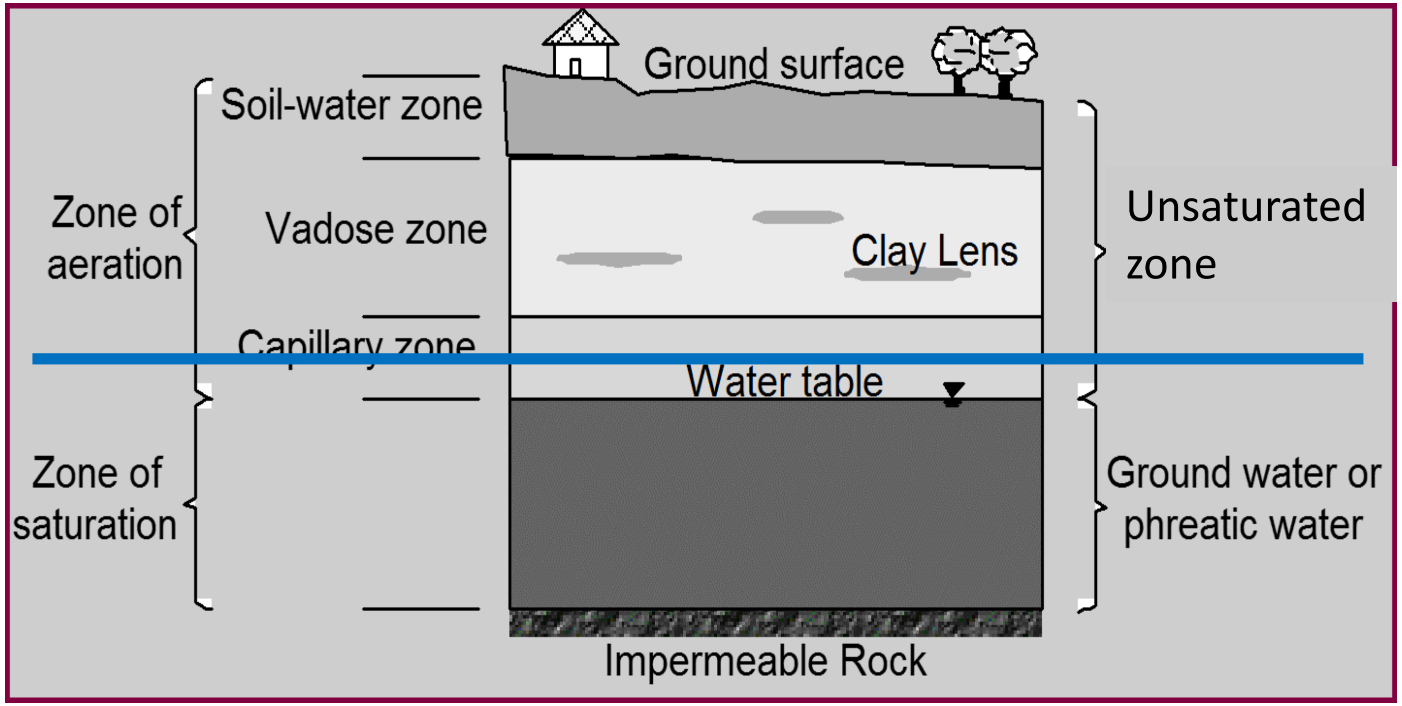



C.V. Theis

VERTICAL DISTRIBUTION OF GROUND WATER







VERTICAL DISTRIBUTION OF GROUND WATER

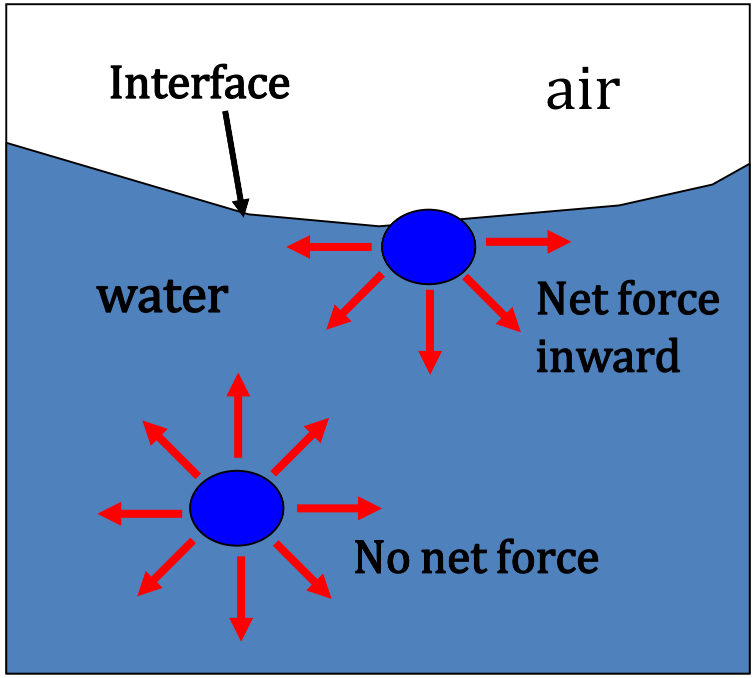
- **Soil water zone**: extends from the ground surface down through the major root zone, varies with soil type and vegetation but is usually a m thickness
- **Vadose zone** (unsaturated zone): extends from the surface to the   water table through the root zone,
- intermediate zone, and the capillary zone
- **Capillary zone**: extends from the water table up to the limit of   capillary rise, which varies inversely with the pore size of the soil and directly with the surface tension

* **Water table**: the level to which water will rise in a well drilled into   the saturated zone

* **Saturated zone**: occurs beneath the water table where porosity is a direct measure of the water contained per unit volume

SURFACE TENSION

- **Below interface**
  - Forces act equally in all directions
- **At interface**
  - Some forces are missing
  - Pulls molecules down and together
  - Like membrane exerting *tension* on the *surface*
- **Curved interface**
  - Higher pressure on concave side
- **Pressure** increase is balanced by surface tension
  - $\sigma = 0.073$ N/m (@ 20°C)
- **Capillary pressure**
  - Relates pressure on both sides of interface



CAPILLARY RISE

Capillary rise is a function of surface tension (σ [FL^{-1}]), fluid specific weight (γ_W [FL^{-3}]) contact angle with the solid surface (θ), and pore diameter (d [L]):

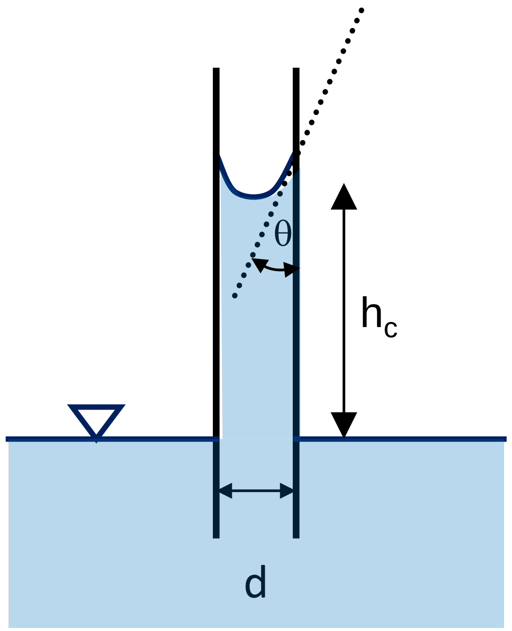
For liquid-vapour interfaces:

$$h_c = 4\sigma \cos(\theta) / \gamma_w d$$

The surface tension of water at 20°C is 7.3×10^{-2} N/m and $\gamma_w = 9.81 \times 10^3$ N/m^3. For water in contact with silicates θ is close to zero so $\cos(\theta)=1$.

$$h_c \approx 3 \times 10^{-5} / d$$

where d is measured in metres.



CAPILLARY RISE OF WATER IN SOILS

Soil Type	Capillary Rise (m)
Clay	>10
Fine Silt	7.5
Coarse Silt	3.0
Very Fine Sand	1.0
Fine Sand	0.50
Medium Sand	0.25
Coarse Sand	0.15
Very Coarse Sand	0.04
Fine Gravel	0.015

$h_c \approx 3 \times 10^{-5} / d$

For $d = 2 \times 10^{-3}$ m

(coarse sand 2 mm)

$h_c \approx 0.015$ m

For $d = 2 \times 10^{-6}$ m

(clay 2 μm)

$h_c \approx 15$ m

TYPICAL AQUIFER SYSTEMS

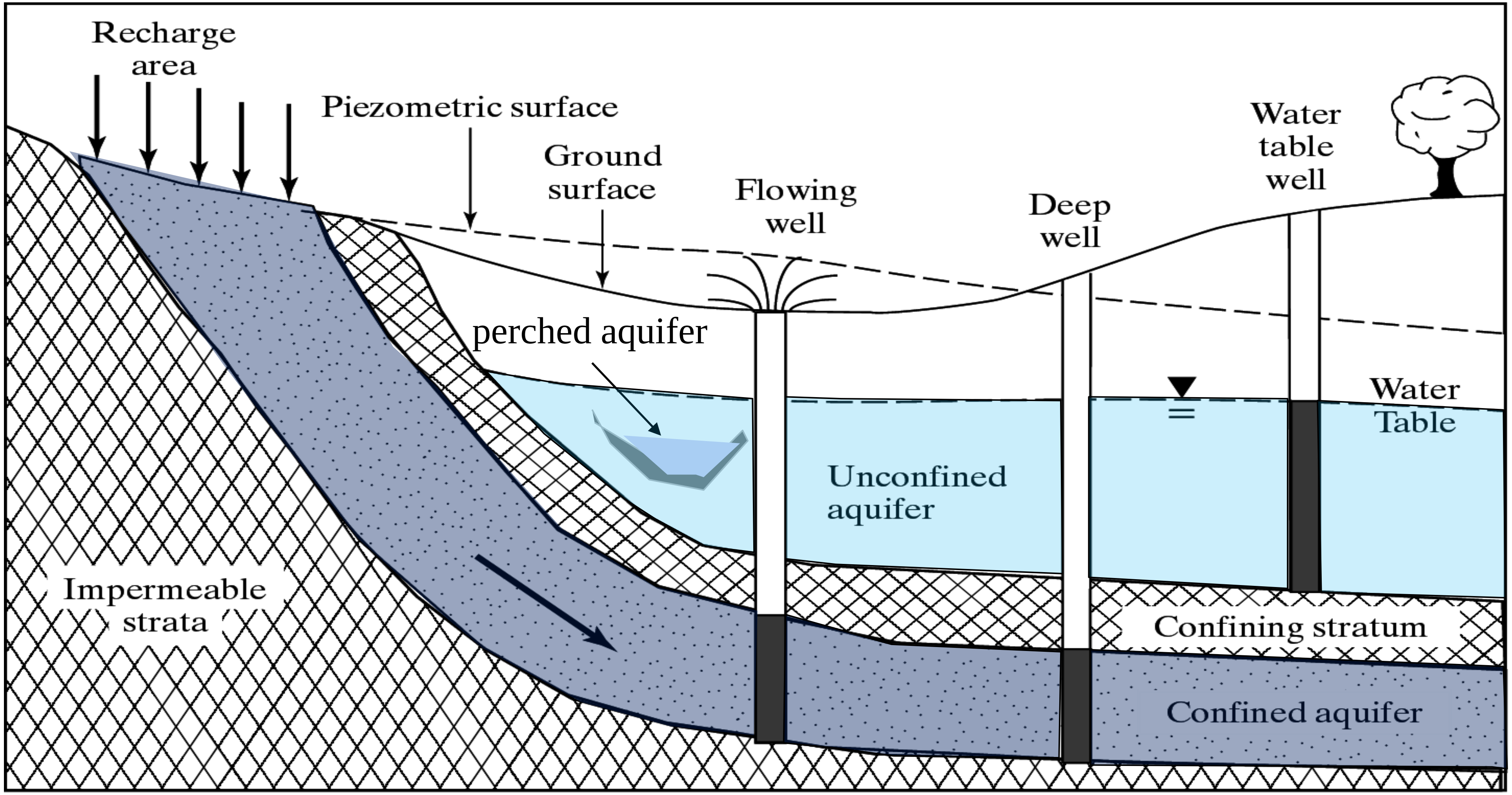
- Aquifer – store and transmit water , unconsolidated deposits – sand, gravel, sandstone etc.
- Unconfined aquifer: an aquifer where the water table exists under atmospheric pressure as defined by levels in shallow wells
- Confined aquifer: an aquifer that is overlain by a relatively impermeable unit such that the aquifer is under pressure and the pressure level rises above the confined unit
- Artesian aquifer: are confined under hydraulic pressure, resulting in free-flowing water, either from a spring or from a well.
- Aquiclude: store , don´t transmit water; clays and less shale, impervious boudaries of aquifer
- Aquitard: transmit don´t store water; shale and less clay; leaky confining layers of aquifers
- Piezometric surface: in a confined aquifer, the hydrostatic pressure level of water in the aquifer, defined by the water level that occurs in a lined penetrating well

Water table: the level to which water will rise in a well drilled into the saturated zone

SPECIAL AQUIFER SYSTEMS

- Leaky confined aquifer: represents a stratum that allows water to flow from above through a leaky confining zone into the underlying aquifer
- Perched aquifer: occurs when an unconfined water zone sits on top of a clay lens, separated from the main aquifer below

CROSS SECTION OF UNCONFINED AND CONFINED AQUIFERS

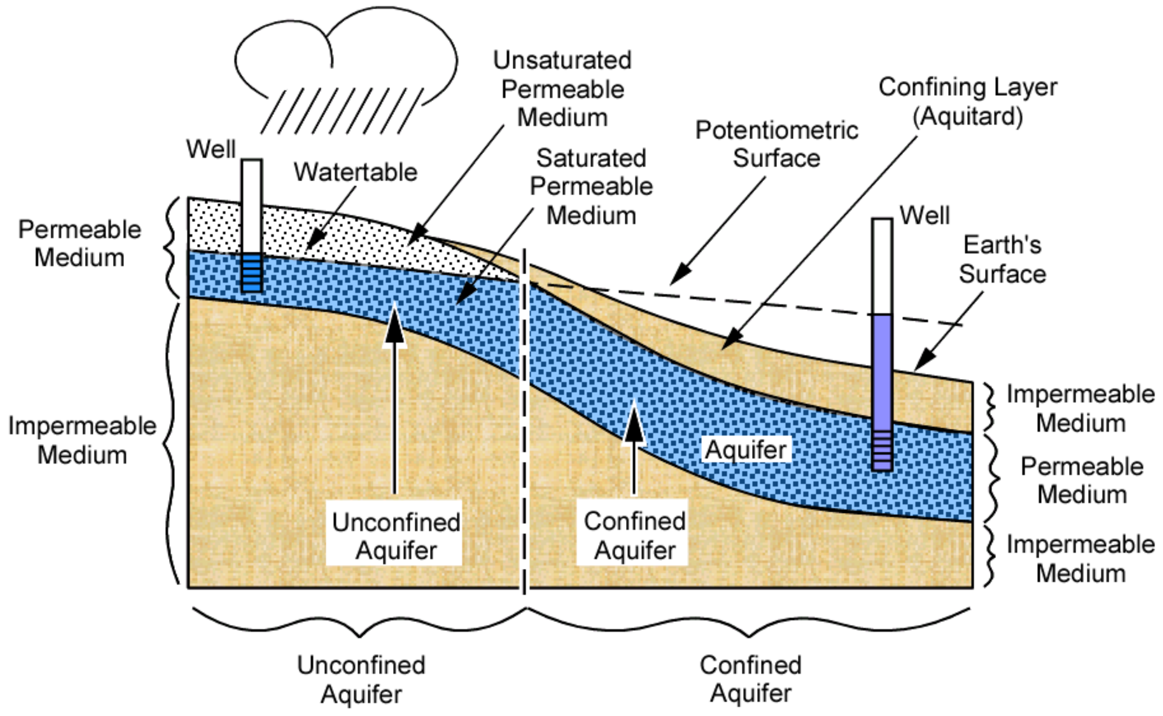


Schematic cross section illustrating unconfined and confined aquifers.

AQUIFER CHARACTERISTICS

1. Matrix type
2. Soil classification
3. Porosity (n)
4. Confined or unconfined
5. Vertical distribution (stratigraphy or layering)

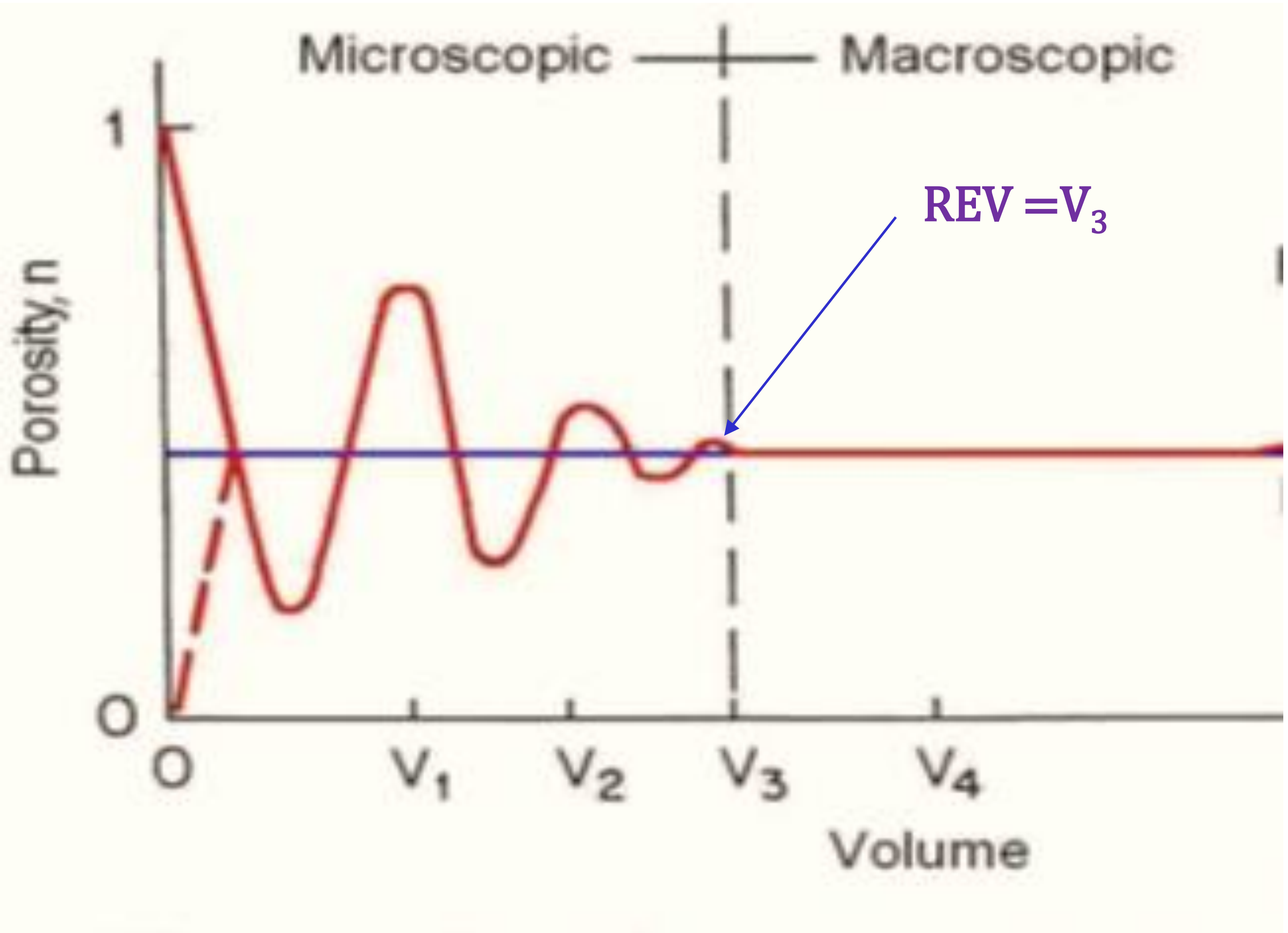
1. Hydraulic conductivity (K)
2. Permeability (k)
3. Transmissivity (T)
4. Storage coefficient or Storativity (S)



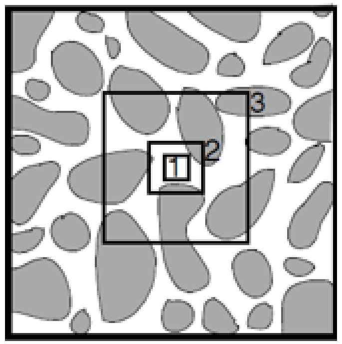
CONTINUUM APPROACH TO POROUS MEDIA - REV

- Pressure, density etc. apply to fluid elements that are large relative to molecular dimensions, but small relative to the size of the flow problem
- We adopt a Representative Elementary Volume (REV) approach
- **REV** must be large enough to contain enough pores to define the average value of the variable in the fluid phase and to ensure that the pore-to-pore fluctuations are smoothed out
- **REV** must be small enough that larger scale heterogeneities do not get averaged out (layering, etc.)

REV



Microscopic
Macroscopic
1
Porosity, n
0
O
V_1
V_2
V_3
V_4
Volume
REV =V_3

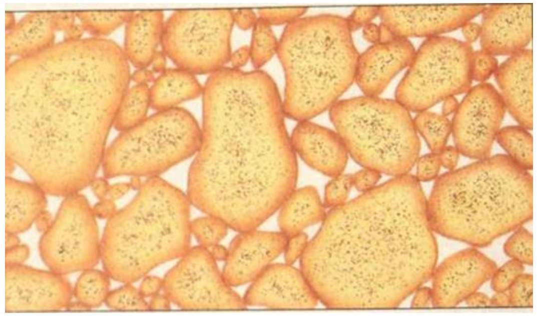


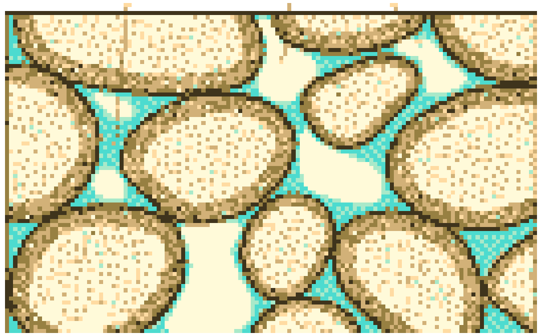
3
2
1

AQUIFER CHARACTERISTICS

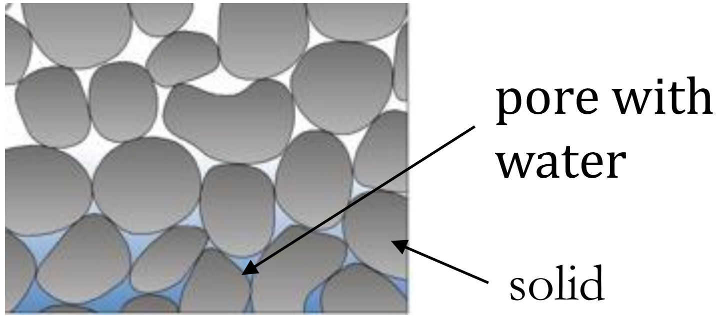
SOIL CLASSIFICATION BASED ON PARTICLE SIZE

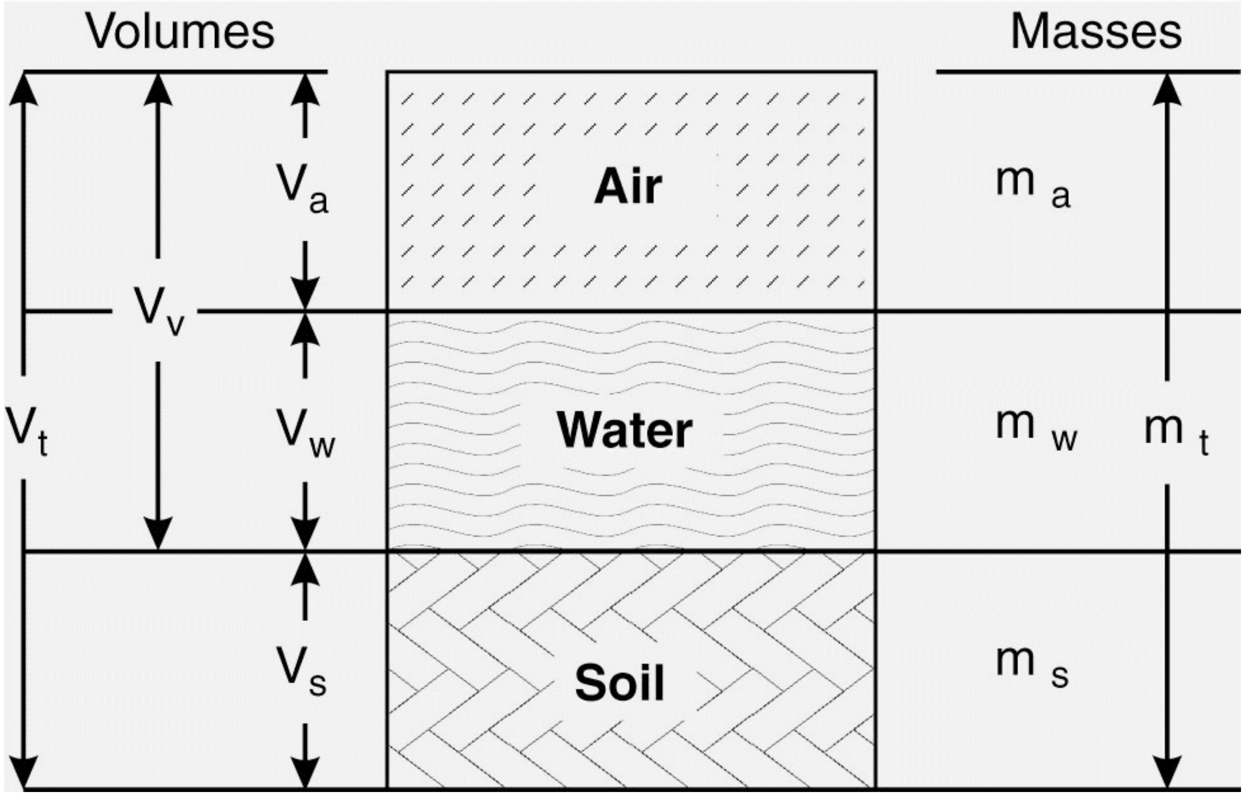
Material	Particle Size, mm
Clay	<0.004
Silt	0.004 - 0.062
Very fine sand	0.062 - 0.125
Fine sand	0.125 - 0.25
Medium sand	0.25 - 0.5
Coarse sand	0.5 - 1.0

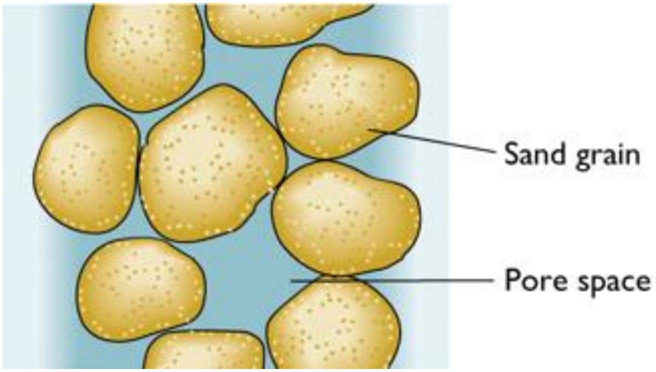


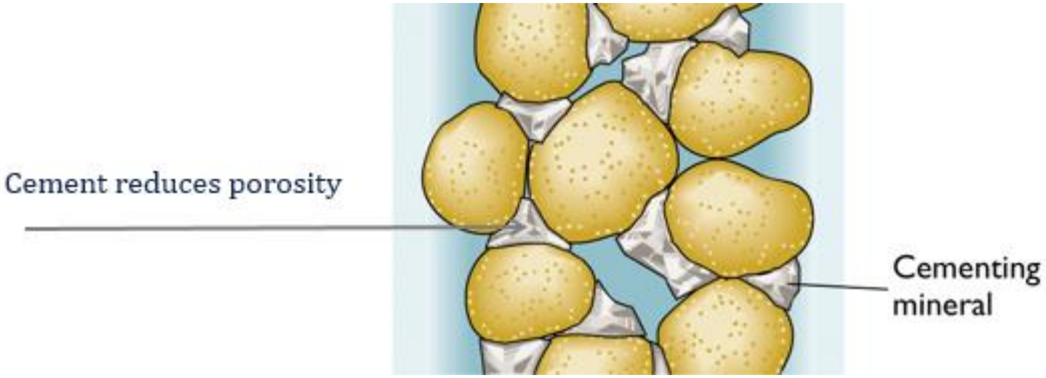


AQUIFER CHARACTERISTICS

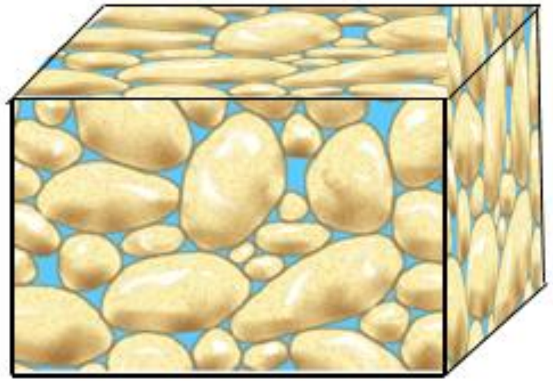








POROSITY

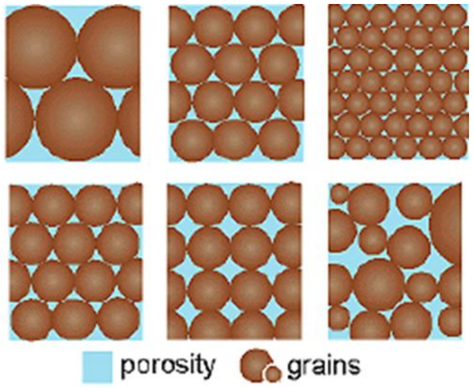


- Porosity - n

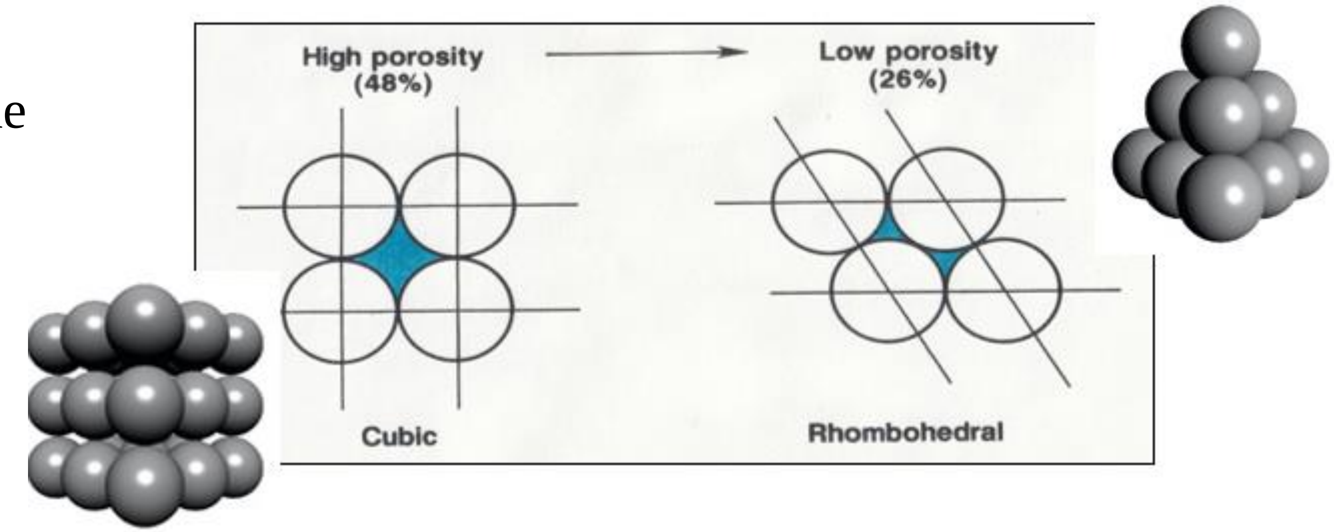
$$n = \frac{V_{pores}}{V_{total}}$$

$$n = \frac{V_{total} - V_{solids}}{V_{total}}$$

$$n_{ef} = \frac{V_{ef\,.pores}}{V_{total}}$$



- Volume of pores is also the **total volume – the solids volume**
- **porosity** is a measure of the capacity of the medium to hold water
- a volume V_T of soil of rock is divide up into the volume of voids V_v and volume of solids V_s
- n_{ef} – effective pores
- V_{ef} – volume of effective pores



RANGE OF POROSITY VALUES FOR MATERIALS

Material	Porosity (%)
Clay	40 – 70
Silt	35 – 50
Fine Sand	40 – 50
Medium Sand	35 – 40
Coarse Sand	25 – 40
Gravel	20 – 40
Sand and Gravel mix	10 – 30
Limestone	0 – 50
Sandstone	5 – 30
Shale	0 – 10
Crystalline Rock	0 – 10

Porosity & Effective Porosity Ranges

Material	Porosity (%)	Eff. Porosity (%)
Silt	34 - 61	0.1 – 10
Clay	34 - 60	0.1 – 10
Sand/Gravel	24 – 55	10 - 55
Limestone/dolomite	5 - 15	0.1 – 5
Shale	1 - 10	0.5 – 5
Sandstone	5 - 15	0.5 – 10

PERMEABILITY, k_p

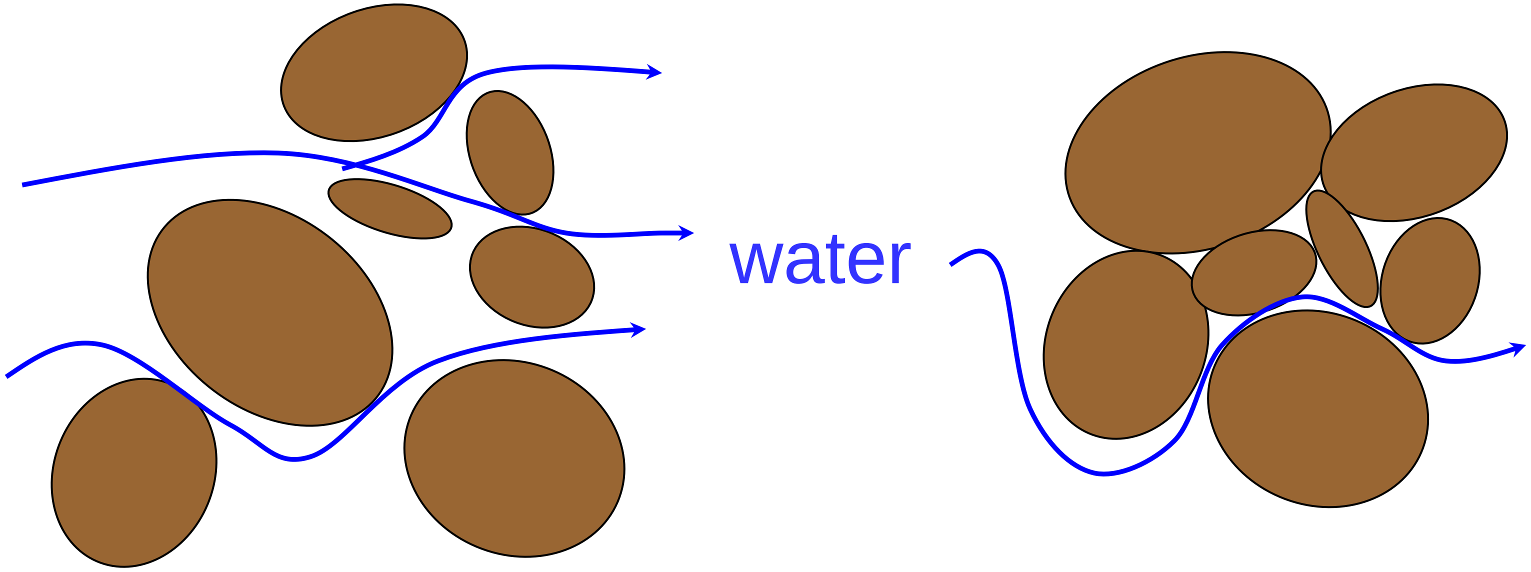
- Permeability - the capacity of a rock to transmit fluid through pores and fractures
  - Interconnectedness of pore spaces
  - Most sandstones and conglomerates are porous and permeable

- with dimensions of area $[L^2]$, depends only on the properties of the porous medium: (How well the pores are connected, and how straight a path a fluid follows - is a property of the rock)
- $\boxed{k_p = Cd^2}$

$$\boxed{k_p = \frac{n_{ef} . D^2}{32}}$$

where C is a dimensionless constant (sometimes called tortuosity) and d is a characteristic pore diameter with dimensions of length, n_{ef} – effective porosity

PERMEABILITY, k_p



<u>Loose soil</u>

- easy to flow

- **high** permeability

<u>Dense soil</u>

- difficult to flow

- **low** permeability

HYDRAULIC CONDUCTIVITY, K

- The hydraulic conductivity K is a measure of how easy the water can flow through the soil.

- The hydraulic conductivity is expressed in the units of velocity (such as cm/sec and m/sec).

- **Hydraulic conductivity** of soils **depends on** several factors:
  - Fluid viscosity (μ): as the viscosity increases, the hydraulic conductivity decreases
  - Pore size distribution
  - Temperature
  - Grain size distribution
  - Degree of soil saturation

HYDRAULIC CONDUCTIVITY

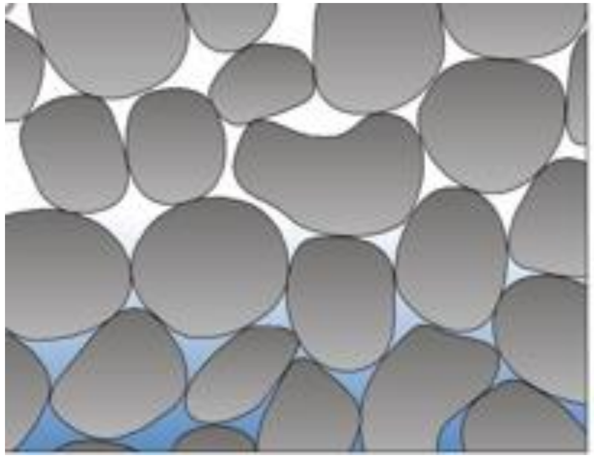
- Specific discharge (q) per unit hydraulic gradient
- Ease with which fluid it transorted through porous medium
- Depends on both matrix and fluid properties
  - Fluid properties:
    - Density ρ, and
    - Dynamic viscosity μ
    - Gravitional constant -g
  - Matrix properties
    - Pore size distribution
    - Pore shape
    - Tortuosity
    - Specific surface area
    - Porosity

$$\boxed{K = k_p \frac{\rho g}{\mu}}$$

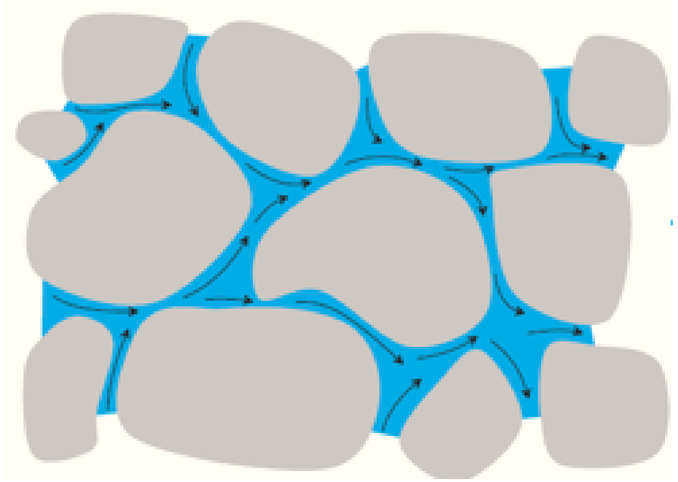
Range of values of K

Medium	K in m/s
Gravel	10^{-3} to 1
Sand	$3X10^{-6}$ to 10^{-2}
Typical BC Forest soil	10^{-7} to 10^{-5}
Bog soils	10^{-9} to 10^{-7}
Marine clay	10^{-12} to 10^{-9}
Basal till	10^{-12} to 10^{-10}
Igneous rock, shale	10^{-13} to 10^{-10}
Sandstone	10^{-10} to 10^{-6}

POROSITY VS. PERMEABILITY



VS



Porosity

Ability to hold water

Permeability

Ability to transmit water

Size, shape, interconnectedness

Porosity ≠ Permeability

Some rocks have high porosity, but low permeability!!

TRANSMISSIVITY

Ease with which water moves through an aquifer

(rate at which water is transmitted through a unit width of aquifer under a unit hydraulic gradient)

The product of K and the saturated thickness of the aquifer, b

$$\mathbf{T = K.b}$$

T: Transmissivity, $[L^2/T]$ e.g., m^2/d

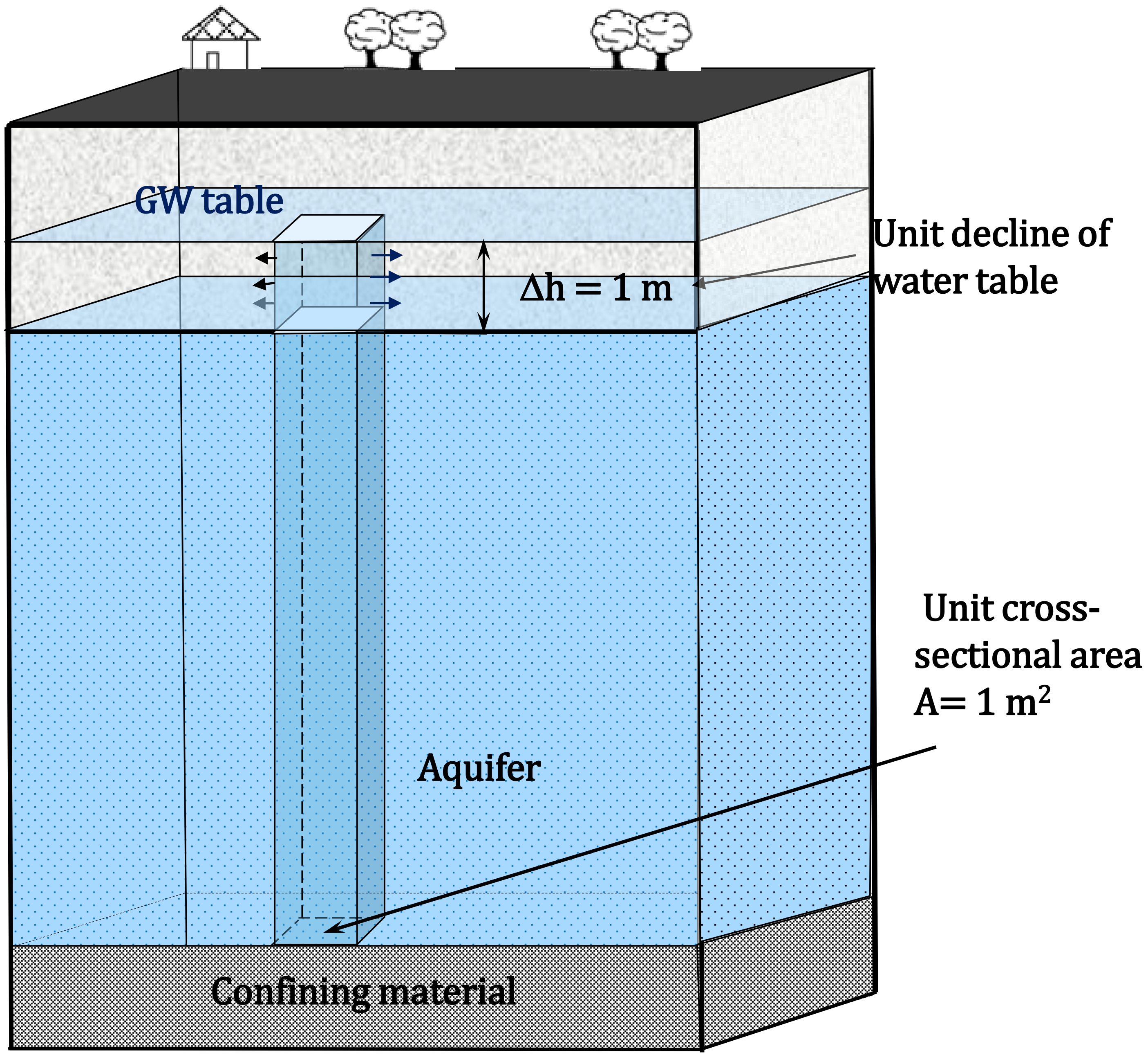
K: Hydraulic conductivity, $[L.T^{-1}]$

b: aquifer thickness, $[L]$

AQUIFER (UNCONFINED) STORAGE

- Storativity (S_y)
- – **Specific yield** ability of an aquifer to store water
- Change in volume of stored water due to change in piezometric head.
- Volume of water released (taken up) from aquifer per unit decline (rise) in piezometric head.

1. In **unconfined aquifer**, main source of water is drainage of water from pores



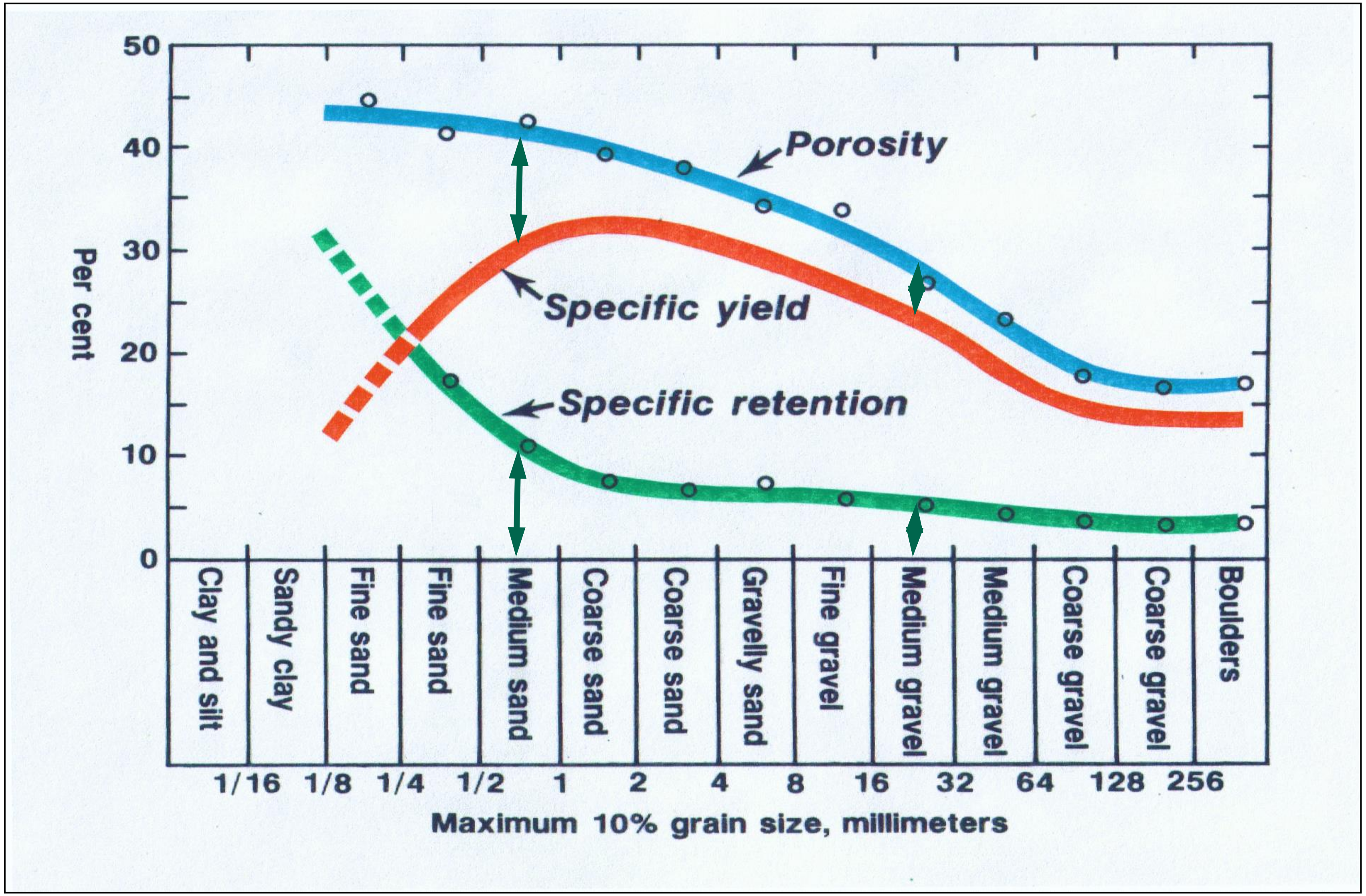
SPECIFIC YIELD AND SPECIFIC RETENTION

- Porosity: maximum amount of water that a rock can contain when saturated.

- Portion of the GW: draining under influence of gravity: **SPECIFIC YIELD - S_y**

- Portion of the GW: retained as a film on rock surfaces and in very small openings: **SPECIFIC RETENTION - S_r**

POROSITY, SPECIFIC YIELD, & SPECIFIC RETENTION



SELECTED VALUES OF POROSITY, SPECIFIC YIELD, AND SPECIFIC RETENTION

[Values in percent by volume]

Material	Porosity	Specific yield	Specific retention
Soil	55	40	15
Clay	50	2	48
Sand	25	22	3
Gravel	20	19	1
Limestone	20	18	2
Sandstone (semiconsolidated)	11	6	5
Granite	.1	.09	.01
Basalt (young)	11	8	3

STORATIVITY (COEFFICIENT OF STORAGE) AND SPECIFIC STORAGE

1. If water is removed from a confined aquifer:
   - Hydraulic head decreases - water level in wells falls
   - Fluid pressure decreases in the aquifer.
   - Porosity decreases as the granular skeleton contracts (aquifer collapses slightly)
   - The volume of water increases

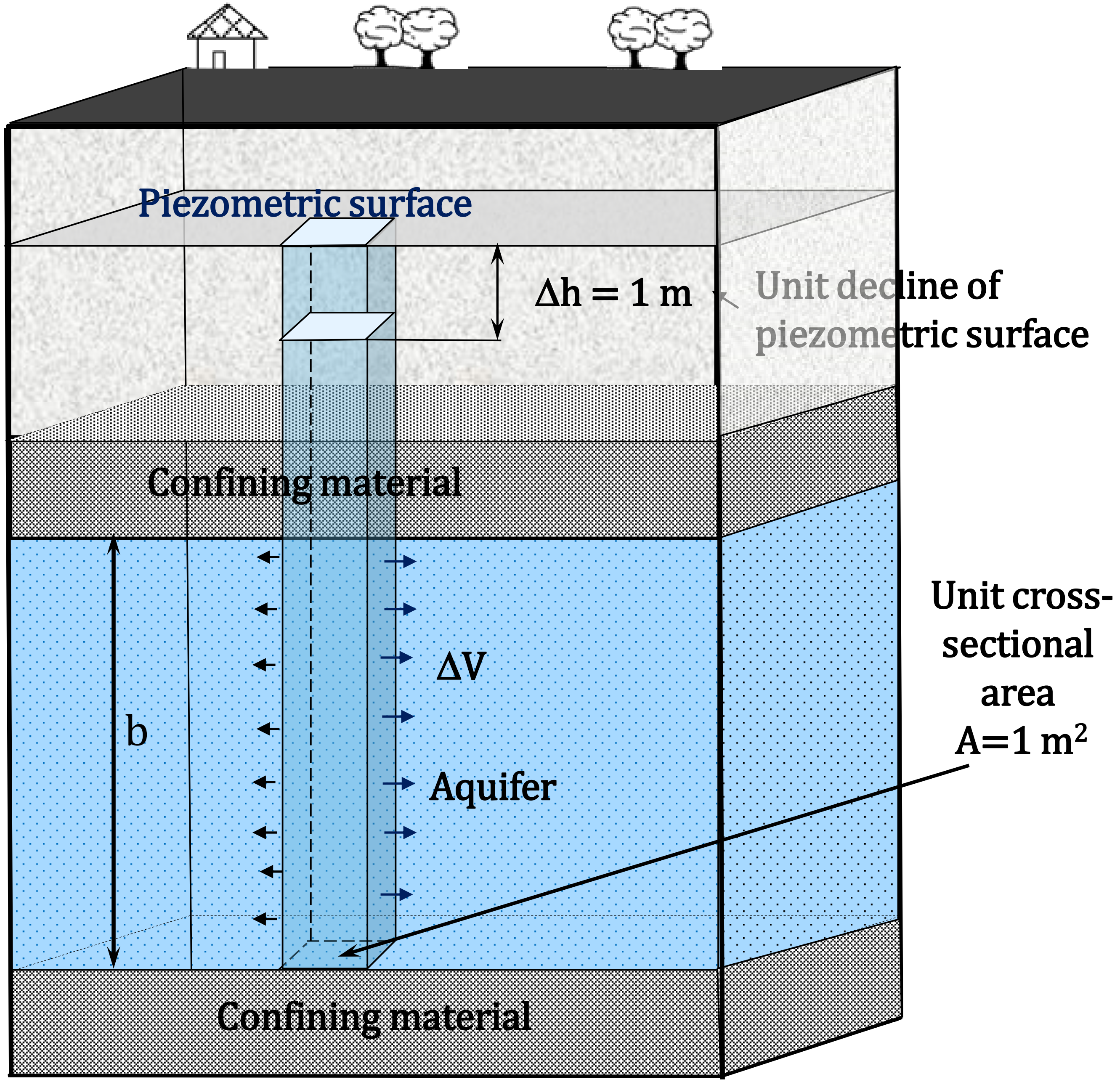
Water is released from storage via:

1. decrease in fluid pressure

2. increase in pressure from overburden

AQUIFER (CONFINED) STORAGE

- Storativity (S) - ability of an aquifer to store water
- Change in volume of stored water due to change in piezometric head.
- Volume of water released (taken up) from aquifer per unit decline (rise) in piezometric head.



- Storativity is a dimensionless property

$S = volume\ of\ water/(unit\ area)\ (unit\ head\ change) = L^3/(L^2 * L) = m^3/m^3$

SPECIFIC STORATIVITY, S_s

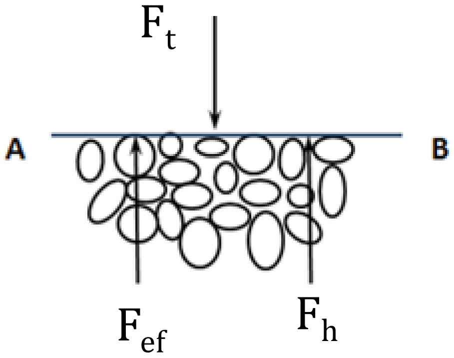
The specific storage of a saturated aquifer is defined as the volume of water released from the storage **per unit volume** of the **aquifet per unit decline in hydraulic head.**

SPECIFIC STORATIVITY, S_s

Karl Von Terzaghi

CONFINED AQUIFER

– Terzaghi, 1925 – effective stress σ_{ef} – the portion of the total stress that is borne by the granular skeleton



Forces above(A) – (B)

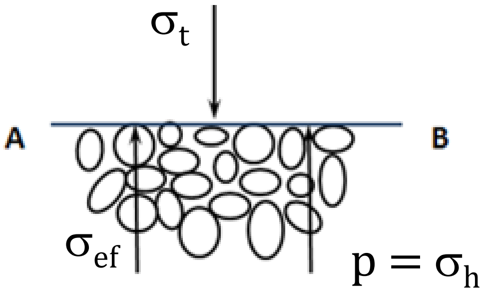
$$F_t = F_{ef} + F_h$$

F_t – Total force above surface (A) – (B)

F_{ef} – effective force(A) – (B)

F_h – hydrostatic force (A) – (B)

Stresses (force/area):



$$\sigma_t = \sigma_{ef} + p$$

In terms of the changes inthese parameters

$$d\sigma_t = d\sigma_{ef} + dp$$

SPECIFIC STORATIVITY, S_s

The change in total stress …… very small ……0

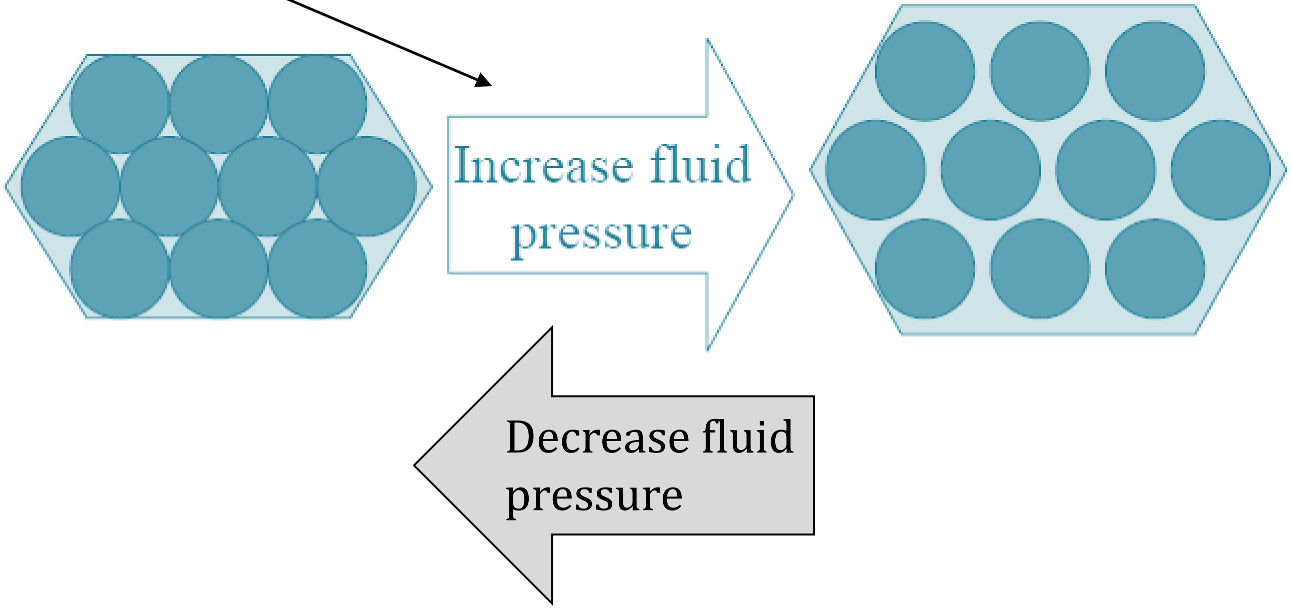
$$d\sigma_t \rightarrow 0$$

$$d\sigma_t = d\sigma_{ef} + dp \quad \rightarrow d\sigma_{ef} + dp = 0$$

Change in hydrostatic pressure ---- change in effective stress:

$$dp = -d\sigma_{ef}$$

an increase in pressure causes the grains to spread apart somewhat



SPECIFIC STORATIVITY, S_s

A. Compressibility of water

$$\beta_w = \frac{-1}{V_w}\frac{dV_w}{dp} \Rightarrow dV_w = -\beta_w . V_w \, dp$$

For saturated aquifer (porosity) $n = \frac{V_w}{V_t} \Rightarrow V_w = nV_t$

$$p = \rho \, g \, h \quad \dots . \; dp = \rho \, g \, dh$$

$$dV_w = -\beta_w . V_w \, dp = -\beta_w \, (n \, V_t)(\rho \, g \, dh)$$

$$dV_w = \beta_w \, n \, \rho \, g$$

For specific storage and unit decline **$V_T = 1$** a **$dh = -1$**

$$dV_w = -\beta_w \, (n \, V_t)(\rho \, g \, dh) = \beta_w \, n \, \rho \, g$$

Then $$dV_w = \beta_w \, n \, \rho \, g$$

dV_w = volume of water produced by the expansion of water caused by decreasing hydrostatic pressure p

SPECIFIC STORATIVITY, S_s

B. Compressibility of aquifer

$$\alpha = -\frac{1}{V_t}\frac{dV_t}{d\sigma_{ef}} \Rightarrow -dV_t = \alpha\, V_t\, d\sigma_{ef}$$

$$V_t = V_w + V_s$$

$$dV_t = dV_w + dV_s$$

$$dV_s \to 0$$

Solid part of porous media

$$dV_t = dV_w$$ for confined aquifer

$$dV_w = -dV_t$$

The negative sign is added since the volumetric reduction dV_t is negative, but the amount of water produced dV_w is possitive

$$dV_w = \alpha\, V_t\, d\sigma_{ef}$$

$$d\sigma_{ef} = -\rho\, g\, dh$$

$$dV_w = \alpha\, V_t\, d\sigma_{ef} = -\alpha\, V_t\, \rho g\, dh$$

Unit volume of aquifer **$V_T = 1$**

Unit decline in hydraulic head **dh = -1**

$$dV_w = \alpha\, \rho\, g$$

SPECIFIC STORATIVITY, S_s

$dV_w = \alpha \rho g$ $\qquad$ $dV_w = \beta_w n \rho g$

The water released from the storage due to a decrease in h is produced by the two mechanism 1) expansion of the water caused by decreasing **p**
2) compaction of the aquifer caused by increasing $\boldsymbol{\sigma_{ef}}$

$$S_s = \alpha \rho g + \beta_w n \rho g$$

$$S_s = \rho g (\alpha + n\beta_w) \qquad (L^{-1})$$

where α - coef. of compressibilty of aquifer,
β_w - coef. of compressibilty of water
n - porosity
ρ – density of water
g – gravity acceleration

And storativity :

$$S = S_S \,.\, b$$

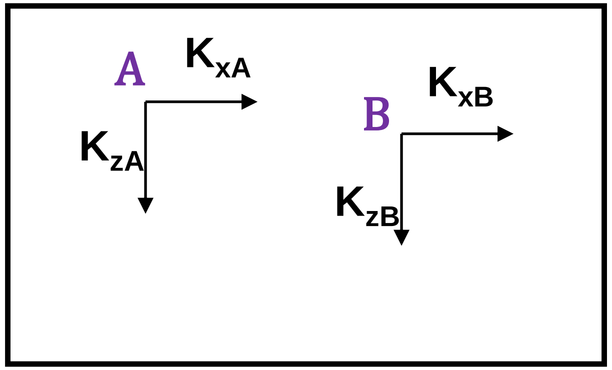
b – aquifer thickness

HOMOGENITY AND ISOTROPY HETEROGENEITY AND ANISOTROPY

- **Homogeneous** aquifer
  - Properties are the same at every point
- **Heterogeneous** aquifer
  - Properties are different at every point
- **Isotropic** aquifer
  - Properties are same in every direction
- **Anisotropic** aquifer
  - Properties are different in different directions
- Often results from stratification during sedimentation

$$K_{horizontal} > K_{vertical}$$

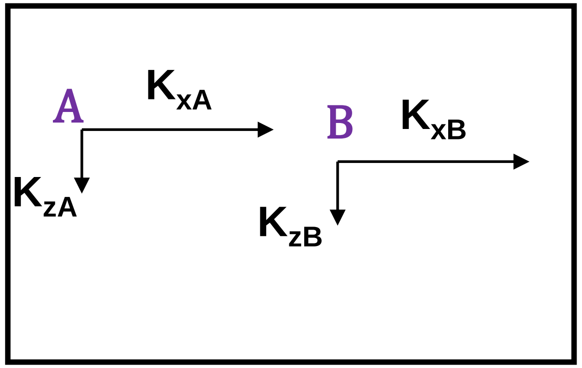
K(x,y) IN TWO DIMENSIONS



1. Homogeneous, isotropic

$K_{xA} = K_{xB}$ $K_{xA} = K_{zA}$

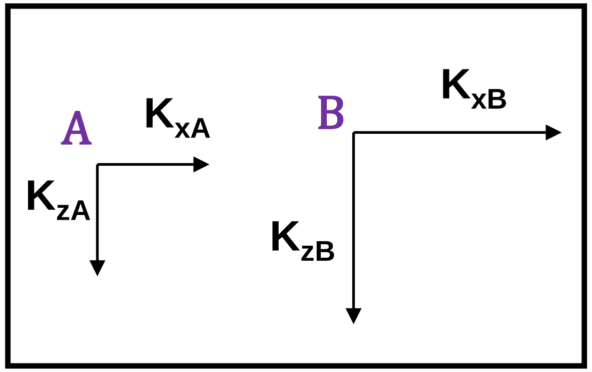
$K_{zA} = K_{zB}$ $K_{xB} = K_{zB}$



2. Homogeneous, anisotropic

$K_{xA} = K_{xB}$ $K_{xA} \neq K_{zA}$

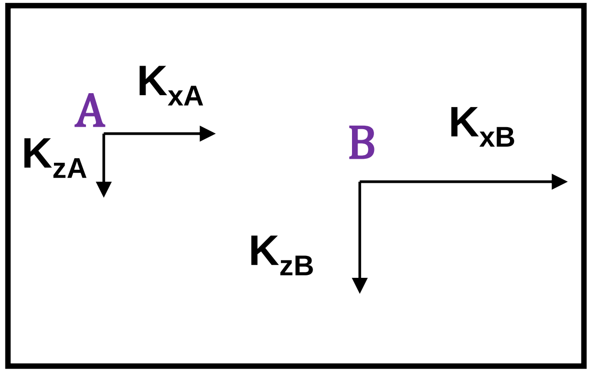
$K_{zA} = K_{zB}$ $K_{xB} \neq K_{zB}$



3. Heterogeneous, isotropic

$K_{xA} \neq K_{xB}$ $K_{xA} = K_{zA}$

$K_{zA} \neq K_{zB}$ $K_{xB} = K_{zB}$



4. Heterogeneous, anisotropic

$K_{xA} \neq K_{xB}$ $K_{xA} \neq K_{zA}$

$K_{zA} \neq K_{zB}$ $K_{xB} \neq K_{zB}$

HYDRODYNAMICS – GROUNDWATER

HYDRAULIC HEAD

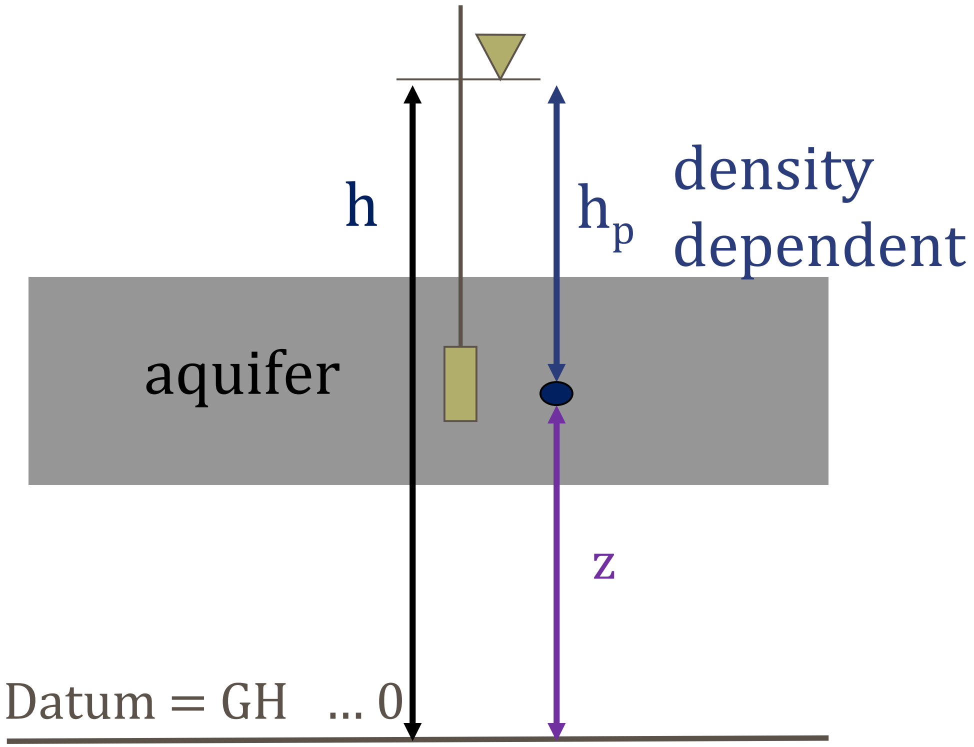
total head = elevation head + pressure head

(pressure head: varying density fluids – important in contamination or salinity)

HYDRAULIC HEAD

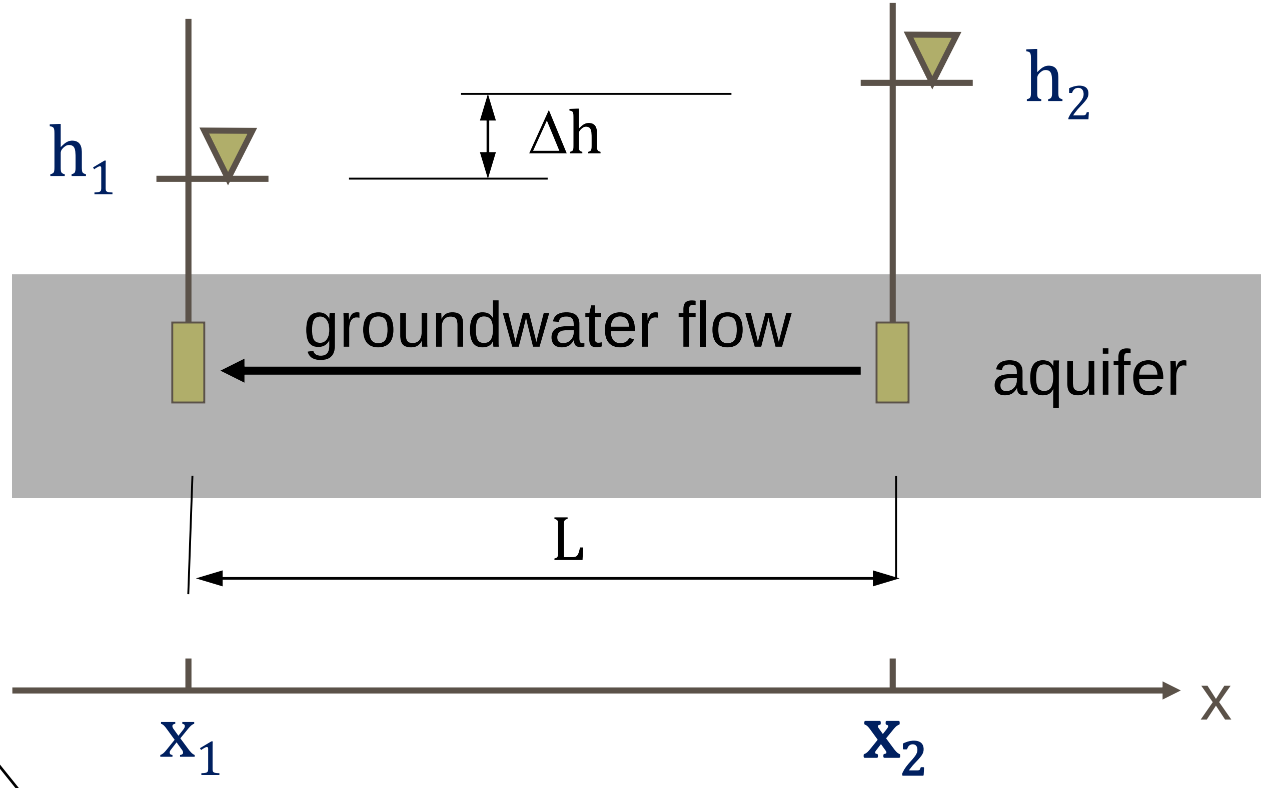
$$\boldsymbol{h} = z + h_p$$

$$h = z + \frac{p}{\gamma} = \boxed{z + \frac{p}{\rho g}}$$



HYDRAULIC GRADIENT

$$I = \frac{h_2 - h_1}{L}$$



$$\nabla h = (h_2 - h_1)/(x_2 - x_1)$$

KINDS AND FORMS OF FLOW (GROUNDWATER)

Unsteady flow ….. $Q = Q(t)$, $v = v(t)$

Steady flow ….. $Q\ (x, y, z) = \text{const.}$

uniform flow … $A = const. \quad v = const.$

non – uniform flow $A \neq const. \quad v \neq const.$

•with **free level** – flow limited by solid walls, free level on surface, motion caused by own weight of liquid

• **pressure** – flow limited by solid walls from all sides, motion caused by difference of pressures (**hydraulic heads**)

Solution of flow:

• space flow (3D numeric models)

• planar flow (2D – simplified solution)

• one dimensional (1D)

Flow regime :

- laminar
- turbulent

HEAD LOSS IN POROUS MEDIA

- Piezometric head $$h_1 = \frac{p_1}{\gamma} + z_1$$
- Energy is lost in the flow through the porous medium due to friction
- Energy equation $$\cancel{\frac{v_1^2}{2g}} + \frac{p_1}{\gamma} + z_1 = \cancel{\frac{v_2^2}{2g}} + \frac{p_2}{\gamma} + z_2 + h_L$$
- Neglect velocity terms $$h_L = \left(\frac{p_1}{\gamma} + z_1\right) - \left(\frac{p_2}{\gamma} + z_2\right) = h_1 - h_2 = \Delta h$$
- Flow is always from higher head to lower head

HEAD LOSS IN POROUS MEDIA

- Piezometric head

- Energy is lost in the flow through the porous medium
  due to friction

- Energy equation

- Neglect velocity terms

- Flow is always from higher head to lower head

TERMS TO REMEMBER

Pressure head: water pressure at a given point, which can be measured by a piezometer
Elevation head: height above GH (z=0)
Total head: the sum of pressure and elevation head
Potential energy: product of the total head and the gravitational constant
Hydraulic gradient: change in the total head per unit distance
Hydraulic conductivity: water flux density per unit volume of water and per unit hydraulic gradient
Macroscopic velocity: the speed of water flow through the cross-sectional area of solid matrix and interstices

LAMINAR AND TURBULENT FLOW

- **laminar – particles of liquid move at parallel paths**
- **turbulent – motion of particles of liquid: irregular and** inordinate, fluctuations of velocity vector in time and space, mixing inside flow
- Criterion – **Reynolds number**

$$\mathbf{Re}_f = \frac{vd_{10}}{\mu}$$

v - velocity

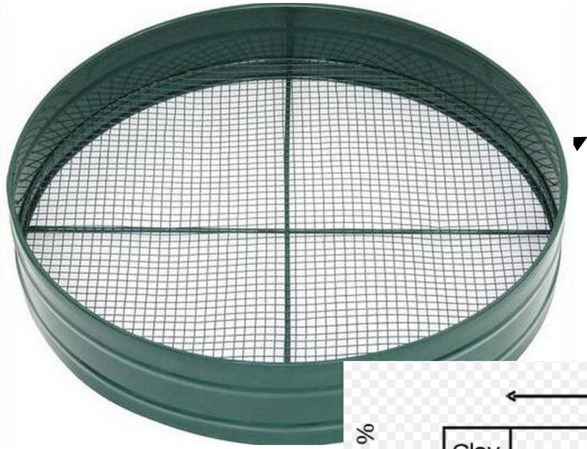
d_{10} = effective grain size diameter

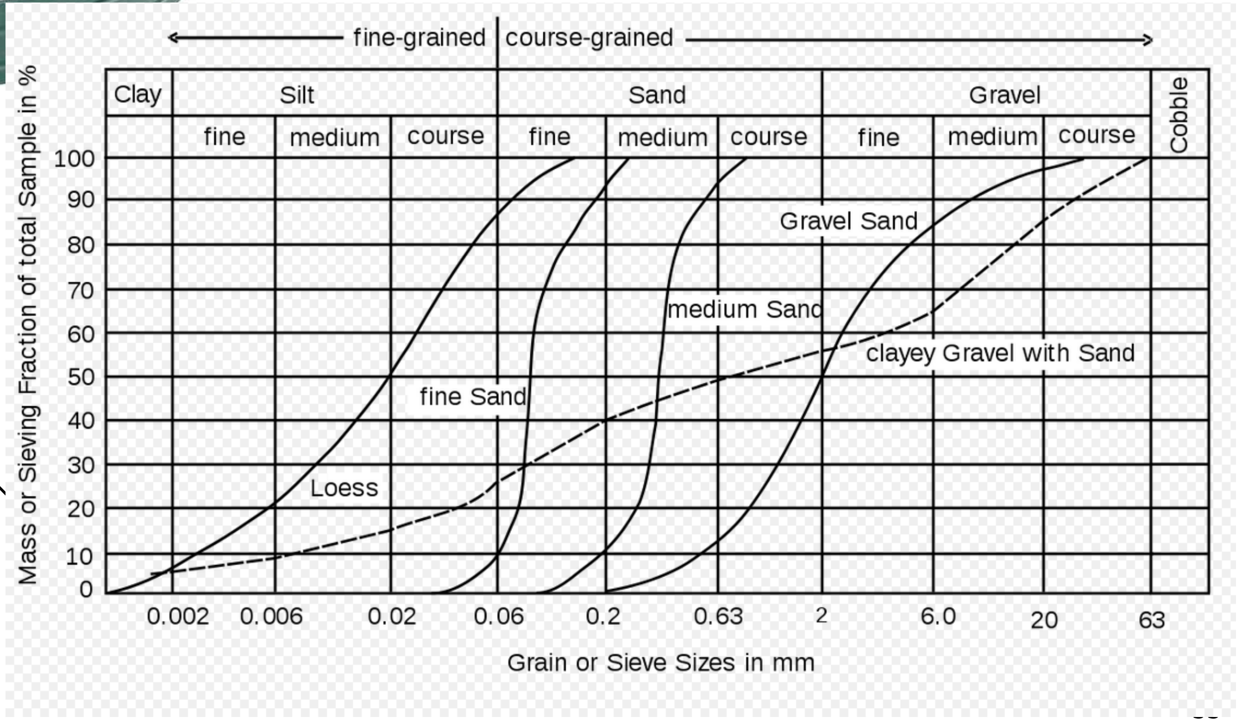
CRITICAL REYNOLDS NUMBER

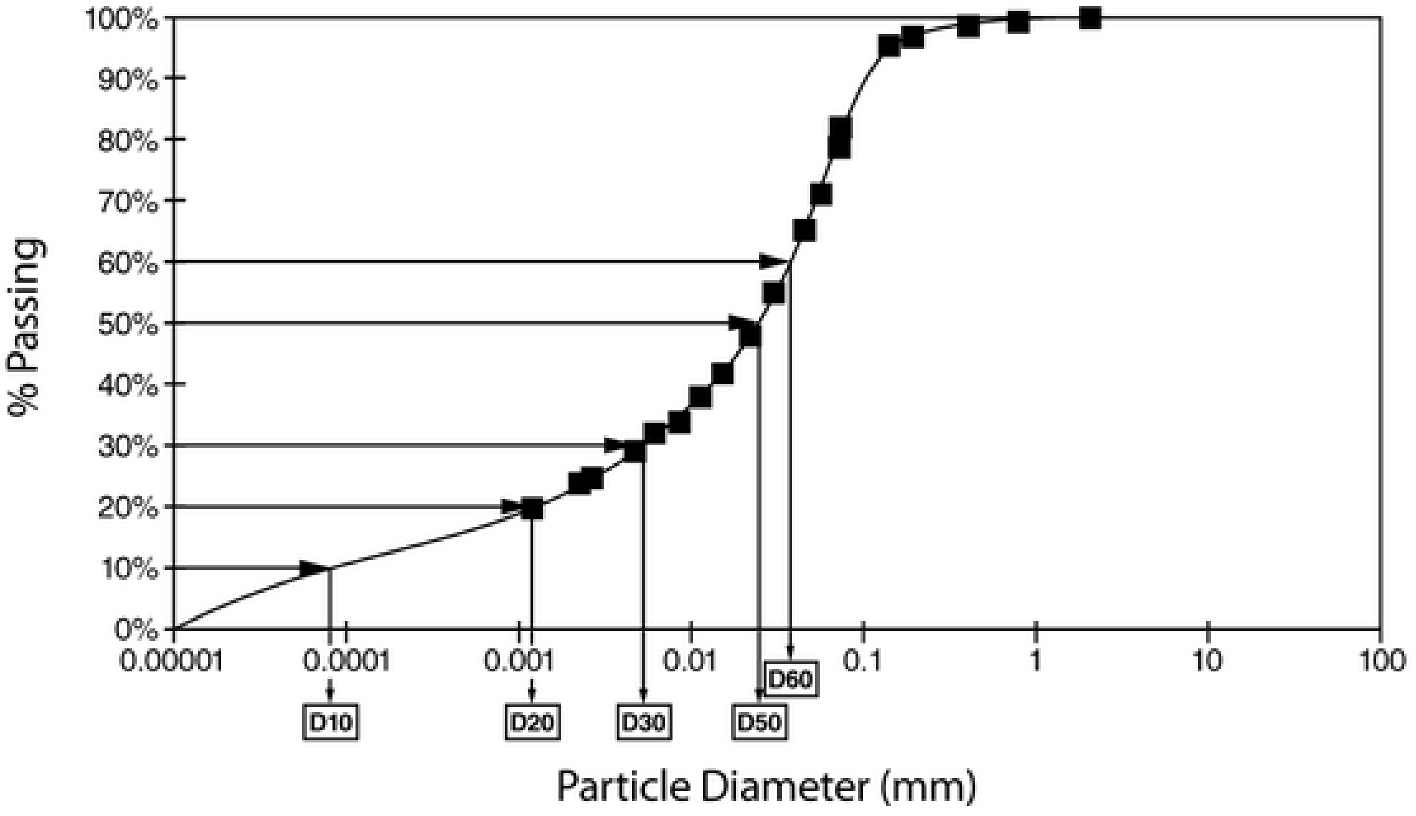
for groundwater flow $\mathrm{Re}_{fcr} = 1$

The *Reynolds number* can be used as a criterion to distinguish between laminar and turbulent flow:

A sieve analysis (or gradation test) is a practice or **procedure used** (commonly used in civil engineering) **to assess the particle size distribution** (also called gradation) of a granular material **by allowing the material to pass through a series of sieves of progressively smaller mesh size** and weighing the amount of material that is stopped by each sieve as a fraction of the whole mass...






100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
% Passing
0.00001
0.0001
0.001
0.01
0.1
1
10
100
D10
D20
D30
D50
D60
Particle Diameter (mm)

DARCY'S LAW

- Water flow through an aquifer.
- Darcy's law (conservation of momentum) was determined experimentally by Darcy, it can be derived from the Navier-Stokes equations
- Analogous to Fourier's law, Ohm's law, or Fick's law
- Darcy's law (conservation of momentum) and the continuity equation (conservation of mass) are used to derive the groundwater flow equation

LINEAR TRANSPORT LAWS

- Fourier’s Law – **Heat** is transferred from a region of higher temperature to a region of lower temperature

Jean B. J. Fourier
1768-1830

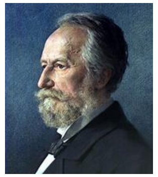
$$Q = -kA\frac{dT}{dx}$$

- Ohm's law – **Electricity** is transferred from a region of higher voltage to a region of lower voltage

Georg Simon Ohm
1789-1854

$$I = -\frac{1}{\rho}A\frac{dV}{dx}$$

- Fick's law – **Mass** is transferred from a region of higher concentration to a region of lower concentration



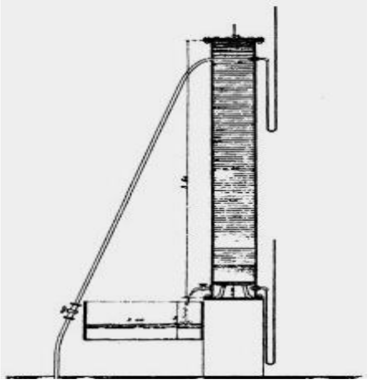
Adolf Eugen Fick
1829-1901

$$J = -DA\frac{dC}{dx}$$

- **Darcy´s law** - ???

Henry Darcy
1803 - 1858

Henry Darcy 1856

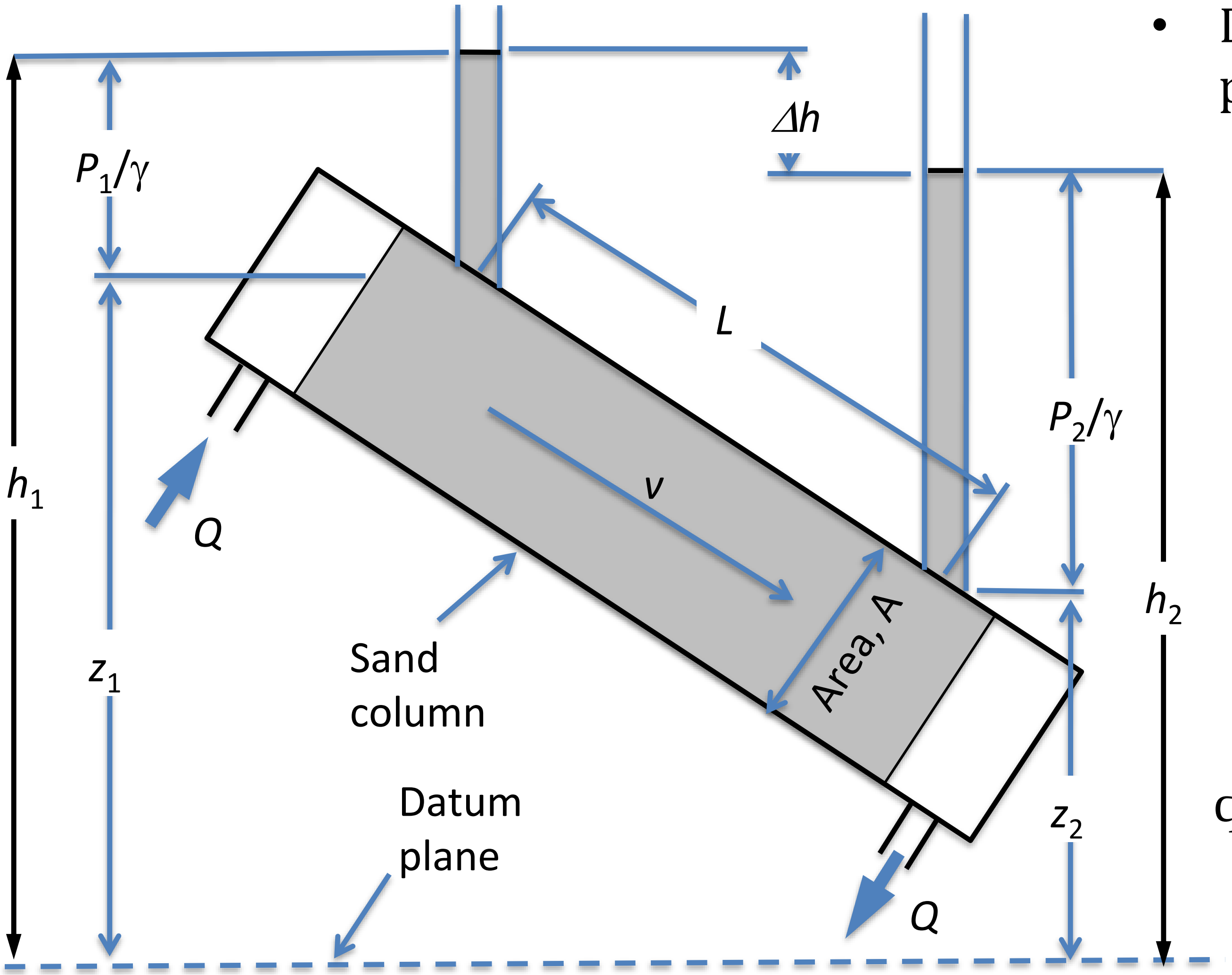


Experimental equipment

Darcy's Experimental Data

NUMÉRO de L'EXPÉRIENCE	DURÉE.	DÉBIT MOYEN par minute.	PRESSION MOYENNE		DIFFÉRENCE des PRESSIONS.	RAPPORT des VOLUMES AUX pressions.	OBSERVATIONS.
			SUR LE FILTRE	SOUS LE FILTRE			
1	2	3	4	5	6	7	8
		l.	m.	m.	m.		
1	15'	18,8	P + 9,48	P − 3,60	13,08	1,44	Fortes oscillations dans le manomètre supérieur.
2	15'	18,3	P + 12,88	P 0	12,88	1,42	*Id.*
3	10'	18,0	P + 9,80	P − 2,78	12,58	1,43	*Id.*
4	10'	17,4	P + 12,87	P + 0,46	12,41	1,40	Faibles.
5	20'	18,1	P + 12,80	P + 0,49	12,33	1,47	Assez faibles.
6	16'	14,9	P + 8,86	P − 0,83	9,69	1,54	Presque nulles.
7	15'	12,1	P + 12,84	P + 4,40	8,44	1,43	Très-fortes.
8	15'	9,8	P + 6,71	P 0	6,71	1,46	Très-faibles.
9	20'	7,9	P + 12,81	P + 7,03	5,78	1,37	Très-fortes.
10	20'	8,63	P + 5,58	P 0	5,58	1,55	Presque nulles.
11	20'	4,5	P + 2,98	P 0	2,98	1,51	*Id.*
12	20'	4,15	P + 12,86	P + 9,88	2,98	1,39	Assez fortes. On a déjà expliqué la cause de ces oscillations.

DARCY'S EXPERIMENT



- Flow through sand filters
- Discharge (Q) proportional to
  - Area, A
  - Head drop, h_1-h_2
  - Inverse of length, L

$$Q \propto K.A \frac{h_1 - h_2}{L}$$

$$\Delta h = h_2 - h_1$$

$$q = v = \frac{Q}{A} = -K\frac{\Delta h}{L}$$

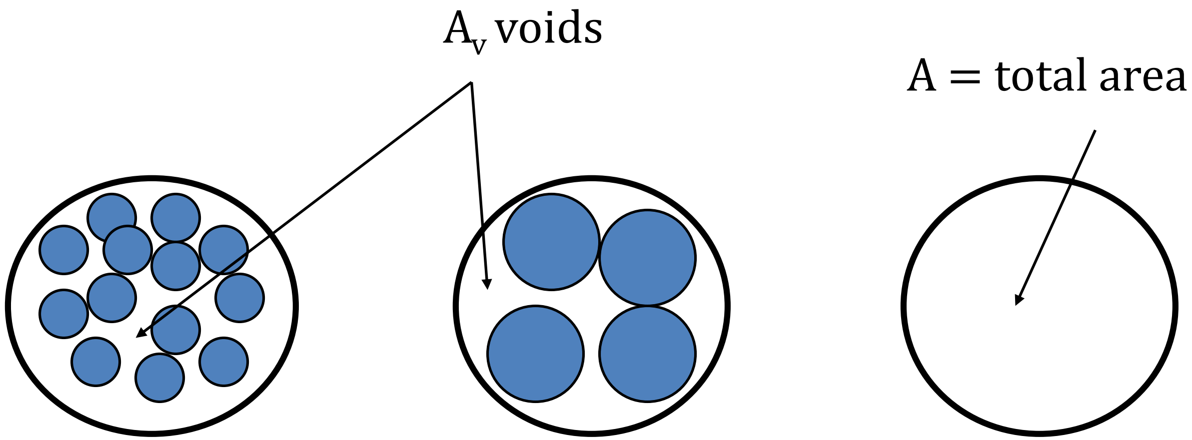
- **GROUNDWATER FLOW**

  - Direction controlled by hydraulic gradient
  - Rate controlled by gradient and hydraulic conductivity

- **Hydraulic gradient** (change in head)
  - flow occurs from high to low head
  - flow is down the hydraulic gradient
  - dh/dz, $\partial h/\partial x$, ∇h etc.

- **DARCY VELOCITY** v_D is a fictitious velocity since it assumes that flow occurs across the entire cross-section of the sediment sample. Flow actually takes place only through interconnected pore channels (voids), at the seepage velocity v_s
- Effective porosity, n_{ef} for

ACTUAL GROUNDWATER VELOCITY (seepage velocity) - v_s



DARCY'S LAW

- $v = K * i$
  - V : Flow velocity

  - K : Hydraulic conductivity

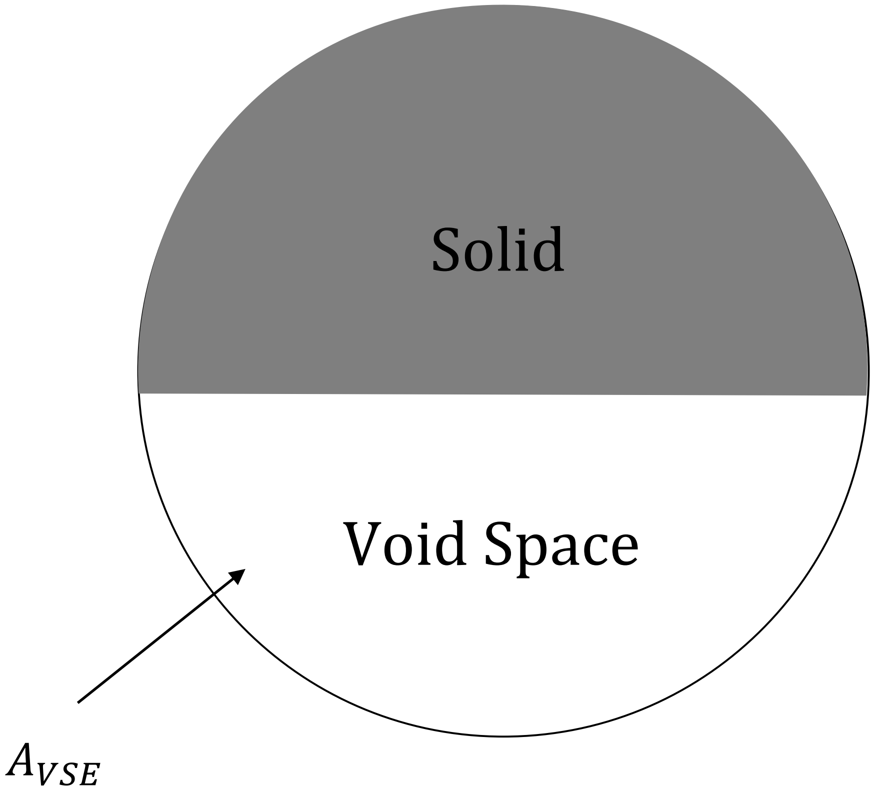
  (The rate at which a soil allows water to move through it)

  - i : Hydraulic gradient; $i = \Delta h / L$

  (Change in hydraulic head per unit of horizontal distance)

VELOCITY THROUGH POROUS MEDIUM

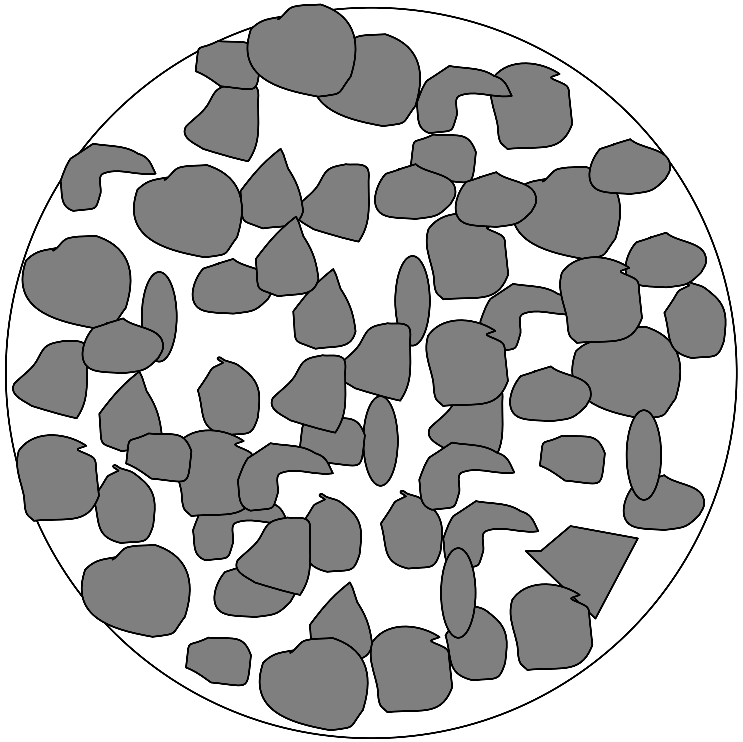
Pipe



"Porosity" = 0.5

$$v_R = \frac{Q}{A_{VSE}} \text{ (real velocity)}$$

Porous Medium



Porosity = 0.5

DARCY & SEEPAGE VELOCITY

- From the Continuity eq.:

$$Q = A\, v_D = A_{VS}\, v_s = A_{VSE}\, v_R$$

  – where:

$\boldsymbol{Q}$ = flow rate

$\boldsymbol{A}$ = total cross-sectional area of material

$\boldsymbol{A_{VS}}$ = area of voids

$\boldsymbol{v_s}$ = seepage velocity

$\boldsymbol{v_D}$ = Darcy velocity

$$v_R = v_D \frac{A}{A_{VSE}} \quad \ldots\ldots\ldots. \quad \rightarrow \quad v_D = n_{ef}\, v_R$$

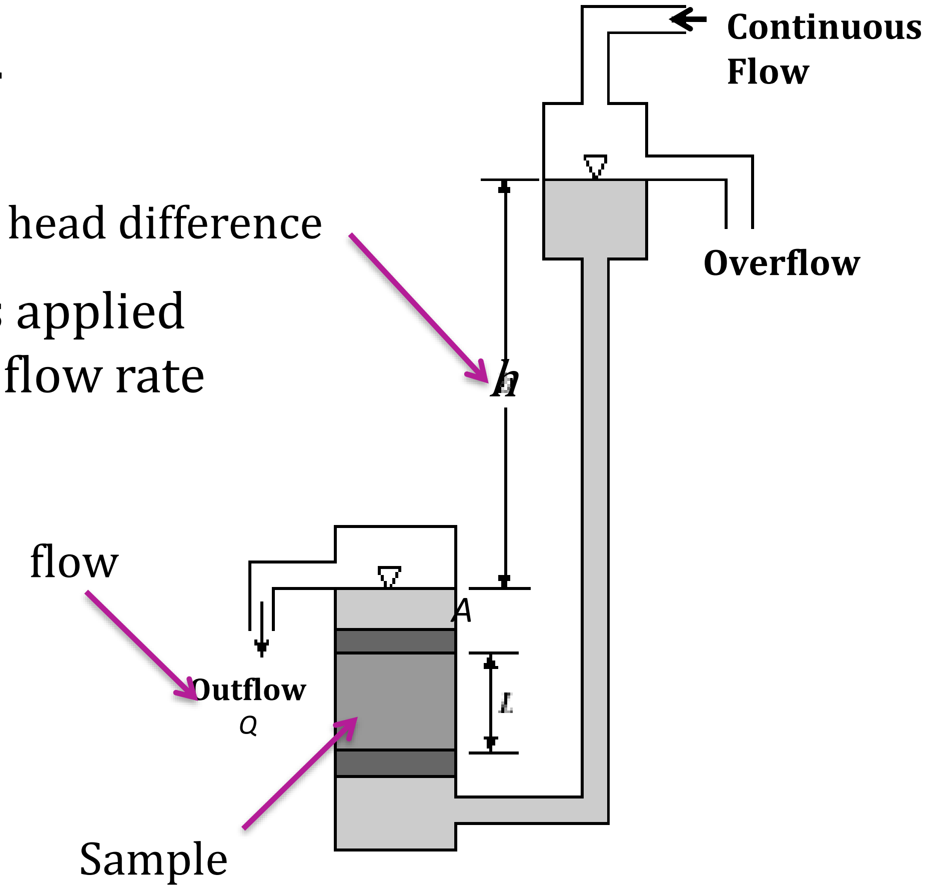
MEASURING CONDUCTIVITY - K

CONSTANT HEAD PERMEAMETER

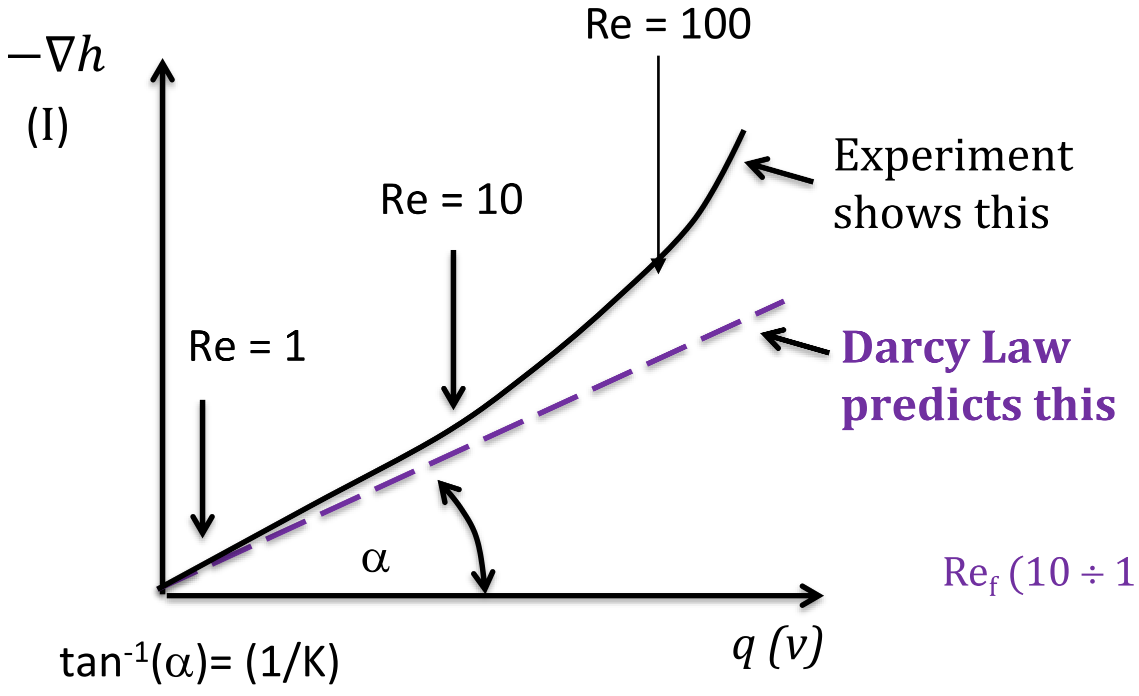
- **Flow is steady**
- Sample: Right circular cylinder
  - Length, L
  - Area, A
- Constant head difference (h) is applied across the sample producing a flow rate Q
- Darcy's Law

$$Q = KA\frac{h}{L}$$

$$\boxed{K = \frac{QL}{Ah}}$$



VALIDITY OF DARCY'S LAW



$Re_f\ (0 \div 1)$ – Darcy eq. is valid

$$v = -K\ I \quad \Rightarrow \quad I = av$$

kde $a = 1/K$

$Re_f\ (1 \div 10)$ Darcy eq. is also valid

$Re_f\ (10 \div 100)$ -Nondarcian flow (Darcy eq. is not valid)

$$I = av + b.\, v^m$$

where **m** $= 1{,}6 \div 2{,}0$

$Re_f > 100$ turbulent flow (Darcy eq. is not valid)

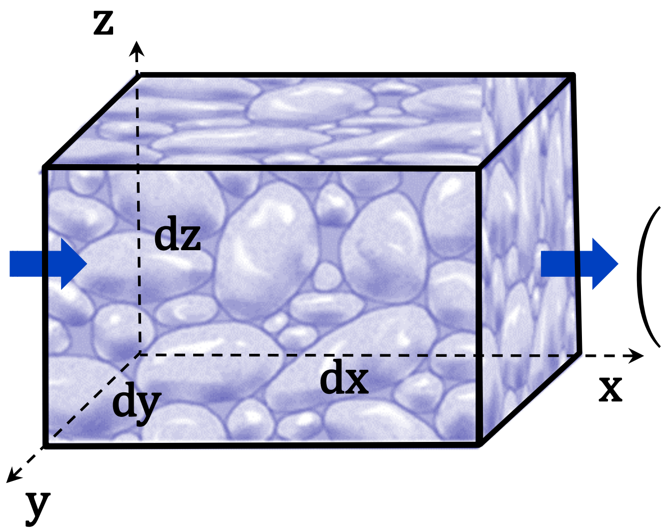
$$I = b\ v^2$$

CONTINUITY EQUATION – POROUS MEDIA

CONTINUITY EQUATION– CONFINED AQUIFER - steady flow

Assumptions
- Saturated aquifer
- Darcy eq. is valid
- Balance of mass
- Steady flow

$(\rho v_x) dydz$



$$\left((\rho v_x) + \frac{\partial(\rho v_x)}{\partial x} dx\right) dz\, dy$$

Mass inflow rate - mass outflow rate = 0

Flow in ....dy.dz:

$$(\rho v_x) dydz$$

Flow out ....dy.dz:

$$\left((\rho v_x) + \frac{\partial(\rho v_x)}{\partial x} dx\right) dz\, dy$$

$$\text{Flow in} = \text{Flow out (Continuity equation)}$$

$$(\rho v_x) dy\, dz - \left((\rho v_x) + \frac{\partial(\rho v_x)}{\partial x} dx\right) dz\, dy = 0 \quad \Rightarrow \quad -\left(\frac{\partial(\rho v_x)}{\partial x}\right) dx\, dz\, dy$$

CONTINUITY EQUATION– CONFINED AQUIFER - steady flow

Balance of mass for x, y, z :

$$-\left(\frac{\partial(\rho v_x)}{\partial x}\right) dx\,dy\,dz - \left(\frac{\partial(\rho v_y)}{\partial y}\right) dx\,dy\,dz - \left(\frac{\partial(\rho v_z)}{\partial z}\right) dx\,dy\,dz \quad = 0 \ldots \ldots / \frac{1}{dx\,dy\,dz}$$

$$-\left(\frac{\partial(\rho v_x)}{\partial x}\right) - \left(\frac{\partial(\rho v_y)}{\partial y}\right) - \left(\frac{\partial(\rho v_z)}{\partial z}\right) \quad = 0$$

Incompressible liquid ρ = const.

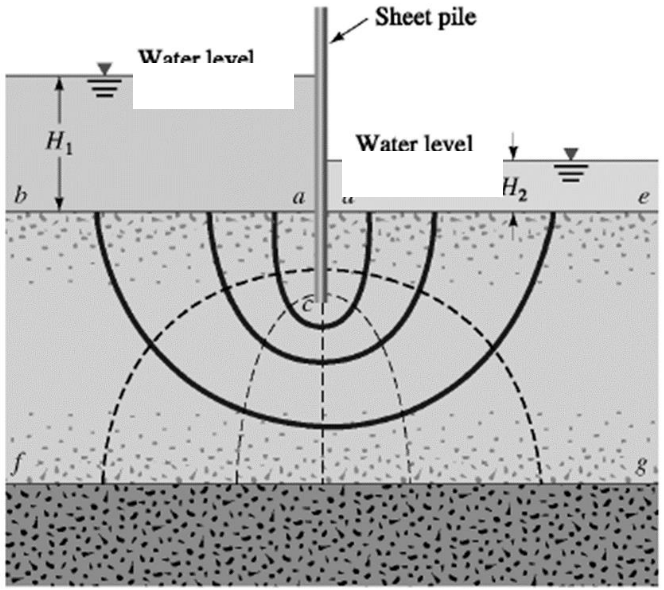
CONTINUTY EQUATION FOR STEADY FLOW

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0$$

LAPLACE EQUATION

Darcy eq. for anisotropic porous media

$$v_x = K_x \frac{\partial h}{\partial x} \qquad v_y = K_y \frac{\partial h}{\partial y} \qquad v_z = K_z \frac{\partial h}{\partial z}$$



From continuity equation

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0$$

With Darcy eq.:

$$\frac{\partial}{\partial x}\left(K_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y \frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z \frac{\partial h}{\partial z}\right) = 0$$

If soil is isotropic ....$K_x = K_y = K_z = K$

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0$$

$\nabla^2 h = 0$ ... **LAPLACE EQUATION**

DUPUIT´S ASSUMPTIONS

For unconfined ground water flow Dupuit developed a theory that allows for a simple solution based off the following assumptions:

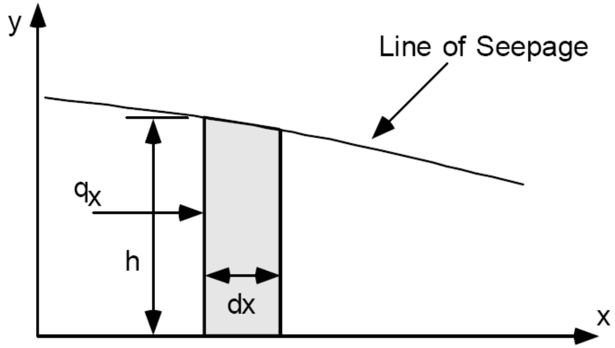
1) The water table or free surface is only slightly inclined

2) Streamlines may be considered horizontal and equipotential lines, vertical

3) Slopes of the free surface and hydraulic gradient are equal

Velocities are horizontal !!!

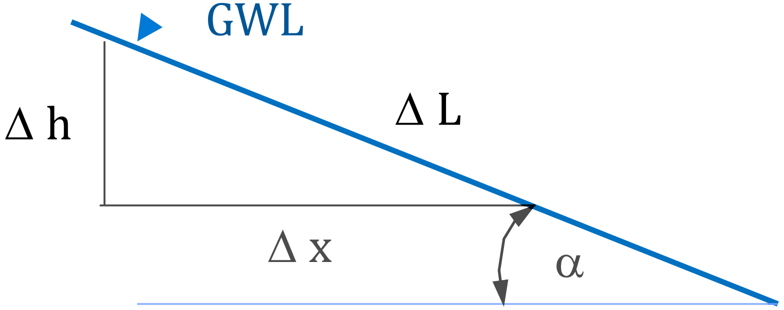
DUPUIT'S ASSUMPTIONS



Comparison:

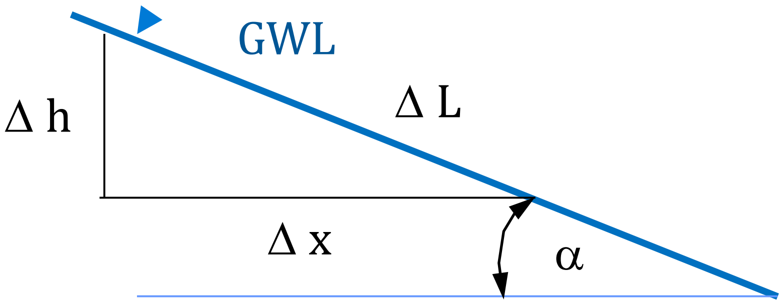
α	sin(α)	tan(α)
0º	0	0
5º	0.087	0.087
10º	0.174	0.176
20º	0.342	0.346
30º	0.500	0.577
40º	0.643	0.839
50º	0.766	1.192

DUPUIT



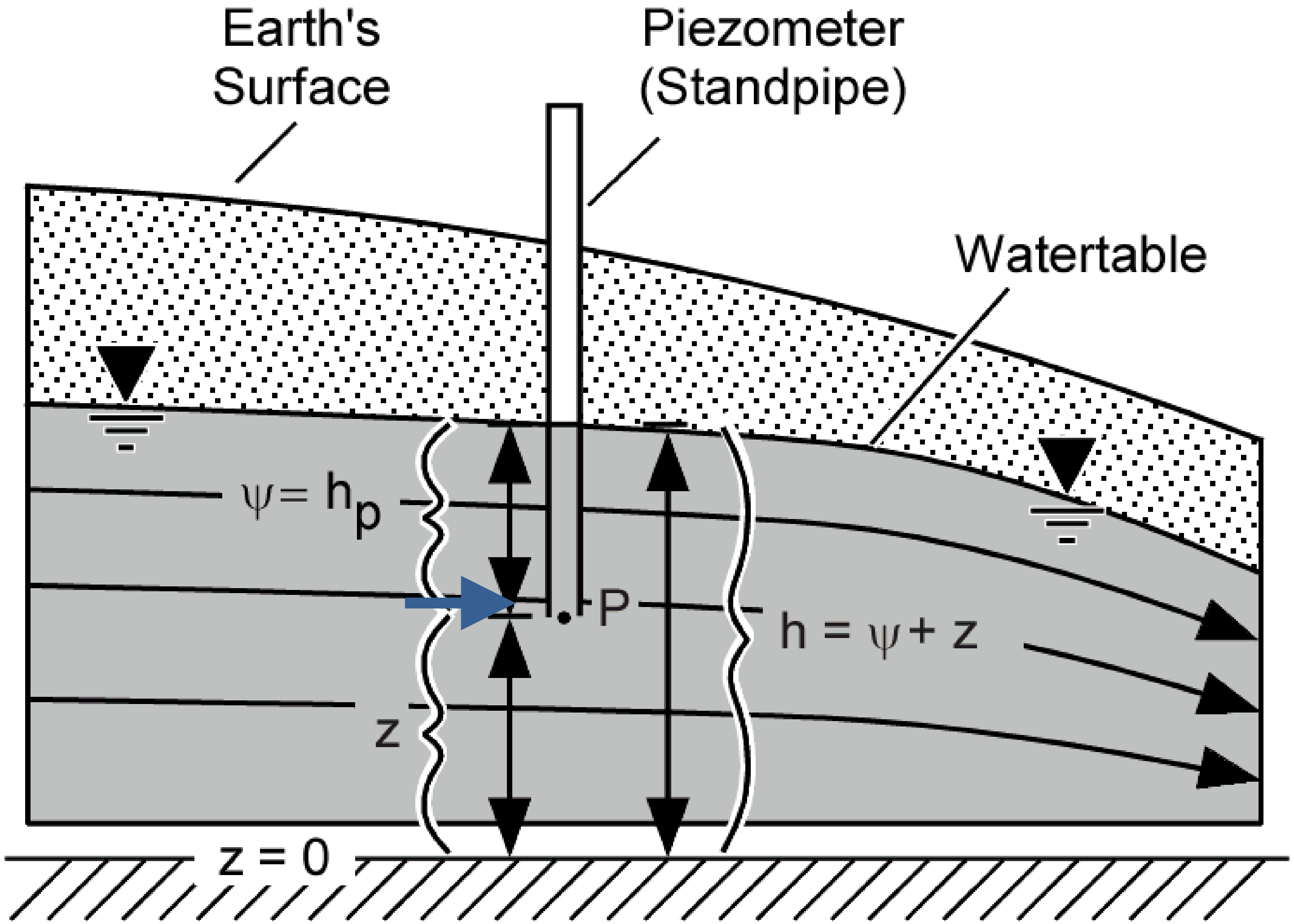
$$I = \frac{\Delta h}{\Delta x} = \tan \alpha$$

ACTUAL



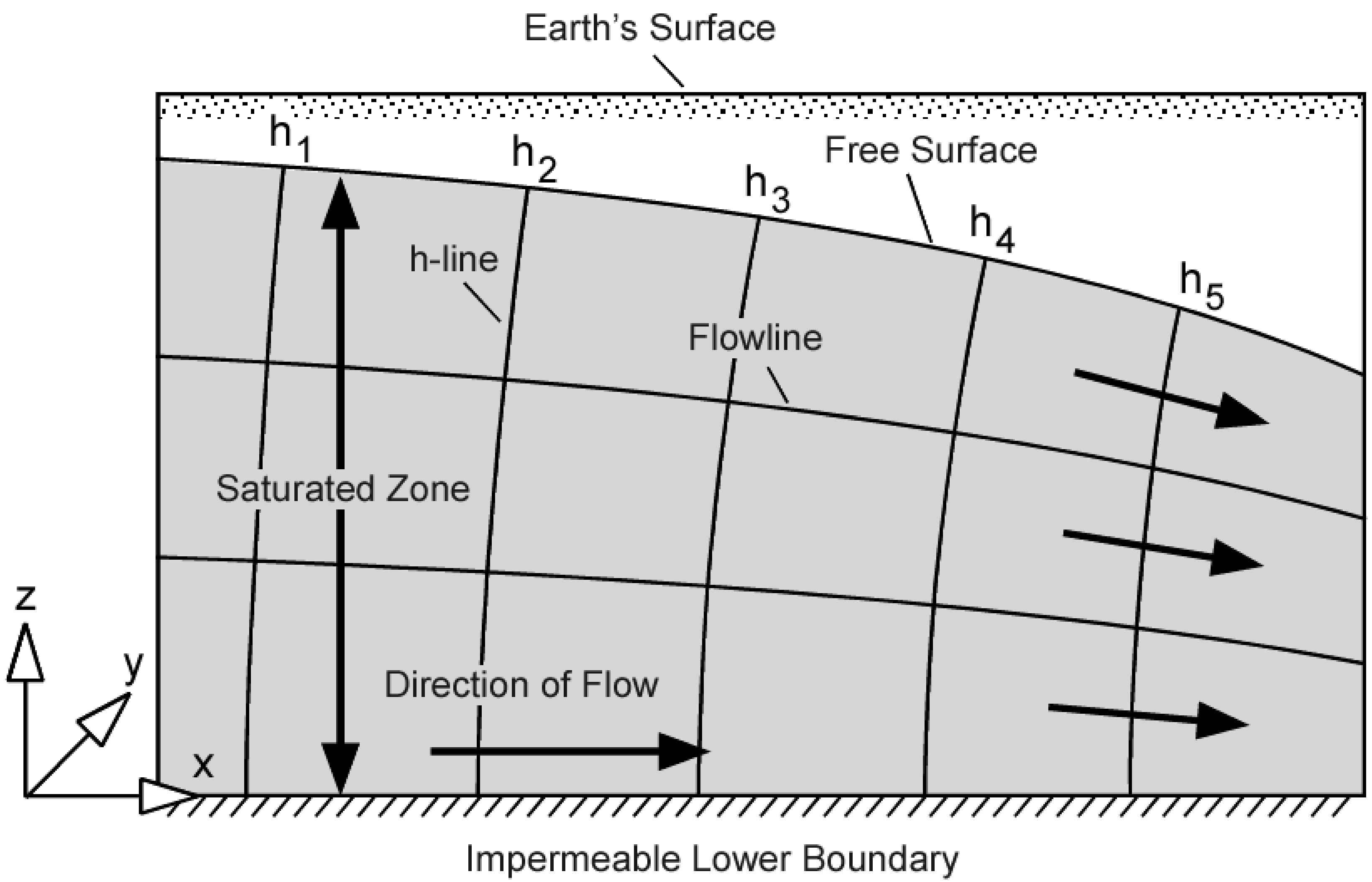
$$I = \frac{\Delta h}{\Delta L} = \sin \alpha$$

THE ELEVATION OF WATER IN THE PIEZOMETER PROVIDES A MEASURE OF h_p



A piezometer measures the hydraulic head at a point.

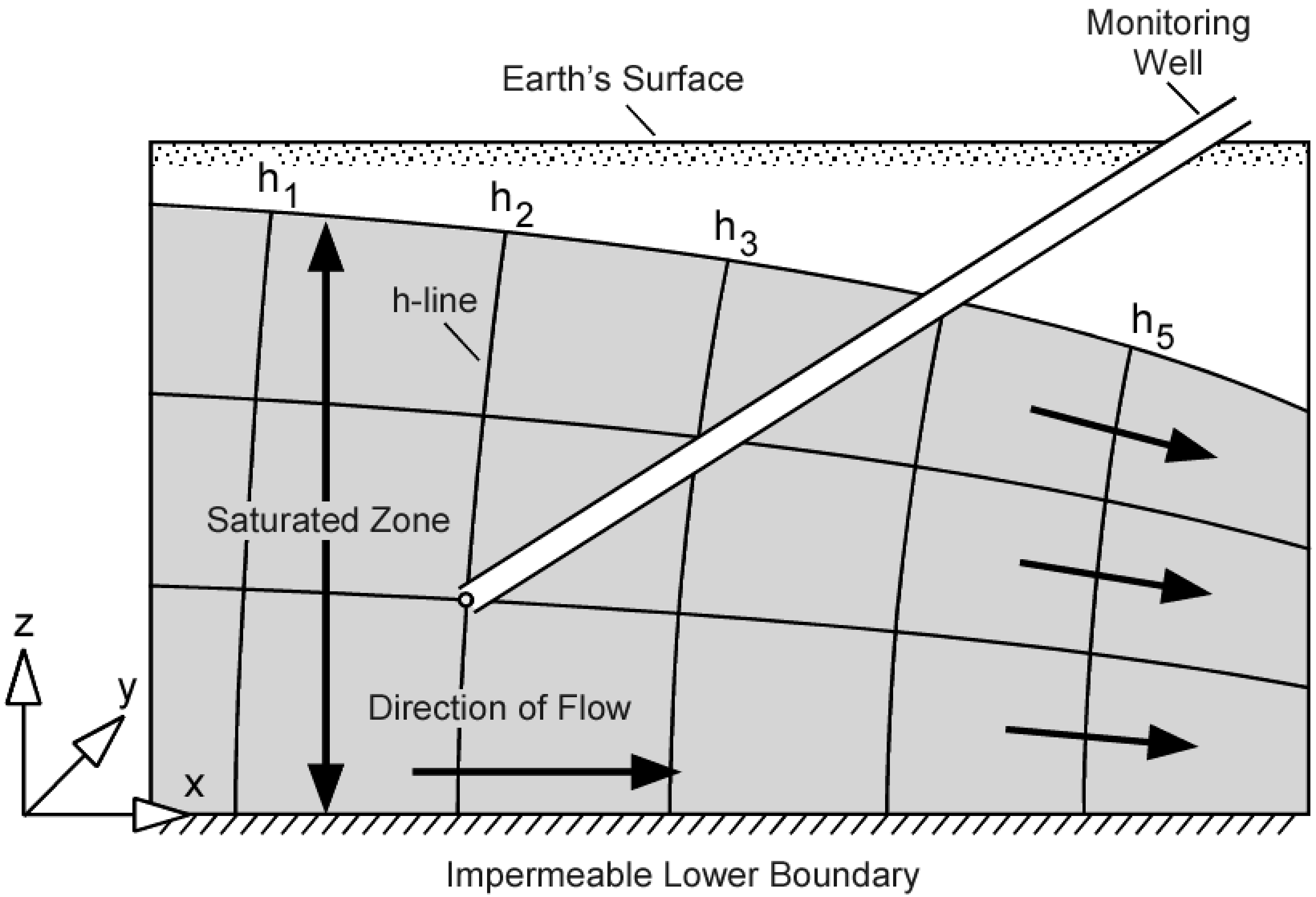
POTENTIOMETRIC SURFACES (H-LINES) FOR UNCONFINED FLOW.

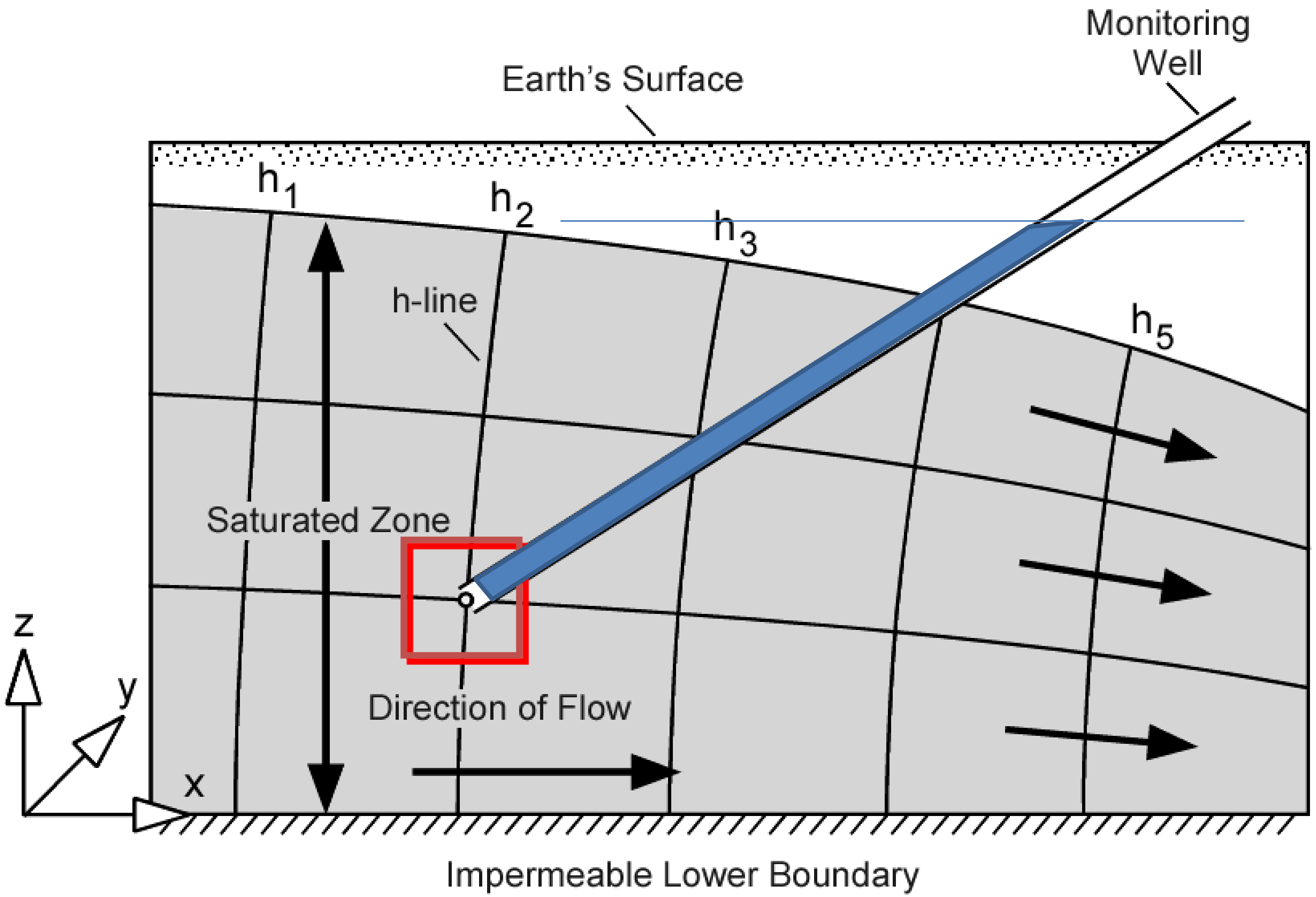


Flowline=streamline **h-line= equipotential line**

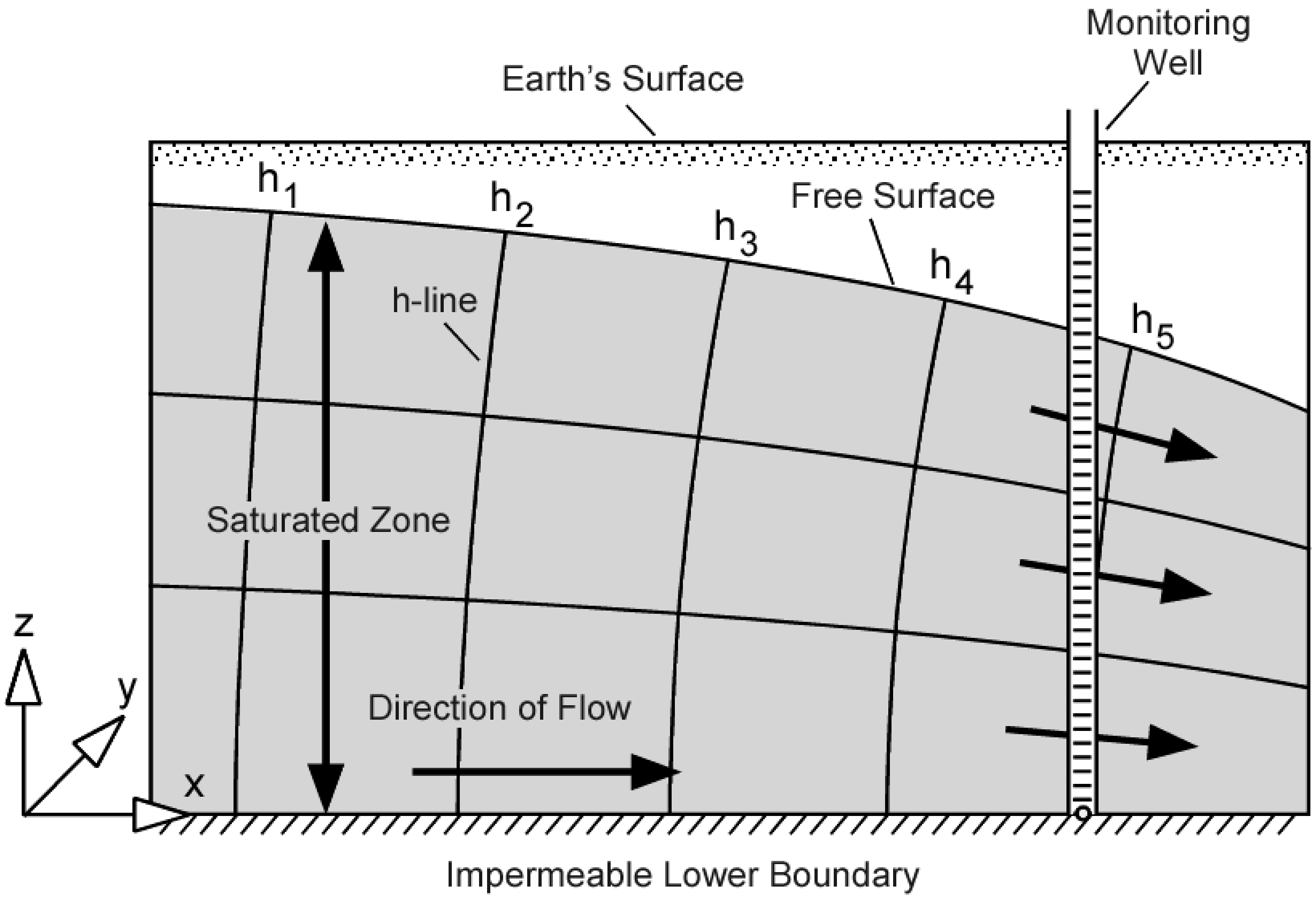
APPLICATIONS

Piezometer:

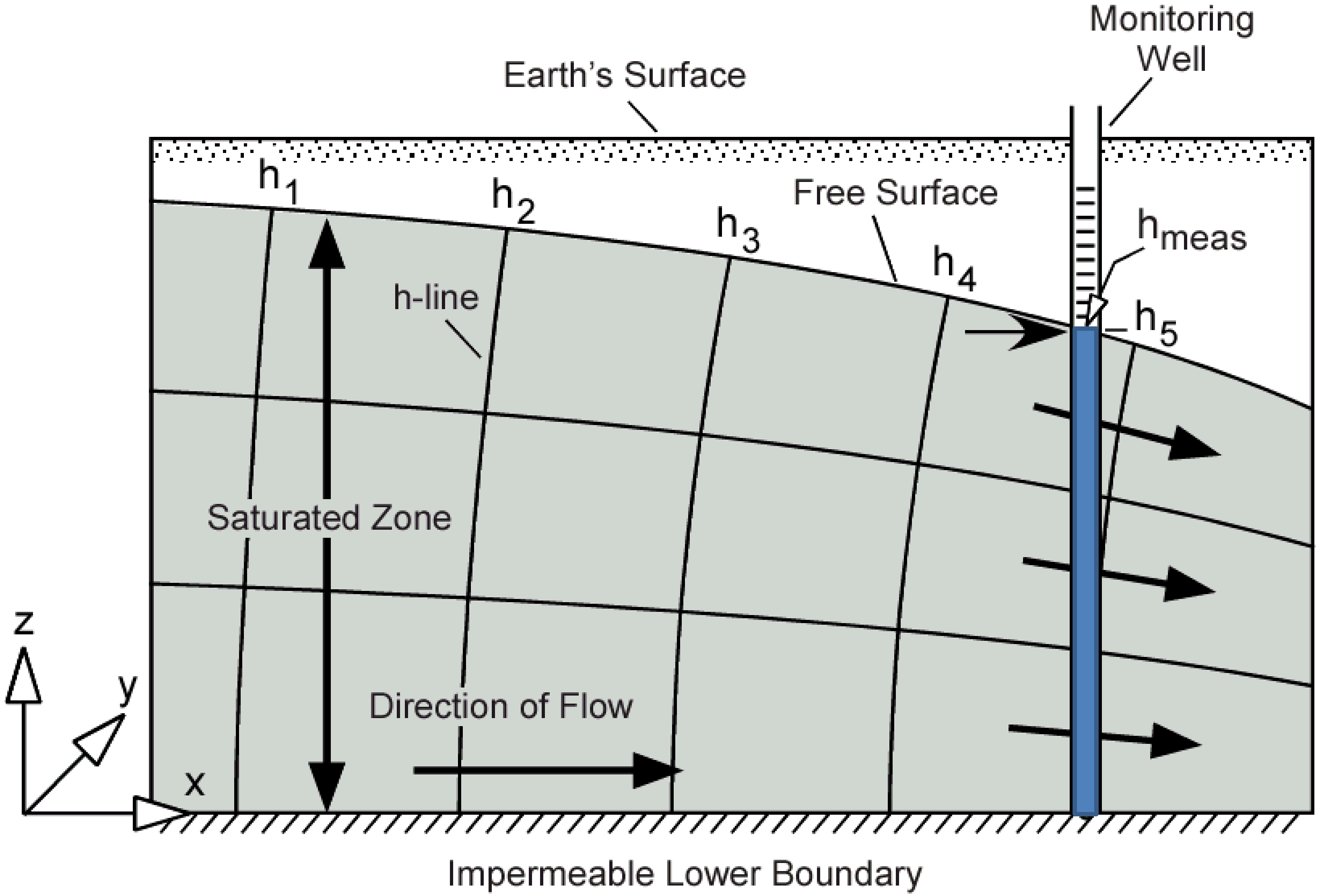


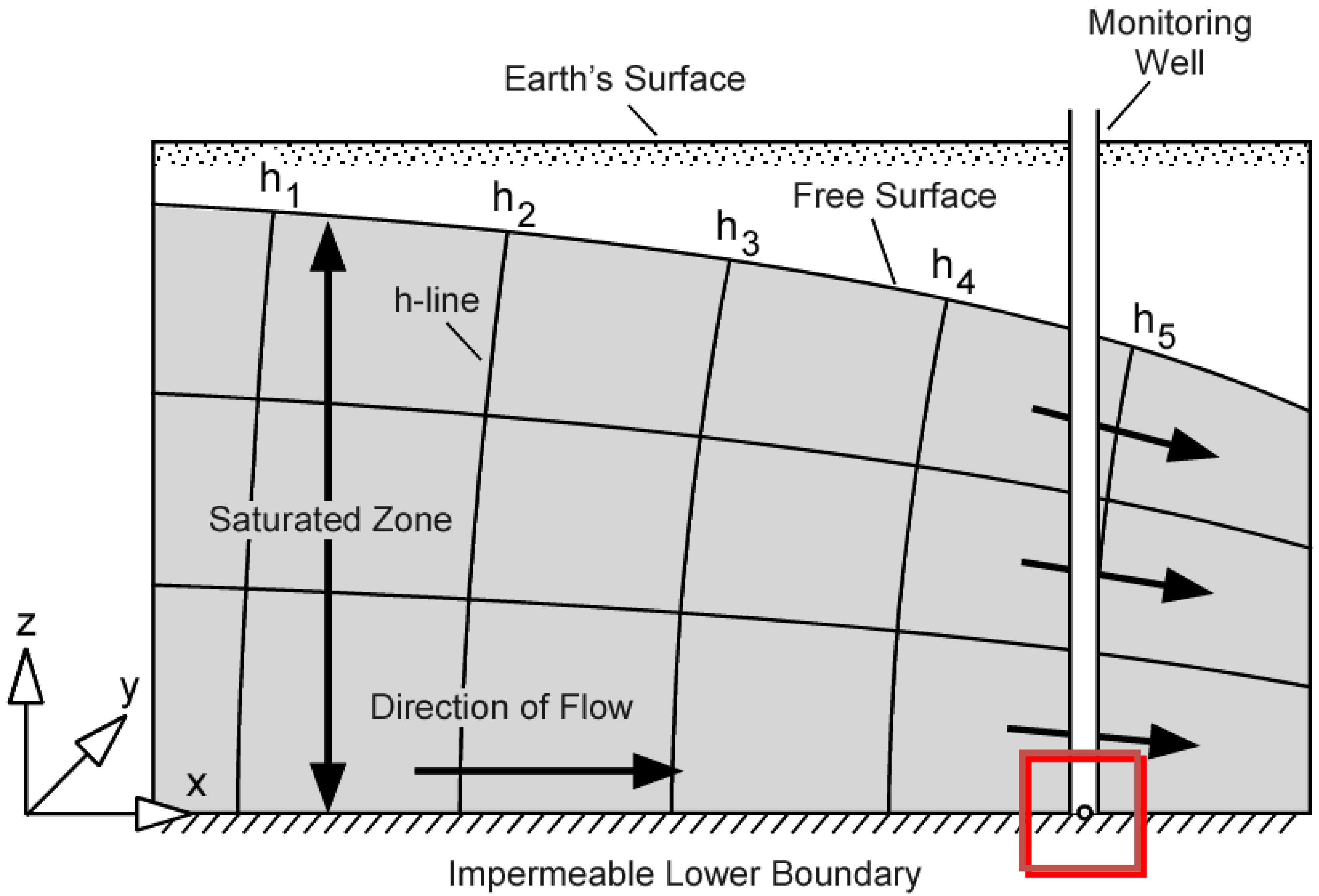


Monitoring Well
Earth's Surface
h1
h2
h3
h5
h-line
Saturated Zone
Direction of Flow
z
y
x
Impermeable Lower Boundary



Monitoring Well
Earth's Surface
Free Surface
h_1
h_2
h_3
h_4
h_5
h-line
Saturated Zone
Direction of Flow
z
y
x
Impermeable Lower Boundary


Monitoring Well
Earth's Surface
Free Surface
h1
h2
h3
h4
h5
hmeas
h-line
Saturated Zone
Direction of Flow
z
y
x
Impermeable Lower Boundary



Monitoring Well
Earth's Surface
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h1
h2
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h4
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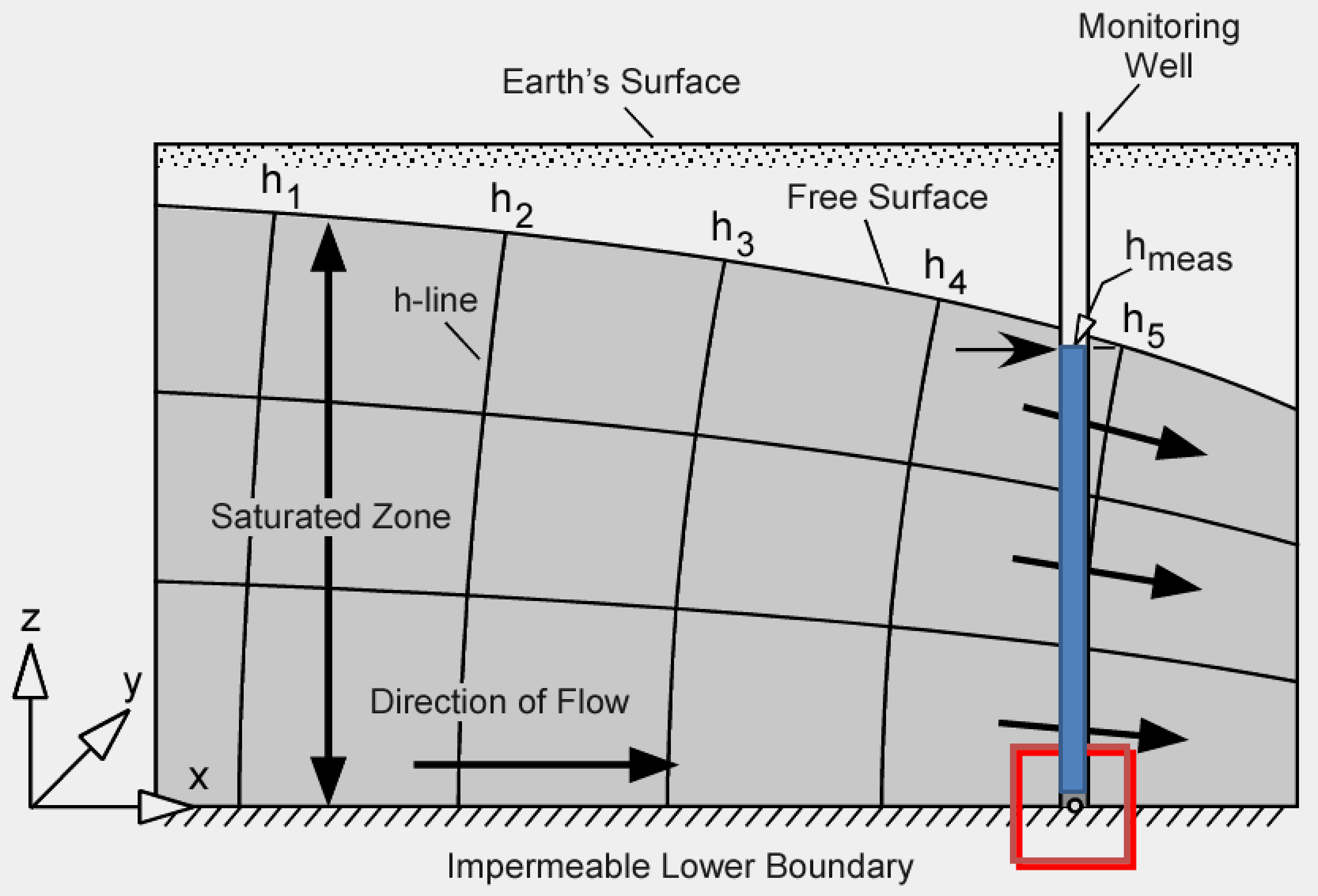
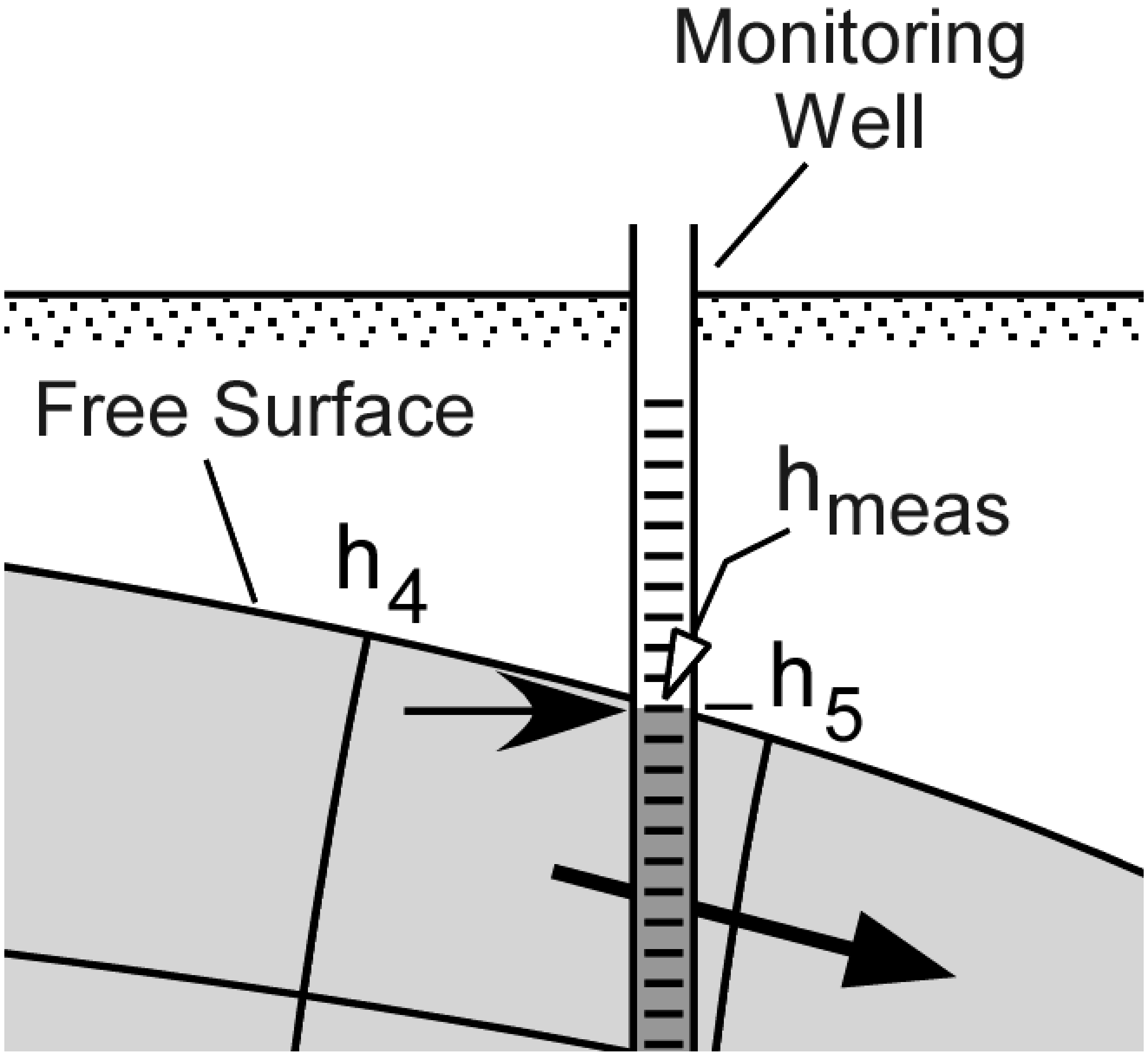
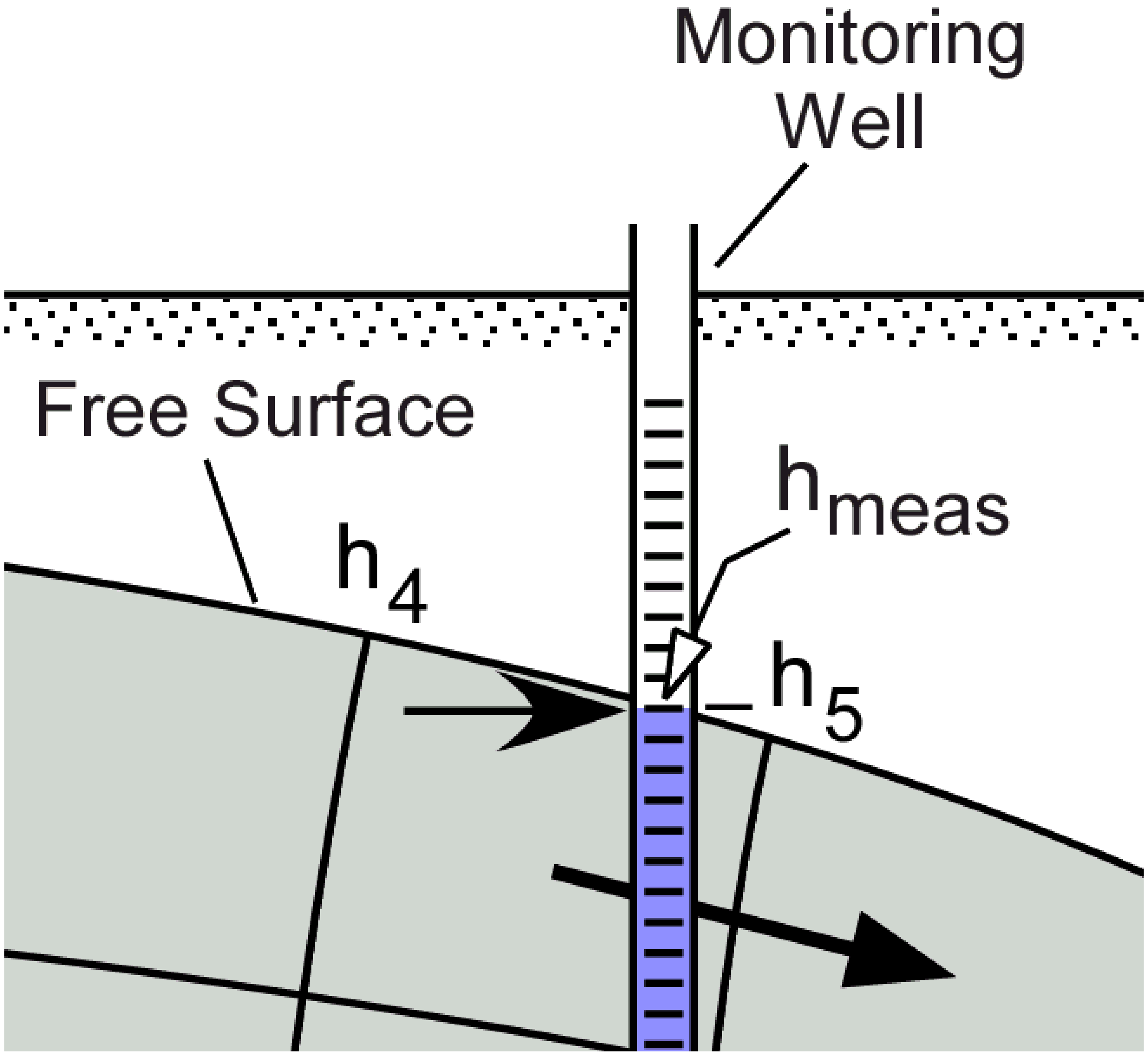
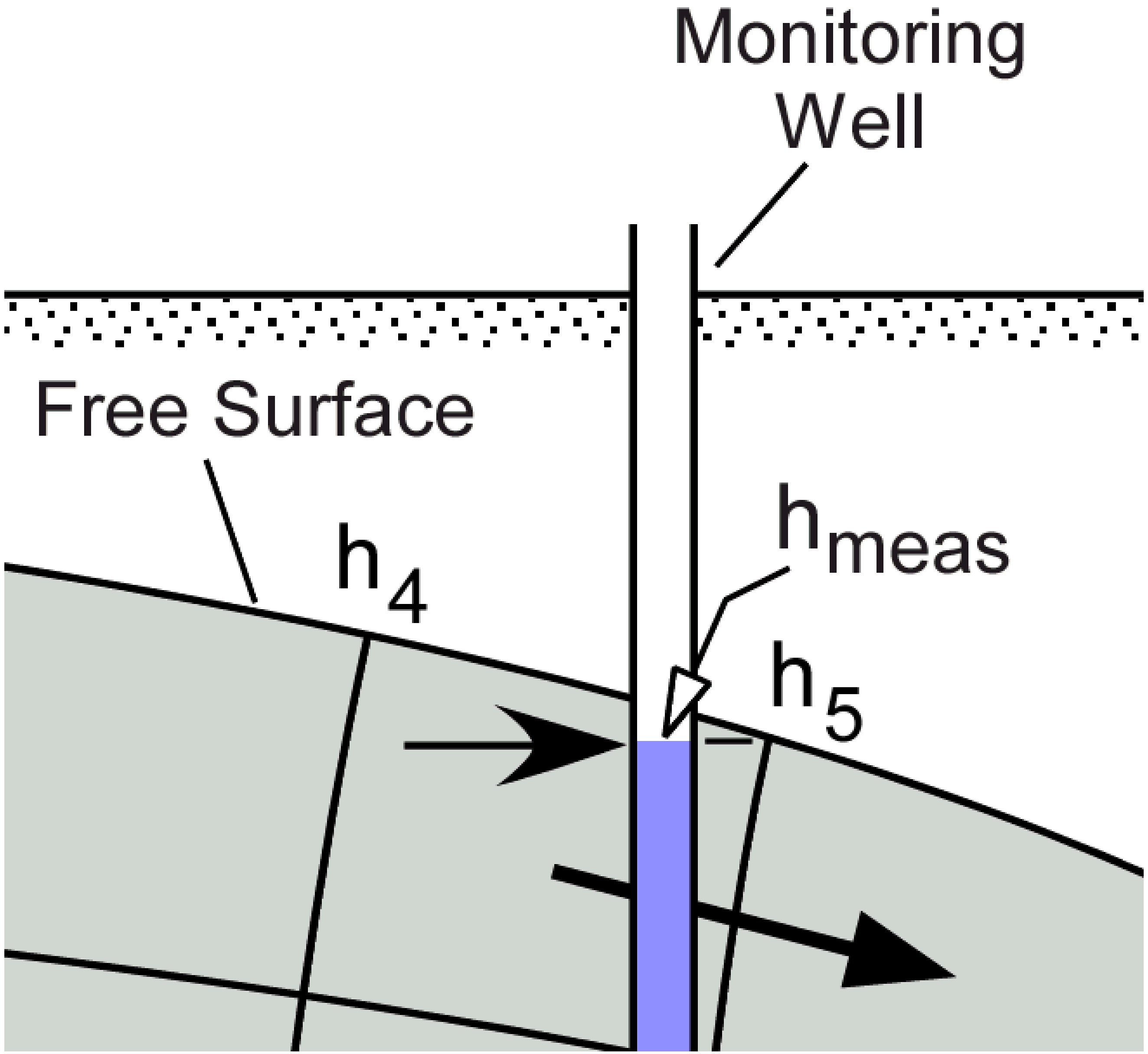
h-line
Saturated Zone
Direction of Flow
z
y
x
Impermeable Lower Boundary

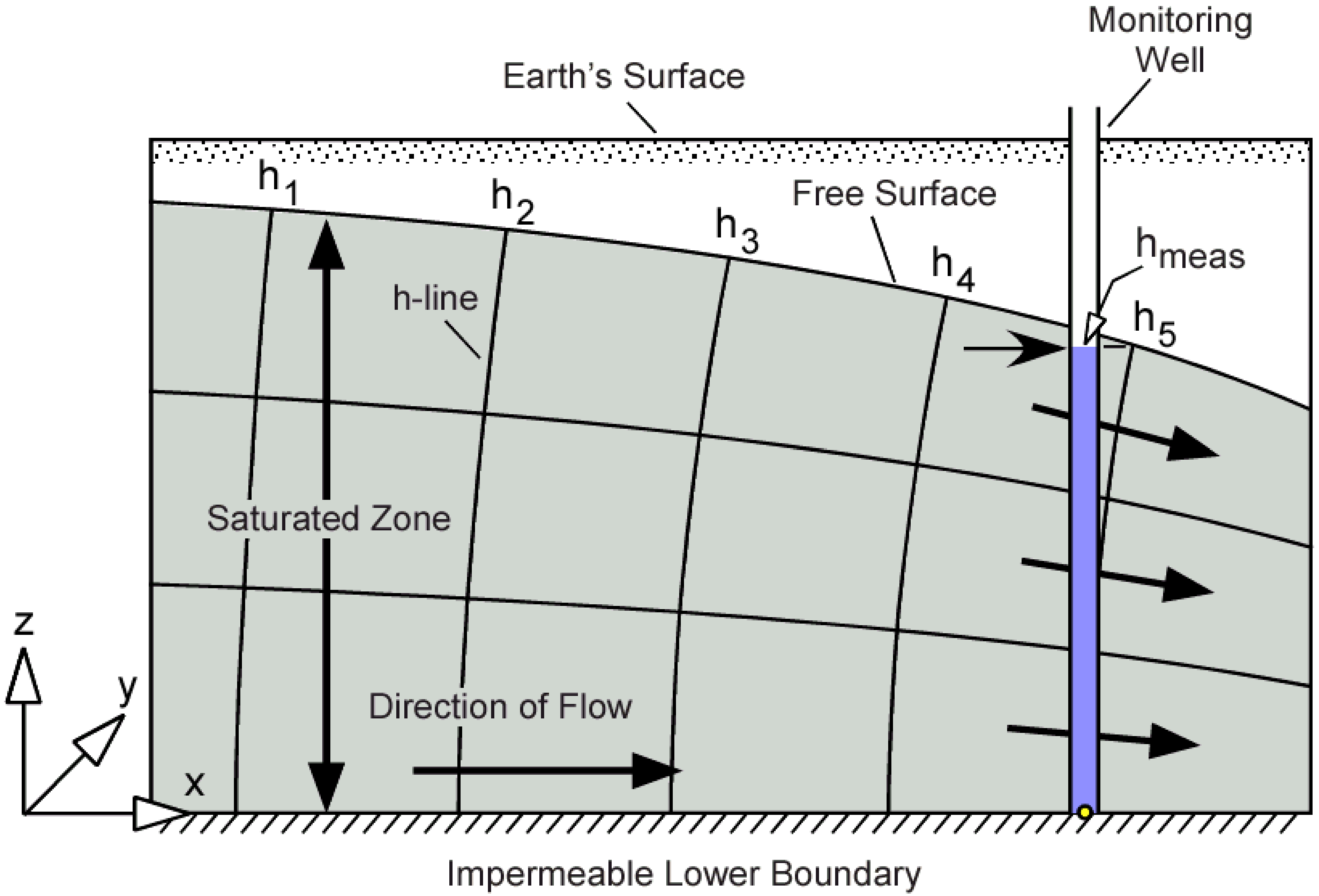

Monitoring Well
Earth's Surface
Free Surface
h_1
h_2
h_3
h_4
h_{meas}
h_5
h-line
Saturated Zone
z
y
x
Direction of Flow
Impermeable Lower Boundary


Monitoring Well
Free Surface
h_4
h_{meas}
h_5


Monitoring Well
Free Surface
h_{meas}
h_4
h_5

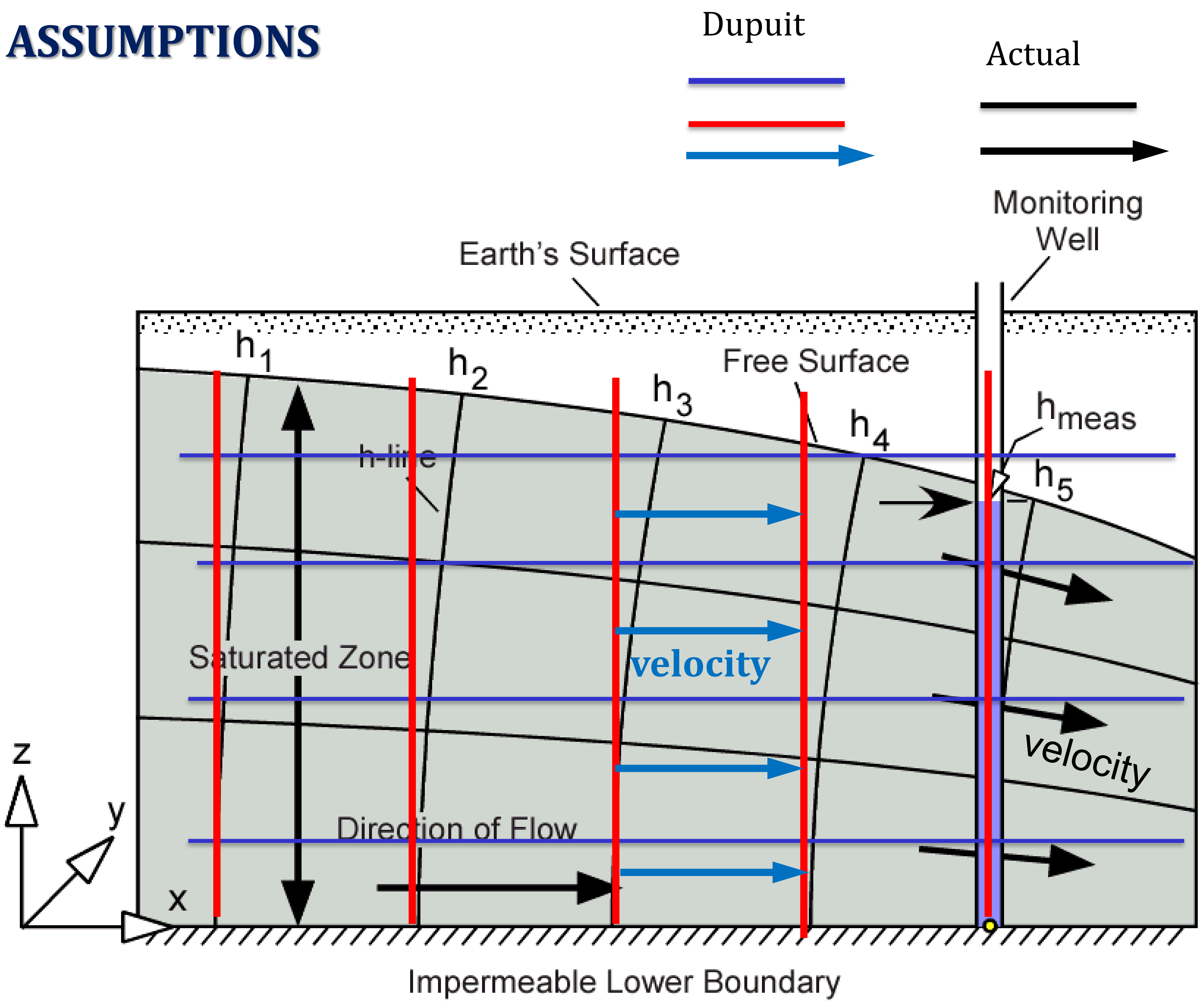

Monitoring
Well
Free Surface
h4
hmeas
h5

Actual

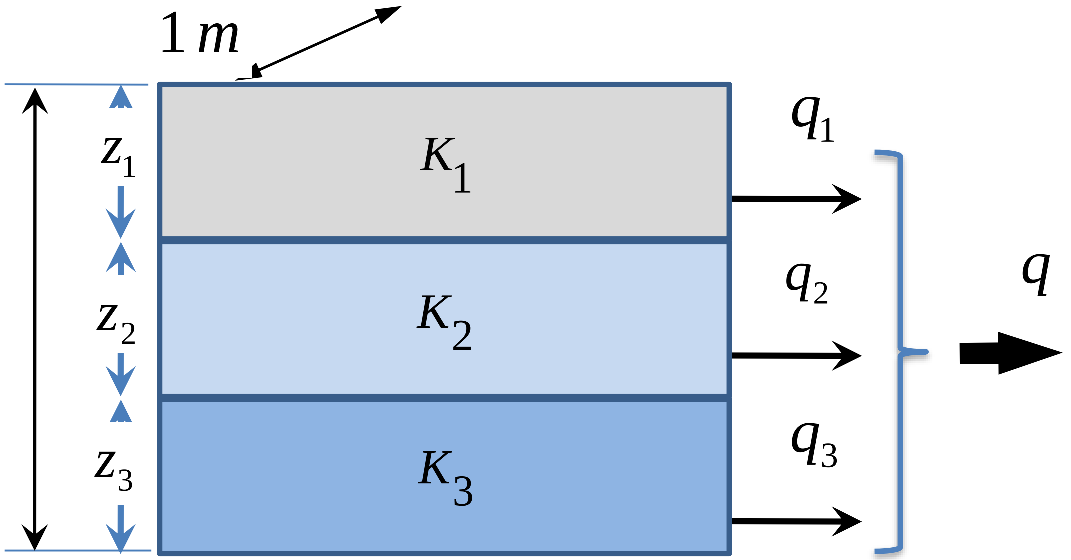


Monitoring Well
Earth's Surface
h1
h2
h3
h4
Free Surface
hmeas
h5
h-line
Saturated Zone
z
y
x
Direction of Flow
Impermeable Lower Boundary

DUPUIT ASSUMPTIONS



LAYERED POROUS MEDIA (FLOW PARALLEL TO LAYERS)



$$q_1 = K_1\, i\, z_1$$

$$q_2 = K_2\, i\, z_2$$

$$q_3 = K_3\, i\, z_3$$

Total horizontal specific discharge, q:

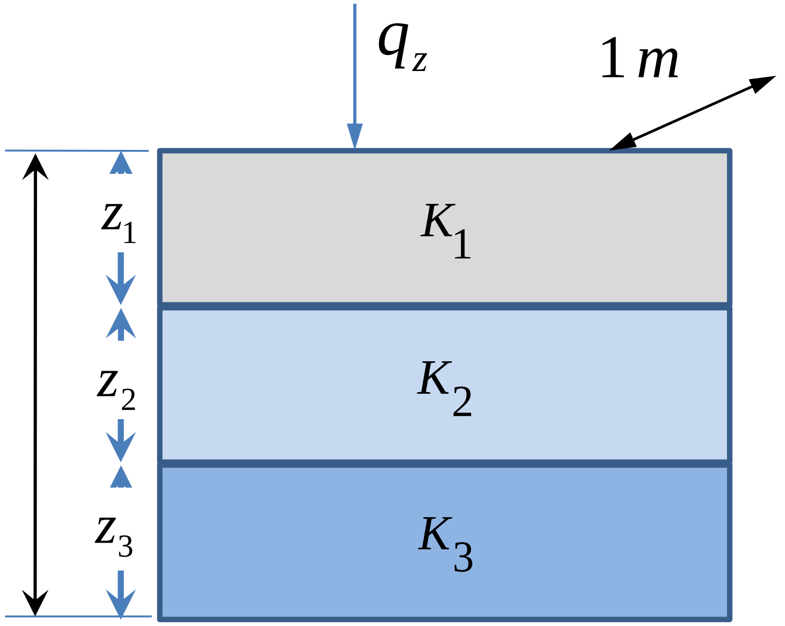
$$q_x = q_1 + q_2 + q_3 = i\left(K_1 z_1 + K_2 z_2 + K_3 z_3\right)$$

Homogeneous aquifer

$$q_x = K_x\, i\,(z_1 + z_2 + z_3) \Rightarrow K_x = \frac{q_x}{i\,(z_1 + z_2 + z_3)} \Rightarrow K_x = \frac{i\left(K_1 z_1 + K_2 z_2 + K_3 z_3\right)}{i\,(z_1 + z_2 + z_3)}$$

$$K_{eq} = \frac{\sum_{i=1}^{n} K_i\, z_i}{\sum_{i=1}^{n} z_i}$$

LAYERED POROUS MEDIA (FLOW PERPENDICULAR TO LAYERS)



$$q_z = K_1 \frac{dh_1}{z_1}$$

dh_1 – pressure loss in the 1st layer

$$dh_1 = \frac{z_1}{K_1} q_z$$

From continuity eq. – q_z = const.

$$dh_1 + dh_2 + dh_3 = \left[\frac{z_1}{K_1} + \frac{z_2}{K_2} + \frac{z_2}{K_2}\right] q_z$$

Homogeneous

$$dh_1 + dh_2 + dh_3 = \left[\frac{z_1 + z_2 + z_2}{K_z}\right] q_z$$

$$q_z = K_z \left[\frac{dh_1 + dh_2 + dh_3}{z_1 + z_2 + z_2}\right] \Rightarrow K_z = q_z \left[\frac{z_1 + z_2 + z_2}{dh_1 + dh_2 + dh_3}\right] \Rightarrow K_z = \frac{z_1 + z_2 + z_3}{\frac{z_1}{K_1} + \frac{z_2}{K_2} + \frac{z_3}{K_3}}$$

$$K_{eq} = \frac{\sum_{i=1}^{n} z_i}{\sum_{i=1}^{n} \frac{z_i}{K_i}}$$

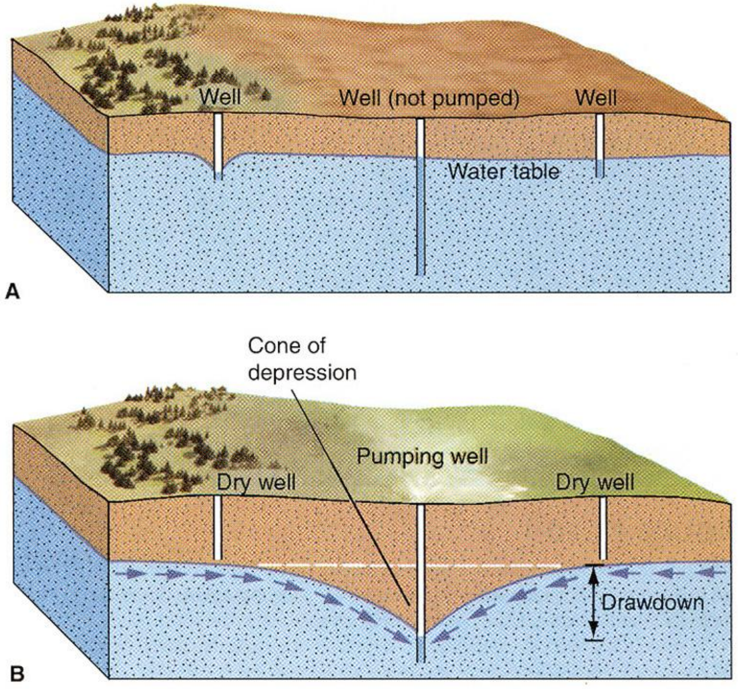
HYDRAULICS OF WELLS

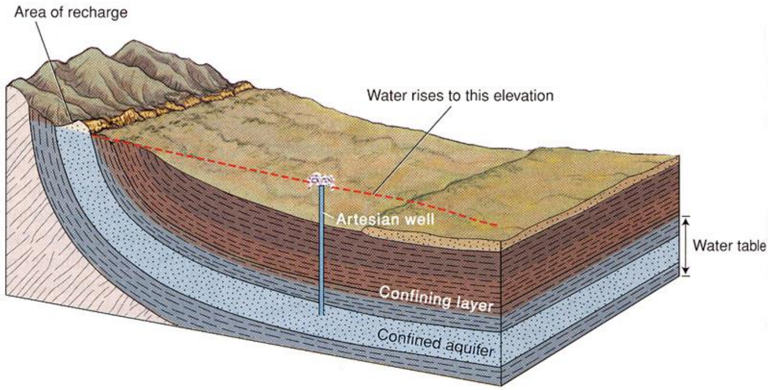
WELL HYDRAULICS

- A **water well** is a hydraulic structure that is designed and constructed to permit economic withdrawal of water from an aquifer
- **Water well construction includes**:
  - Selection of appropriate drilling methods
  - Selection of appropriate completion materials
  - Analysis and interpretation of well and aquifer performance

WELLS

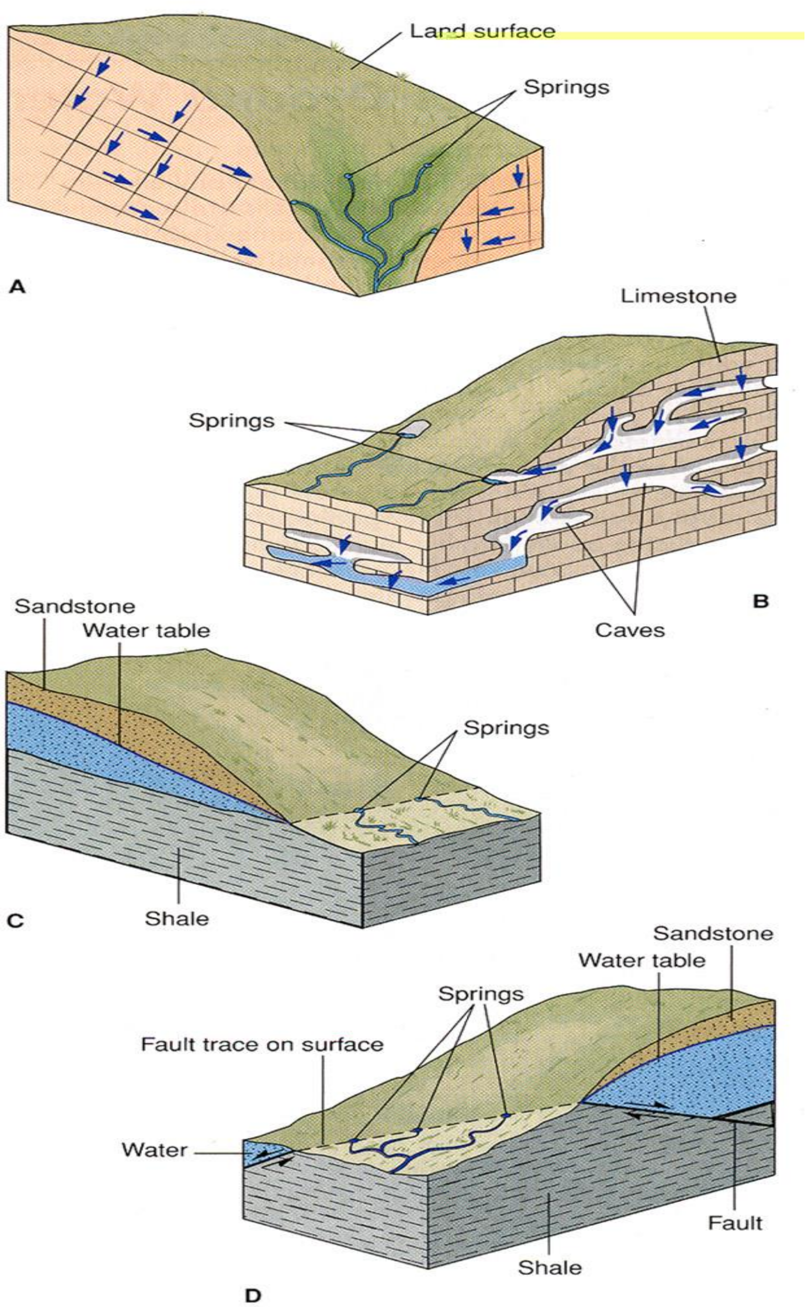
- ***Well*** - a deep hole dug or drilled into the ground to obtain water from an aquifer
  - For wells in unconfined aquifers, water level before pumping is the water table
  - Water table can be lowered by pumping, a process known as *drawdown*
  - Water may rise to a level above the top of a confined aquifer, producing an *artesian well*





SPRINGS

- *Spring* - a place where water flows naturally from rock or sediment onto the ground surface



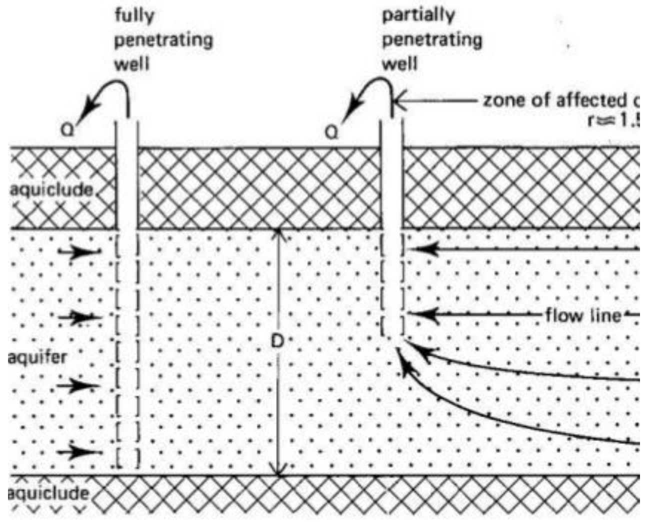
WELL

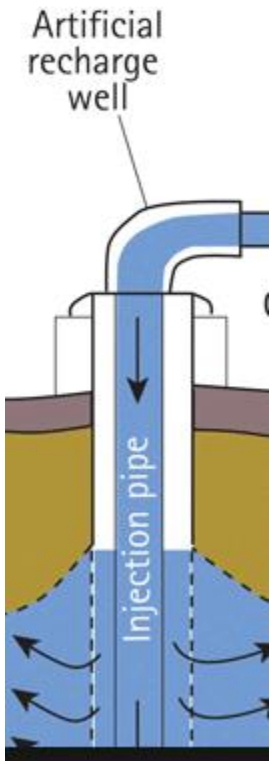
A) - Unconfined aquifer
- Confined aguifer
- (Artesian aquifer)

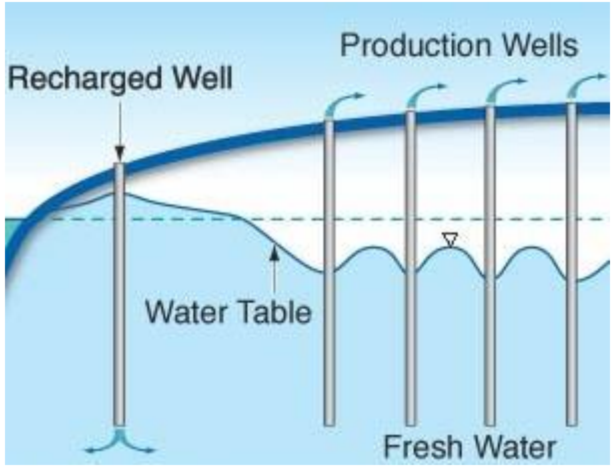
B) – Fully penetrating
- Partially penetrated

C) – Dug
- Drilled

D) – Discharge
(Production, pumped)
- Recharge

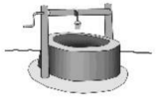






ADVANTAGES AND DISADVANTAGES

Dug Well



Advantages:

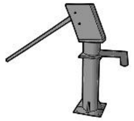
- High degree of involvement of the local community during the whole process
- Under supervision, **no skilled workers are required**
- Simple equipment sufficient for both construction and maintenance
- **Low cost** for construction
- Involvement of private sector possible (local well diggers)
- **Yield can be increased** after construction
- **Reservoir included** (large diameter)

Disadvantages:

- **Long construction phase**
- Dangerous excavation
- Application restricted to regions with rather **soft geological formation** and relatively **high groundwater levels**
- Alteration of groundwater level can adversely affect the surrounding environment
- People (i.e. children) can fall in if the well is uncovered

ADVANTAGES AND DISADVANTAGES

Drilled Well



Advantages:

- **Quicker** and cheaper to sink than hand-dug wells
- Less susceptible to contamination
- **No dewatering** during sinking required
- Safer in construction and use
- The **well** itself **needs barely maintenance**
- Many simple drilling techniques available suiting many geological conditions

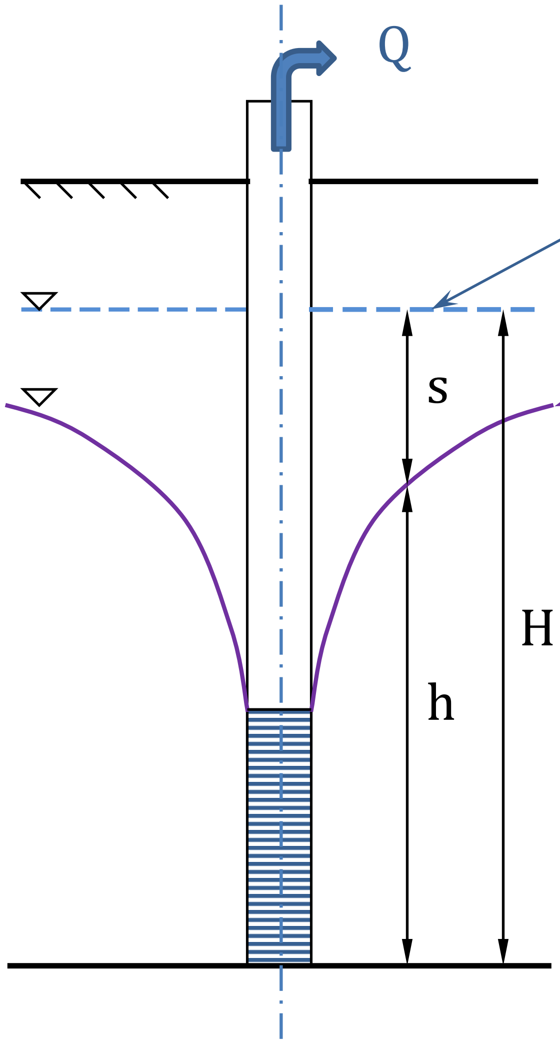
Disadvantages:

- Skilled staff and experts required for drilling
- **Pump required**
- Lower yield than hand-dug wells (smaller diameter)
- **Overexploitation** may lead to adverse effects on the environment
- More technical equipment and skills necessary for construction
- **No integrated storage** capacity

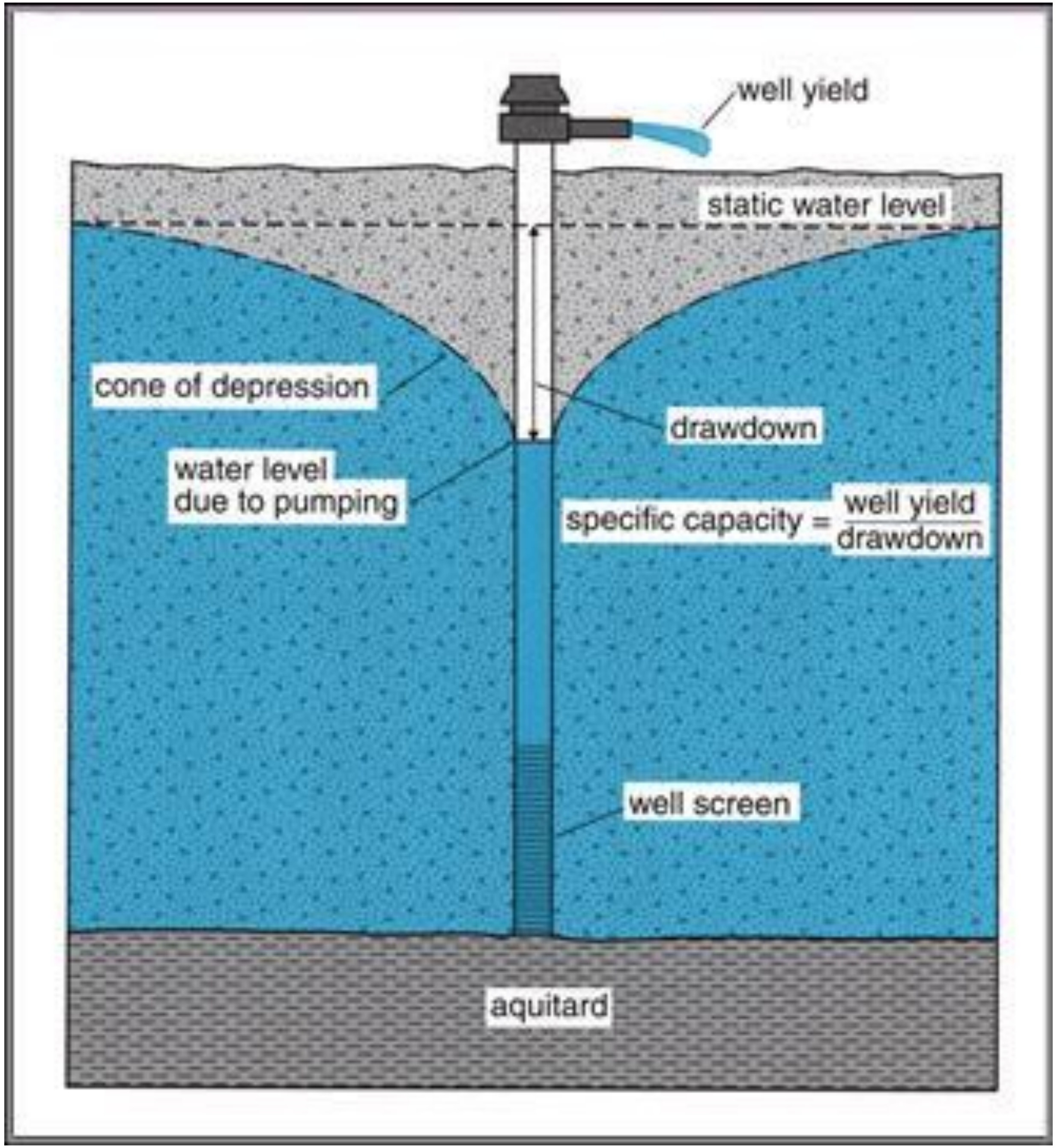
https://www.youtube.com/watch?v=MeeYy-dVzJU

https://www.youtube.com/watch?v=iXdq65xzsus

PUMPING WELL TERMINOLOGY

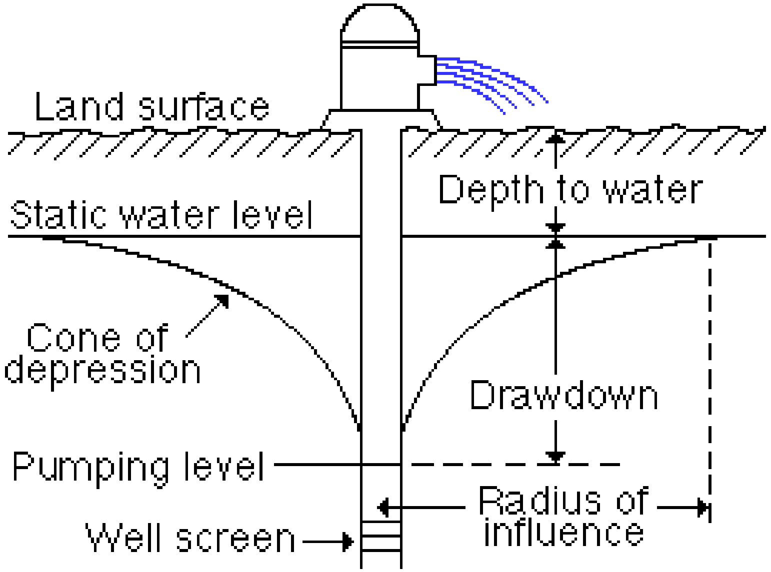


- **Static Water Level** [SWL] (H) is the equilibrium water level before pumping commences
- **Pumping Water Level** [PWL] (h) is the water level during pumping
- **Drawdown** (s = H - h) is the difference between SWL and PWL
- **Well Yield** (Q) is the volume of water pumped per unit time
- **Specific Capacity** (Q/s) is the yield per unit drawdown

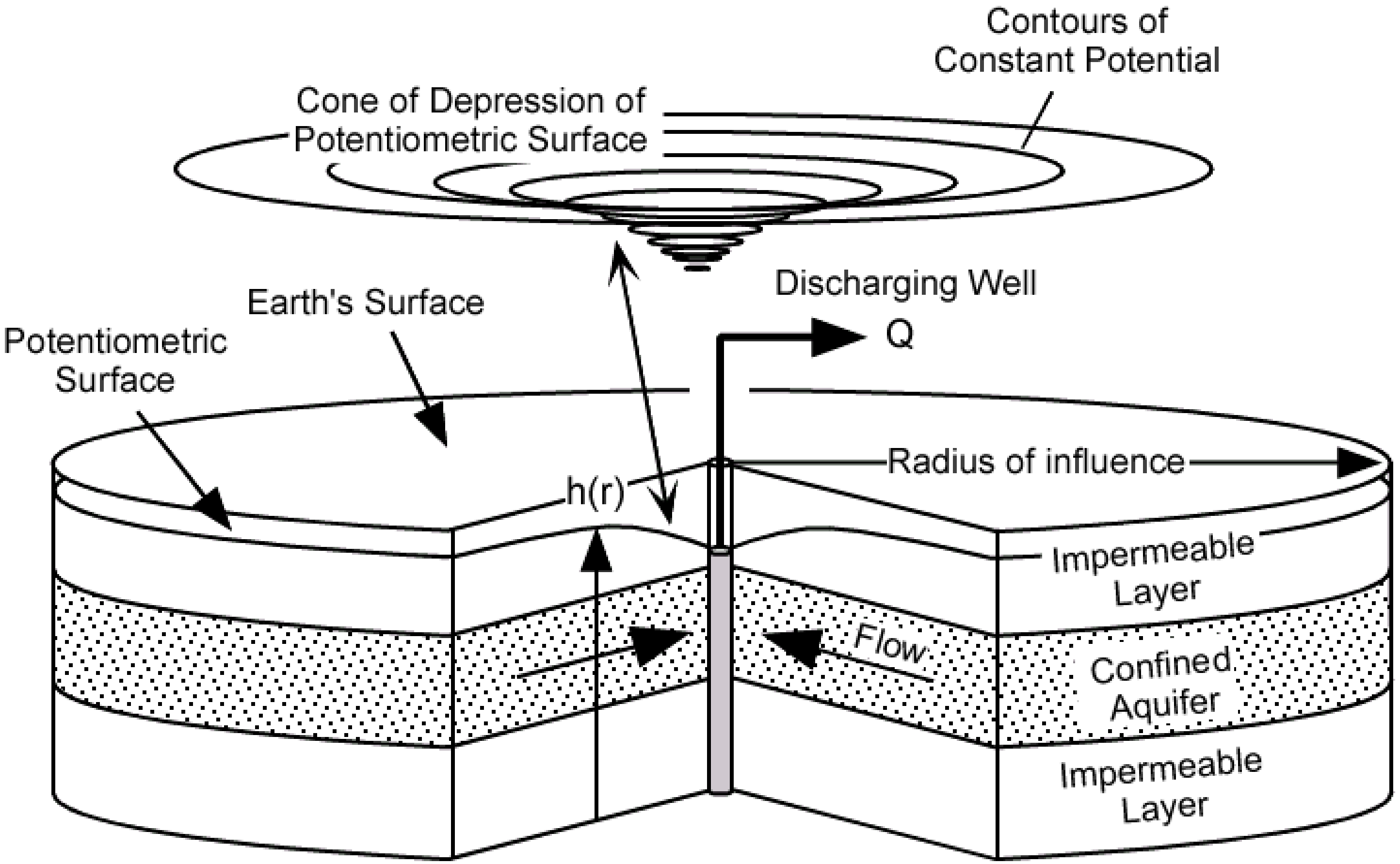


Some useful terms to know:

- ✓ **Cone of depression**
- ✓ **Drawdown, s**
- ✓ **Radius of influence**, **R**
- ✓ **Specific capacity**, **q**



Details on the geometry of drawdown and the "cone of depression".



SCHEMATIC OF A TYPICAL WELL INSTALLATION

Well screen
perforated pipe or slotted pipe

- **Well screen** (holds back sediments while allowing water to infiltrate the well)

Filter pack (sand / gravel)
extends at least 0.5 m above well screen

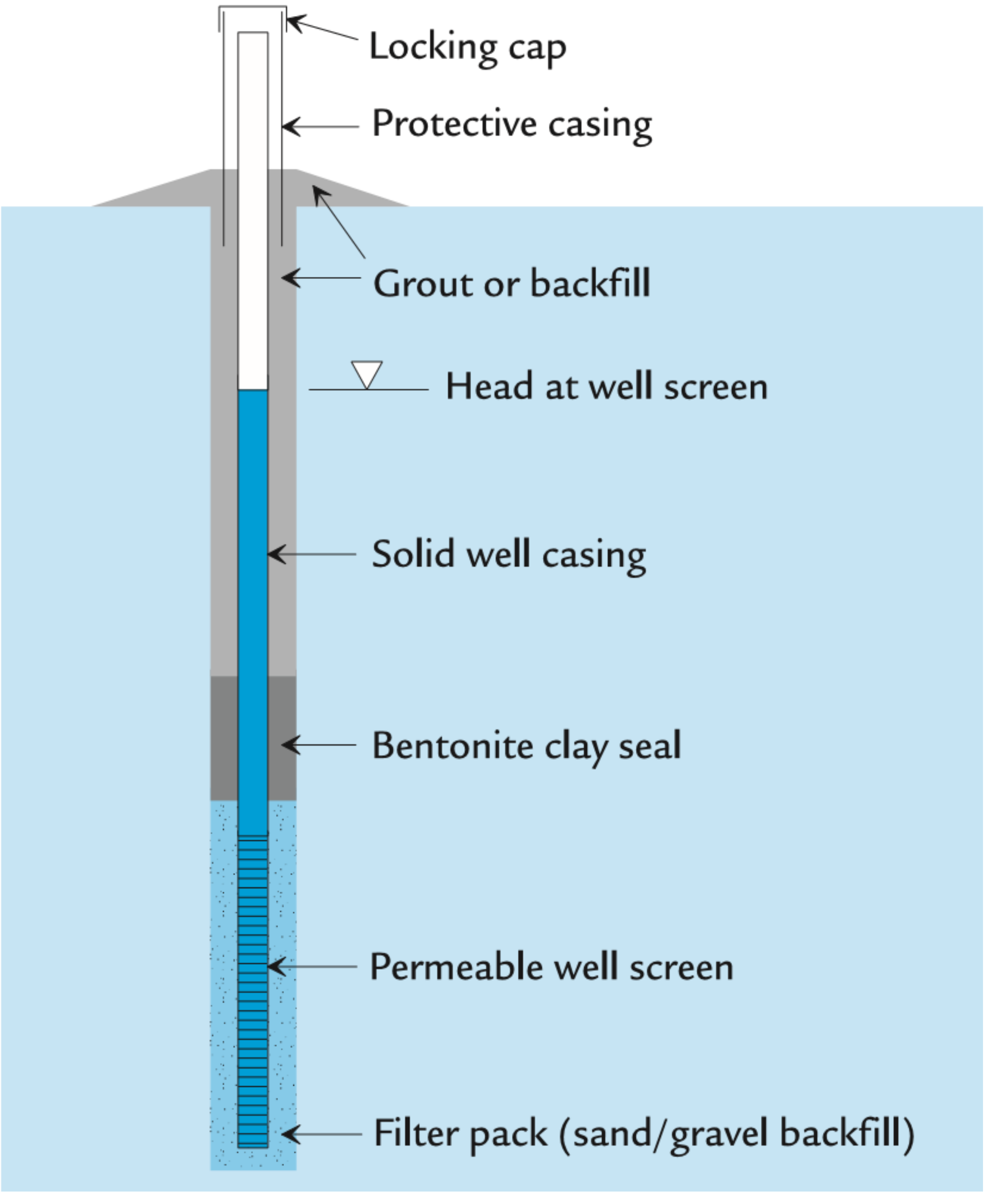
- (prevents the well screen from becoming clogged)

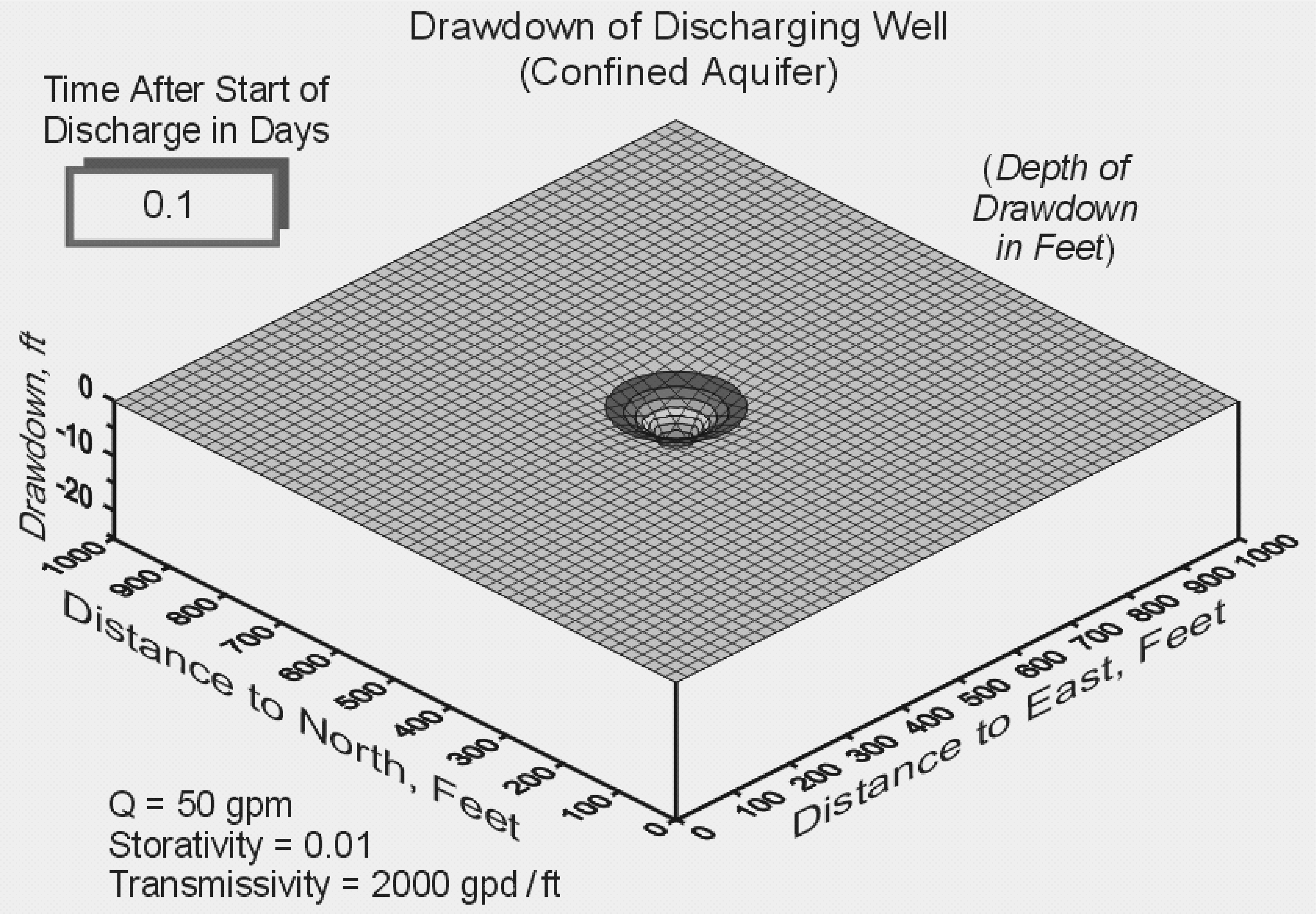
Casing
PVC or Steel

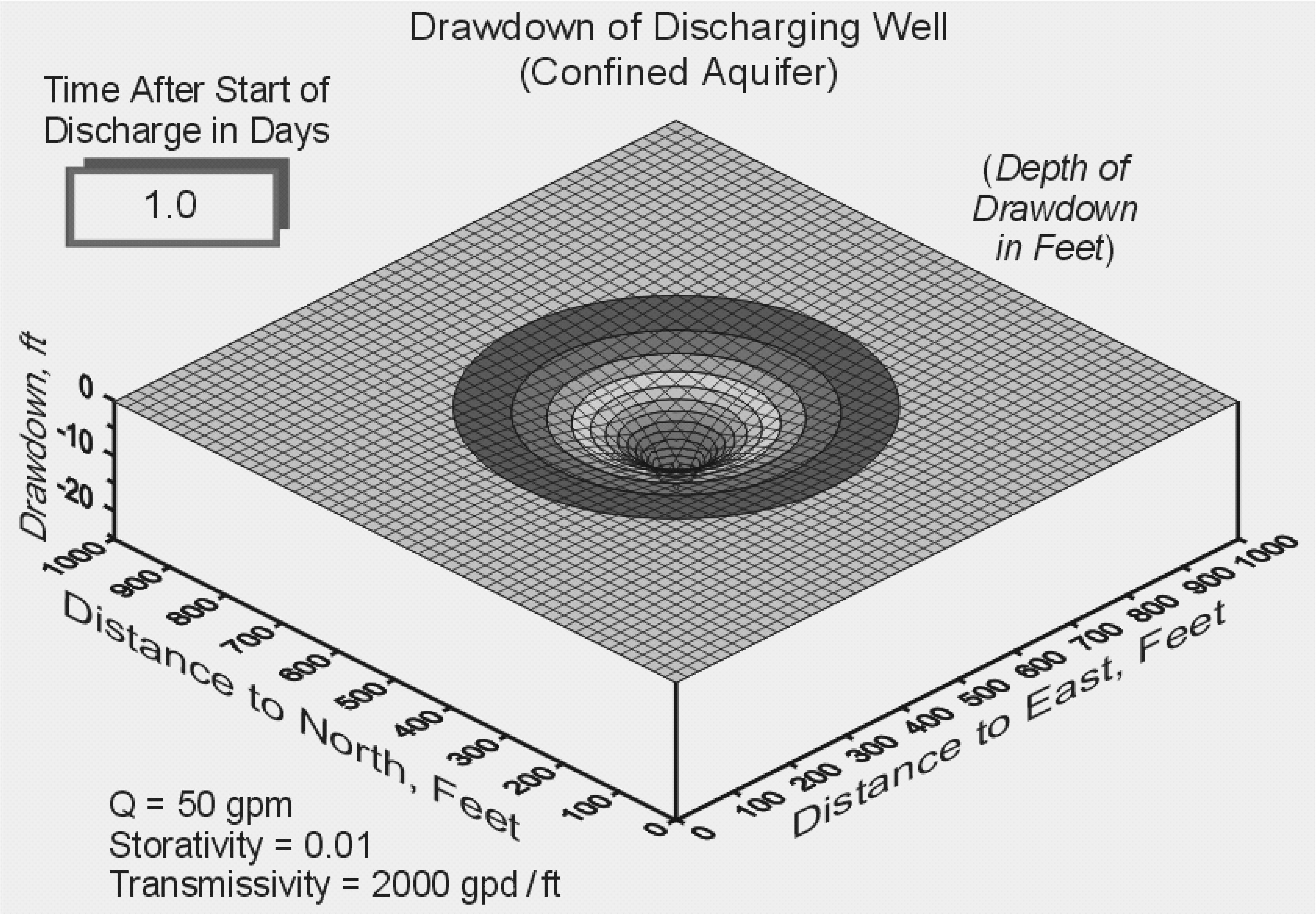
(prevents the well from collapse)

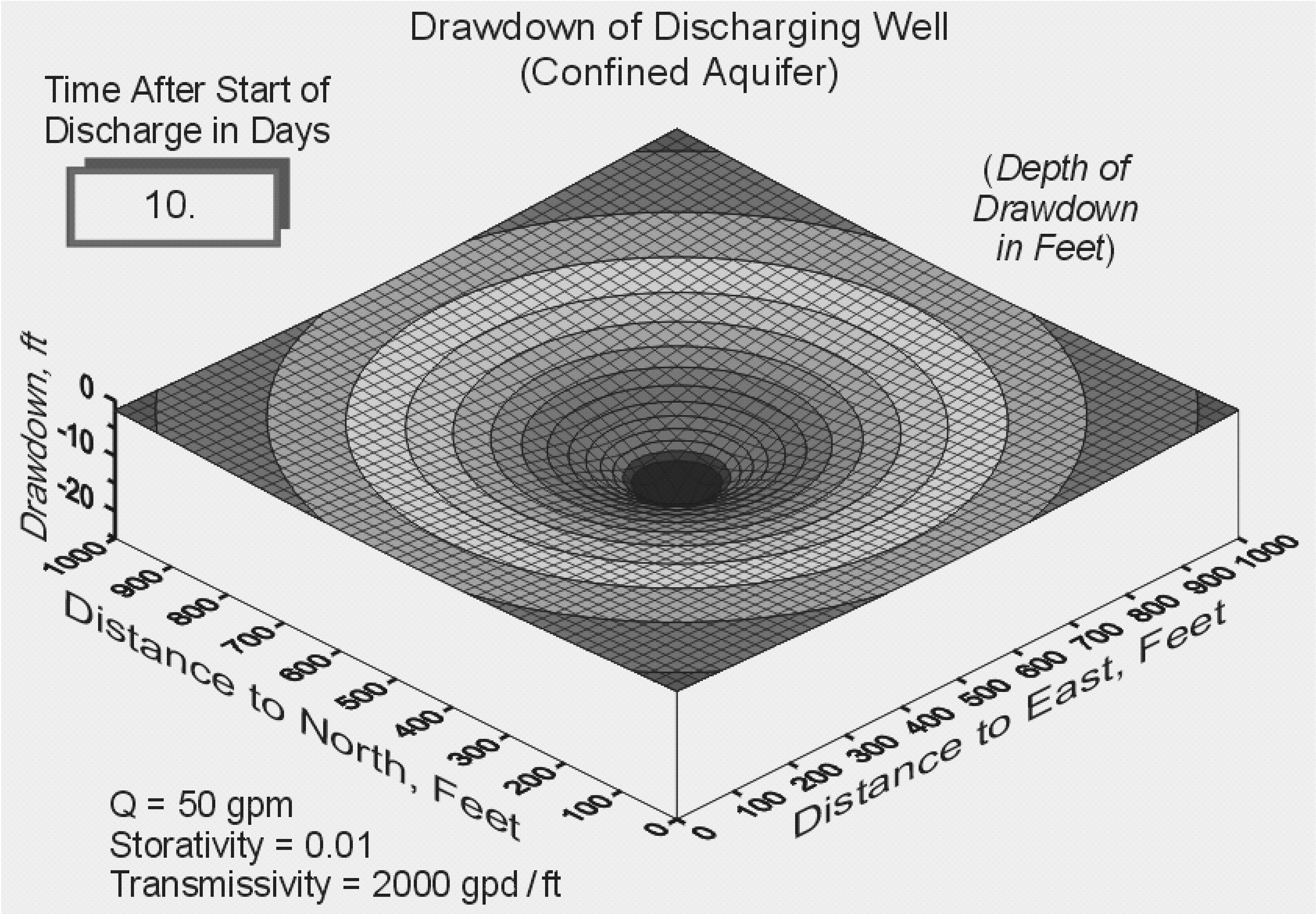
Seal
Grout, bentonite, cement

Well development
purge and surge
pump till clean




Drawdown of Discharging Well
(Confined Aquifer)
Time After Start of Discharge in Days
0.1
(Depth of Drawdown in Feet)
Drawdown, ft
0
-10
-20
1000
900
800
700
600
500
400
300
200
100
0
Distance to North, Feet
0
100
200
300
400
500
600
700
800
900
1000
Distance to East, Feet
Q = 50 gpm
Storativity = 0.01
Transmissivity = 2000 gpd / ft


Drawdown of Discharging Well
(Confined Aquifer)
Time After Start of Discharge in Days
1.0
(Depth of Drawdown in Feet)
Drawdown, ft
0
-10
-20
1000
900
800
700
600
500
400
300
200
100
0
Distance to North, Feet
0
100
200
300
400
500
600
700
800
900
1000
Distance to East, Feet
Q = 50 gpm
Storativity = 0.01
Transmissivity = 2000 gpd / ft


Drawdown of Discharging Well
(Confined Aquifer)
Time After Start of
Discharge in Days
10.
(Depth of
Drawdown
in Feet)
Drawdown, ft
0
-10
-20
1000
900
800
700
600
500
400
300
200
100
0
Distance to North, Feet
0
100
200
300
400
500
600
700
800
900
1000
Distance to East, Feet
Q = 50 gpm
Storativity = 0.01
Transmissivity = 2000 gpd / ft

AQUIFER CHARACTERISTICS

PUMP TESTS allow estimation of transmission and storage characteristics of aquifers

- Transmissivity ($T = Kb$) is the rate of flow through a vertical strip of aquifer (thickness b) of unit width under a unit hydraulic gradient
- Storage Coefficient ($S = S_y + S_s b$) is storage change per unit volume of aquifer per unit change in head
- Radius of Influence (R) for a well is the maximum horizontal extent of **the cone of depression** when the well is in equilibrium with inflows

BASIC ASSUMPTIONS

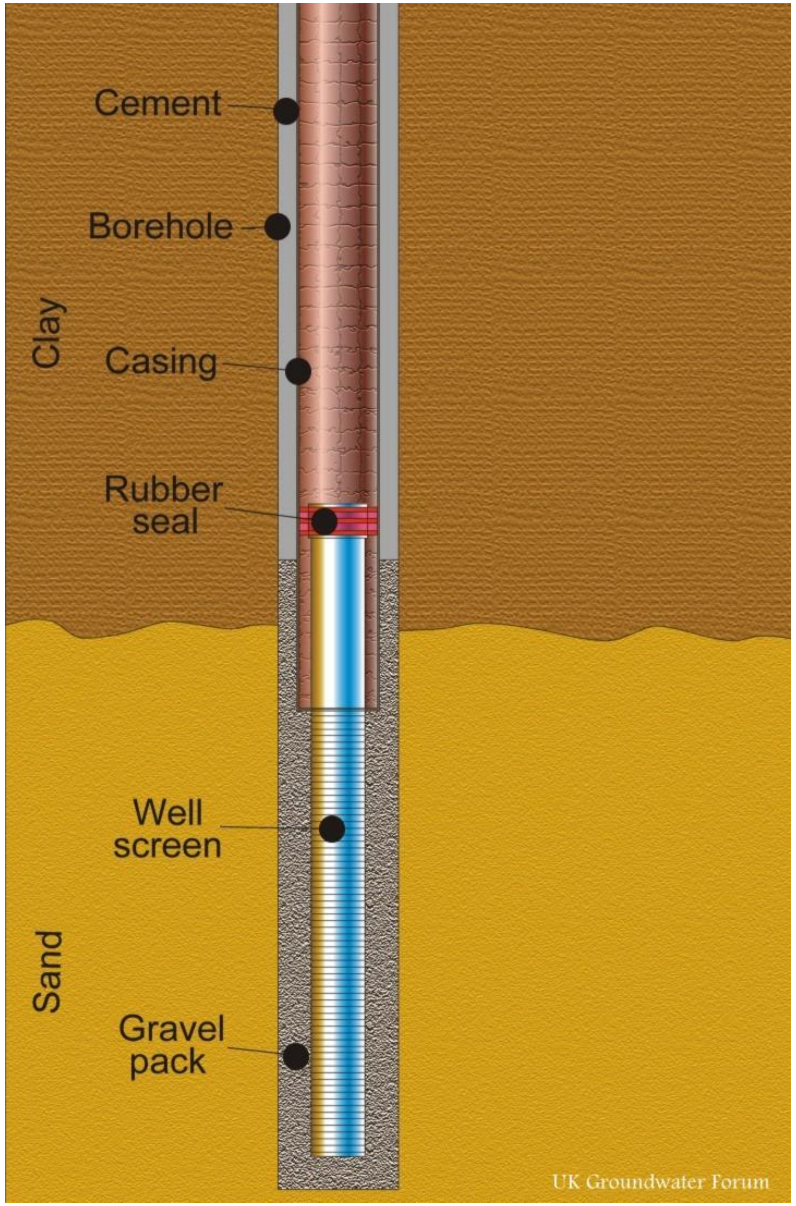
- The aquifer is **homogeneous** and **isotropic**.
- All **flow is radial** toward the well.
- Ground water flow is **horizontal.**
- **Darcy's law** is valid.
- Ground water has a **constant density** and **viscosity.**
- The pumping well and the observation wells are **fully penetrating** aquifer.
- The pumping well has an **infinitesimal diameter** and is 100% efficient.

WELL COMPONENTS

Borehole diameter
Depth and length of screen
Filter pack
Seal necessary

BOREHOLE DIAMETERS

Piezometers: 2.5 – 5 cm
Monitoring wells: 5 – 20 cm
Domestic supply: 10 – 40 cm
Public water supply: 20 cm



STEADY FLOW - WELL

PUMP TEST PLANNING

- Pump tests will not produce satisfactory estimates of either aquifer properties or well performance unless the data collection system is carefully is addressed in the design.
- Several preliminary estimates are needed to design a successful test:
  - Estimate the maximum drawdown at the pumped well
  - Estimate the maximum pumping rate
  - Evaluate the best method to measure the pumped volumes
  - Plan discharge of pumped volumes distant from the well
  - Estimate drawdowns at observation wells
  - Simulate the test before it is conducted
  - Measure all initial heads several times to ensure that steady-conditions prevail
  - Survey elevations of all well measurement reference points

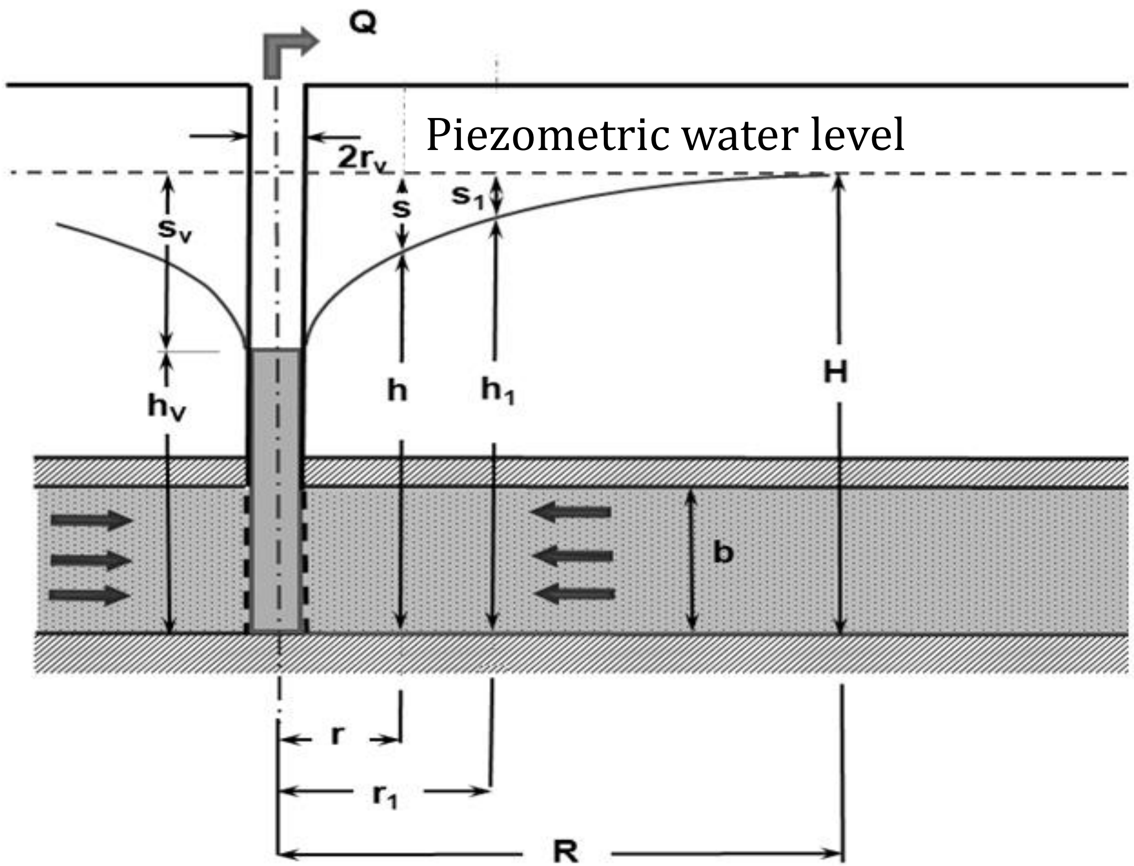
ADVANTAGES OF PUMPING TESTS

- Measure **parameters *in situ***.
- **Average parameters** over a large volume.
- **Measure T** and **S** simultaneously.

DISADVANTAGES OF PUMPING TESTS

- **High cost**.
- Non-uniqueness of T and S results.

STEADY RADIAL FLOW CONFINED FLOW



Hydraulic head:

h=z+p/γ

$$Q = S\,.\,v_r$$

$$S = 2\pi\, r\, b \qquad v_r = -K\frac{dh}{dr}$$

$$Q = S\,.\,v_r = 2\pi\, r\, b\, K\frac{dh}{dr}$$

Boundary condition

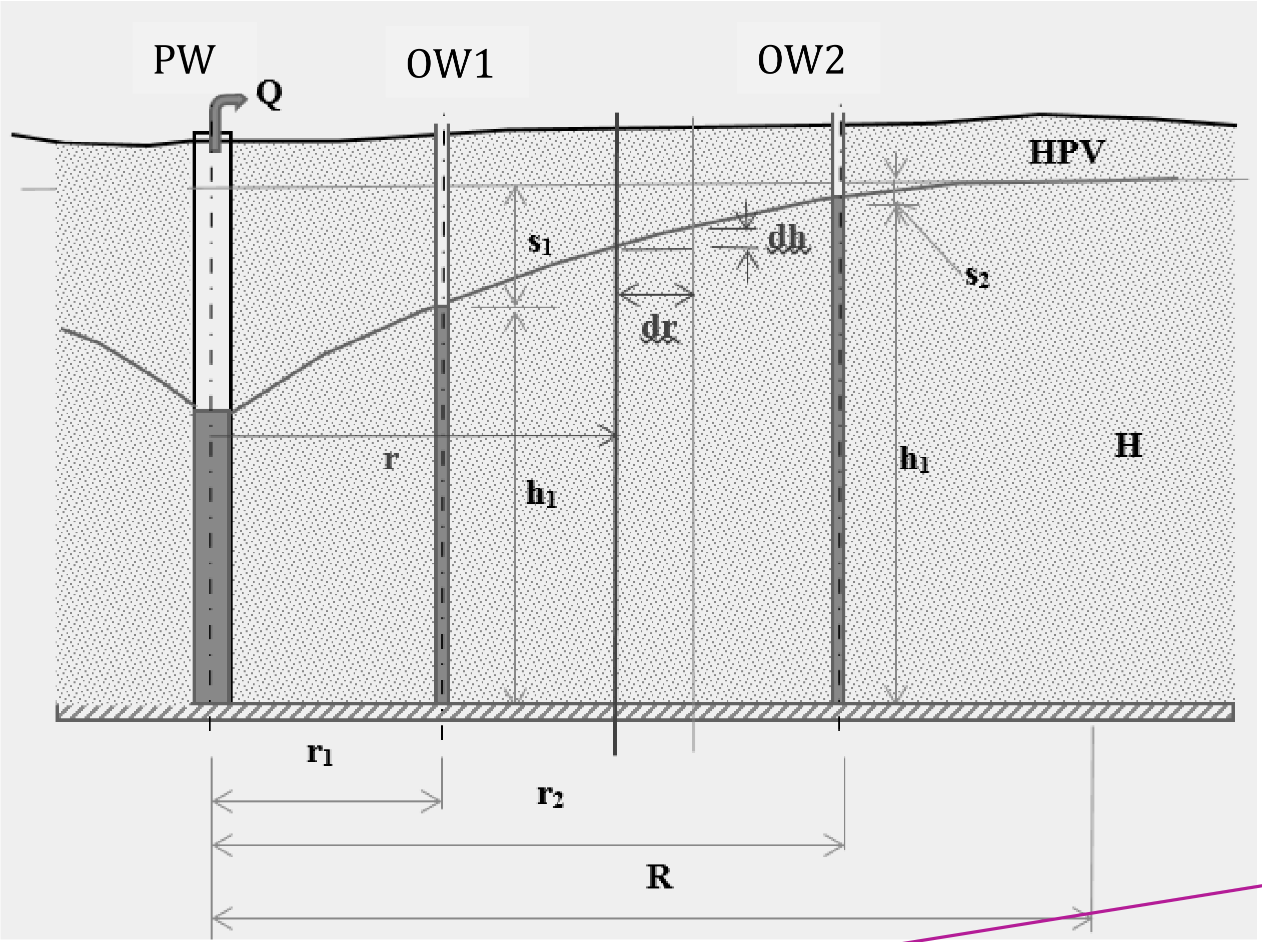
$$r = r_v \quad \ldots\ldots\, h = h_v$$

$$r = R \quad \ldots\ldots\, h = H$$

$$\int_{h_v}^{H} dh = \frac{Q}{2\pi K b}\int_{r_v}^{R}\frac{dr}{r} \quad \Rightarrow \quad H - h_v = s_v = \frac{Q}{2\pi K b}\ln\frac{R}{r_v}$$

This is the Thiem equation

STEADY UNCONFINED RADIAL FLOW



$$Q = S.v_r$$

$$S = 2.\pi r h$$

$$v_r = -K.I = -K.\frac{dh}{dr}$$

$$Q = 2.\pi r h K \frac{dh}{dr}$$

$$\int_{h_V}^{H} h\,dh = \frac{Q}{2\pi K}\int_{r_V}^{R}\frac{dr}{r}$$

$$\left[\frac{h^2}{2}\right]_{h_V}^{H} = \frac{Q}{2\pi K}\left[\ln r\right]_{r_V}^{R} \Rightarrow H^2 - h_V^2 = \frac{Q}{\pi K}\ln\frac{R}{r_V}$$

This is the Thiem equation

SUMMER SEMESTER

Lectures:

April, 9   - Well hydraulics

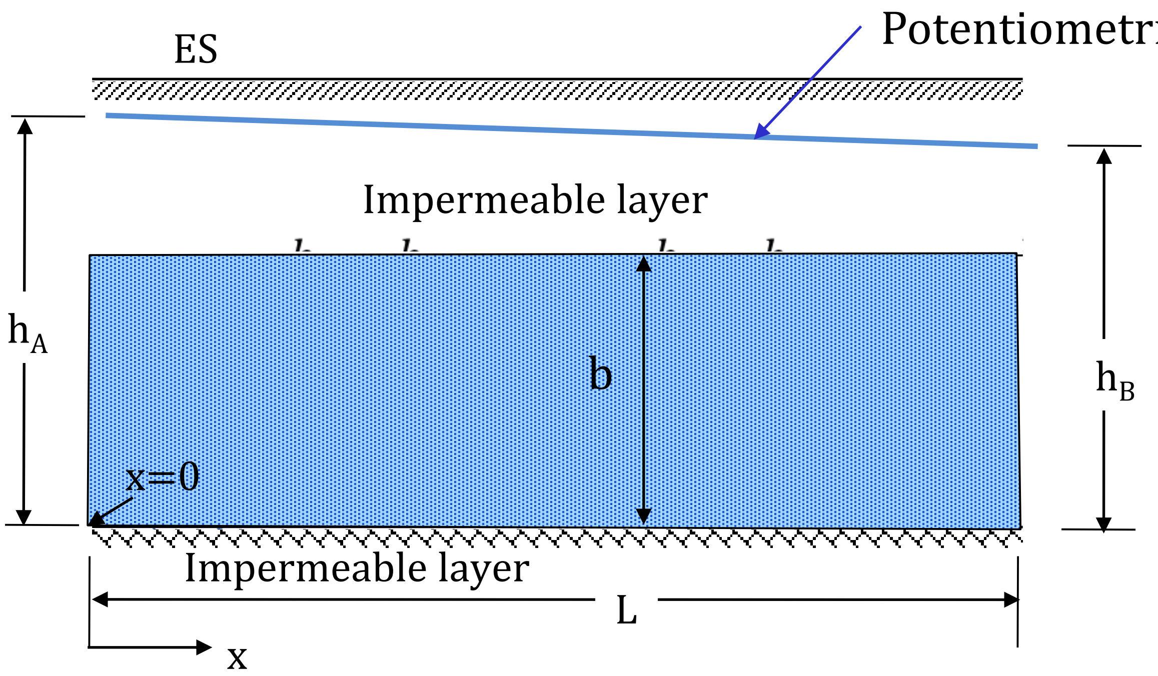
April, 16 - ?? Test (1 hr)

April, 23 – external lecturer (attendance + 5 points)
+ results from test
after the lecture I will write you the results

April, 29 – Test (1 hr)

SEEPAGE – 1D

GROUNDWATER FLOW IN A CONFINED AQUIFER STEADY FLOW



Darcy´s equation

$$Q = K.A\frac{h_A - h_B}{L}$$

Hydraulic gradient

$$\mathrm{I} = \frac{dh}{dx}$$

In case of 1-D GW flow in an isotropic, homogeneous aquifer, the flow rate per unit width (q) is

$$\mathrm{q} = -Kb\frac{dh}{dx}$$

Eq. can be rewritten in the form

$$\mathrm{dh} = \frac{q}{bK}\,dx$$

This equation can be integrated

$$\int_{h_A}^{h_B} dh = -\frac{q}{bK}\int_{0}^{L} dx$$

Boundary conditions

x = 0 ………. x=L
$\mathrm{h} = \mathrm{h}_A$………. $\mathrm{h} = \mathrm{h}_B$

$$h_B - h_A = -\frac{q}{bK}\,L$$

The flow rate per unit width is

$$q = K\ b\ \frac{h_A - h_B}{L}$$

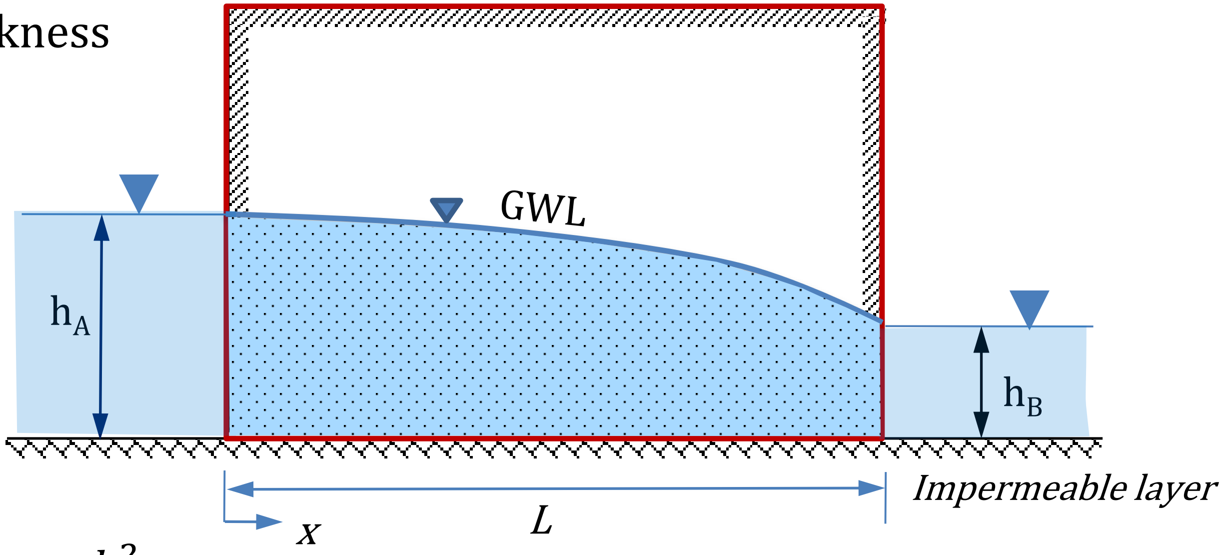
GROUNDWATER FLOW IN AN UNCONFINED AQUIFER

With the Dupuit asumprions, the flow per unit thickness

$$q = v\,h(x) = (-K\frac{dh}{dx})\,h(x)$$

Po úpravě

$$q\,dx = -K\,h(x)\,dh$$



Integration - **$x = 0$ $h=h_A$ $x=x$ $h(x)=h$**

$$q\int_{x=0}^{x} dx = -K\int_{h=h_A}^{h} h\,dh \Longrightarrow q\,x = K\frac{h_A^2 - h^2}{2}$$

Then for GWL

$$h^2 = h_A^2 - \frac{2qx}{K} \Longrightarrow h = \sqrt{h_A^2 - \frac{2qx}{K}}$$

And specific discharge $x = L$ $h = h_B$

$$q\,x = K\frac{h_A^2 - h^2}{2}$$

L, h_B

$$q = K\frac{h_A^2 - h_B^2}{2L}$$

For GWL

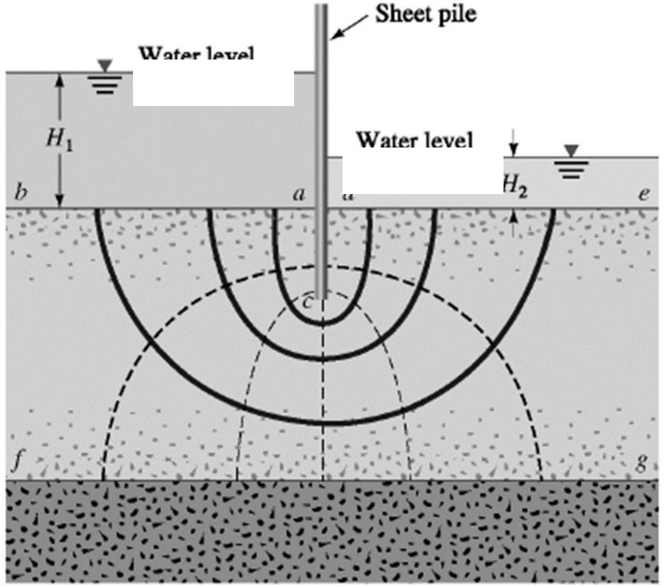
$$h^2(x) = h_A^2 + (h_B^2 - h_A^2)\frac{x}{L}$$

SEEPAGE – 2D FLOW NETS

LAPLACE EQUATION

Darcy eq. for anisotropic porous media

$$v_x = -K_x \frac{\partial h}{\partial x} \quad v_y = -K_y \frac{\partial h}{\partial y} \quad v_z = -K_z \frac{\partial h}{\partial z}$$



From continuity equation

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0$$

With Darcy eq.:

$$\frac{\partial}{\partial x}\left(K_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y \frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z \frac{\partial h}{\partial z}\right) = 0$$

If soil is isotropic ....$K_x = K_y = K_z = K$

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0$$

$\nabla^2 h = 0$ **... LAPLACE EQUATION**

2-D SEEPAGE – LAPLACE EQUATION

Laplace eguation is a very important equation in engineering
It represents loss of fluid flow through porous medium

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0$$

Exat solution of Laplace´s equation for 2-D seepage can be obtained for cases with **simple boundary conditions**
For most practical geotechnical problems, it is simpler to solve this equation graphically by drawing **FLOW NETS.**

It is assumed:

- the soil is **homogeneous** and **isotropic** with respect to permeability
- The pore fluid is incompressible

SEEPAGE TERMINOLOGY

- **FLOW NET** consists of two sets of curves – equipotentials and flow lines (stream lines) – that intersect each other at 90°
- Along an equipotentional, the total head is constant
- A pair of adjacent stream lines define a **FLOW CHANNEL** through which the rate of flow of pore fluid is constant
- The loss of head between two successive equipotentials is called the **EQUIPOTENTIAL DROP**

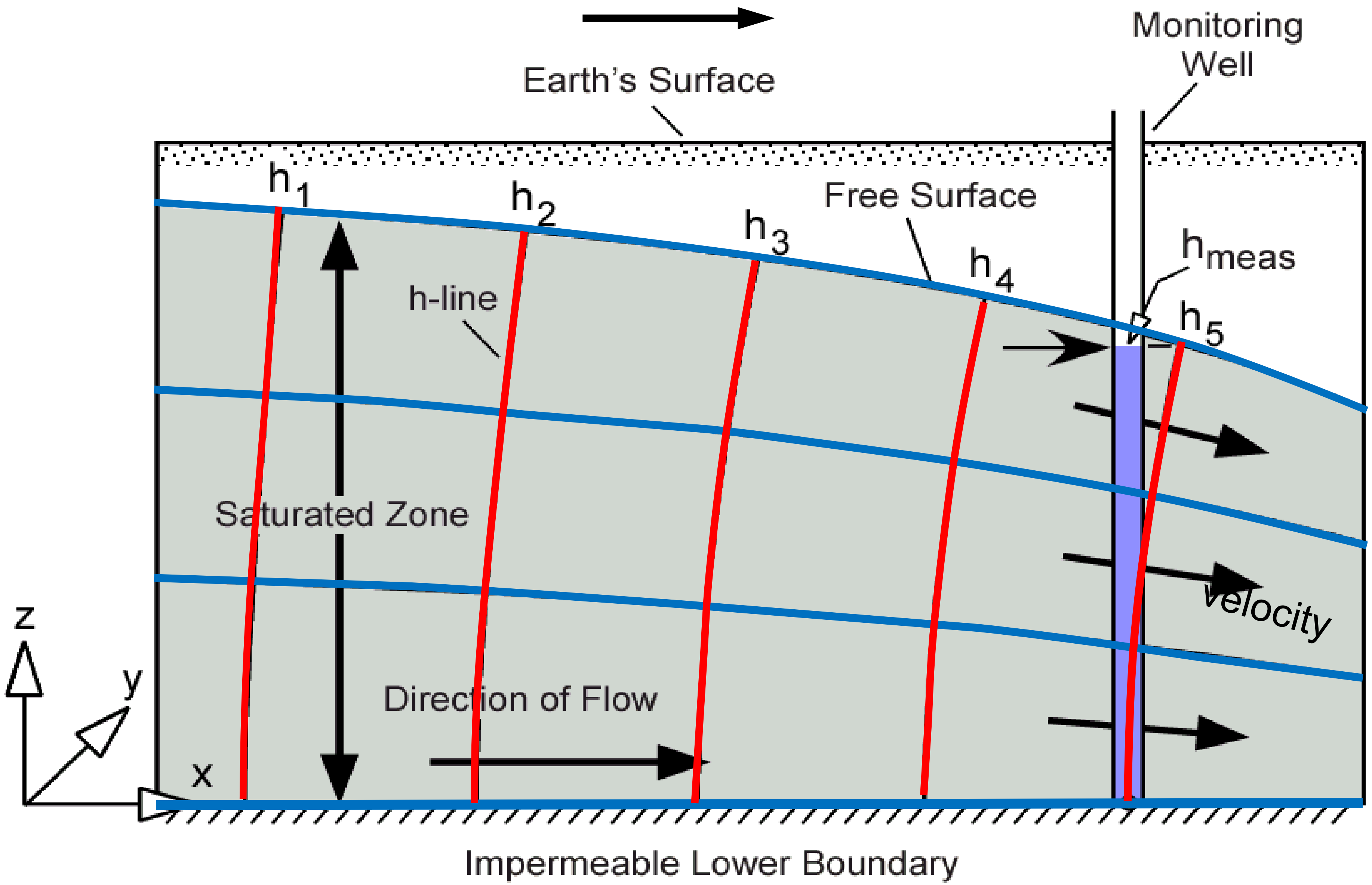
Stream line is simply the path of a water particle (molecule).

From upstream to downstream, **total head steadily decreases** along the stream line.

Actual

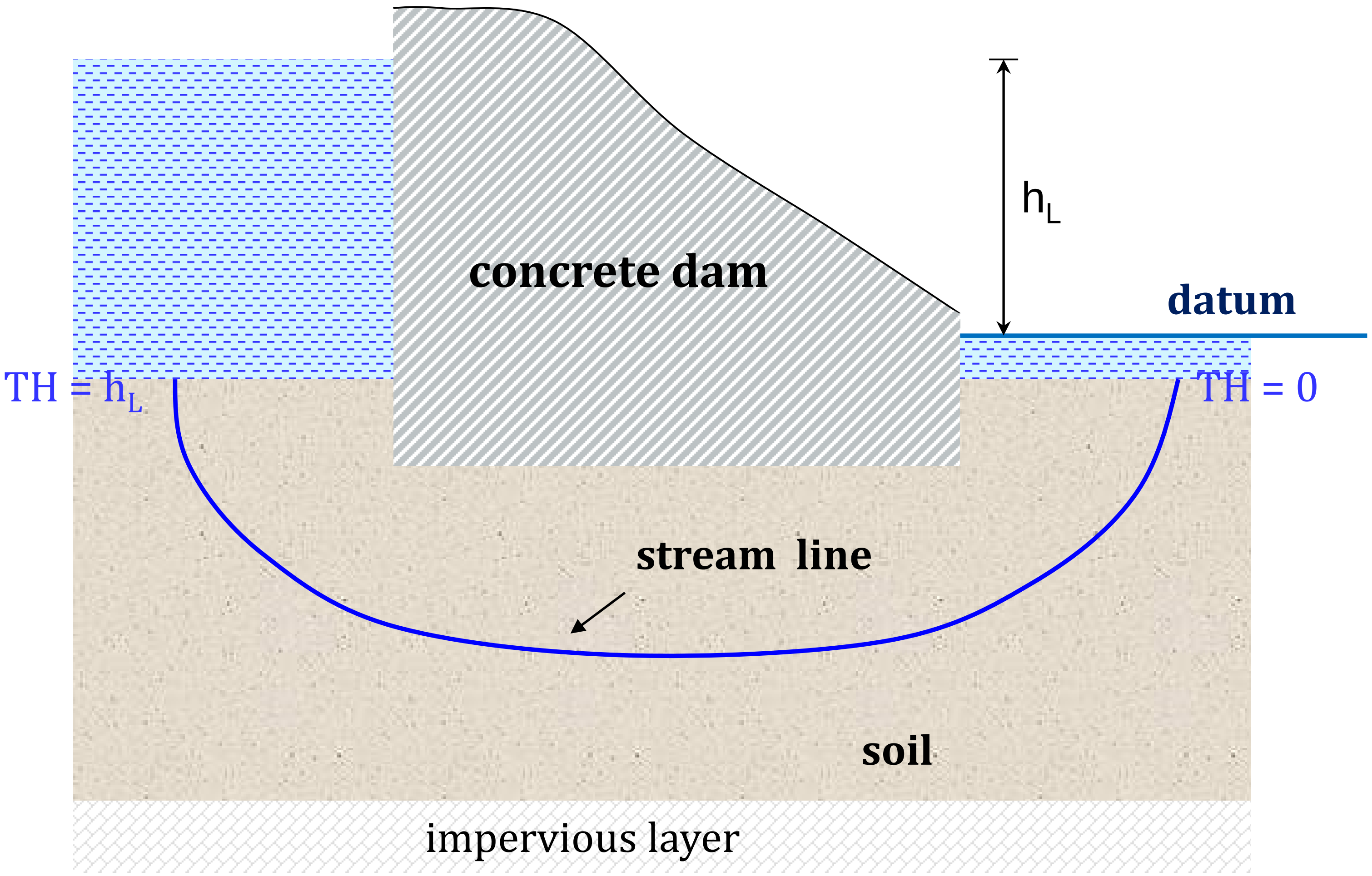
stream lines

equipotentional lines



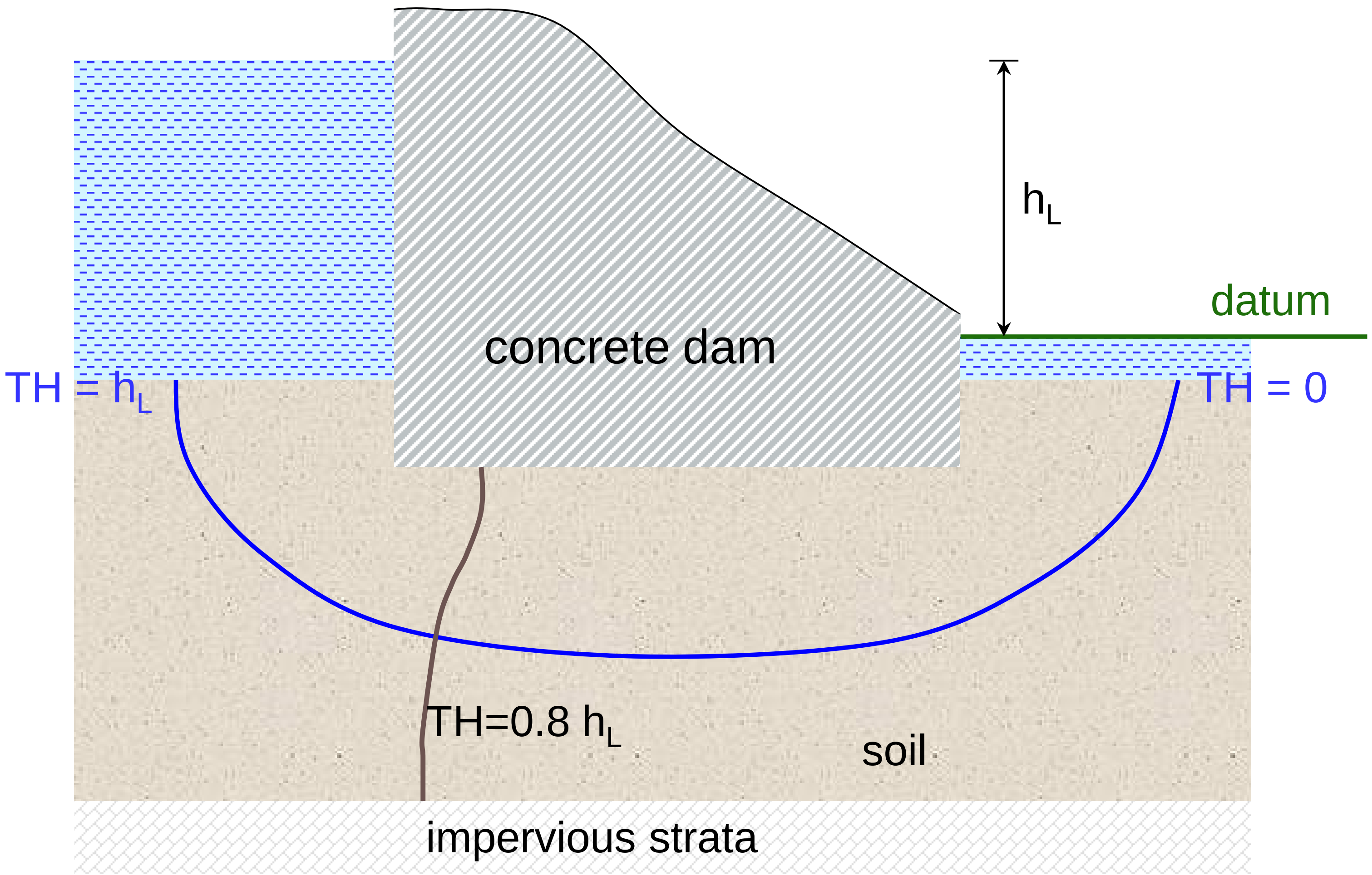
SEEPAGE – FLOW NETS

STREAM LINE



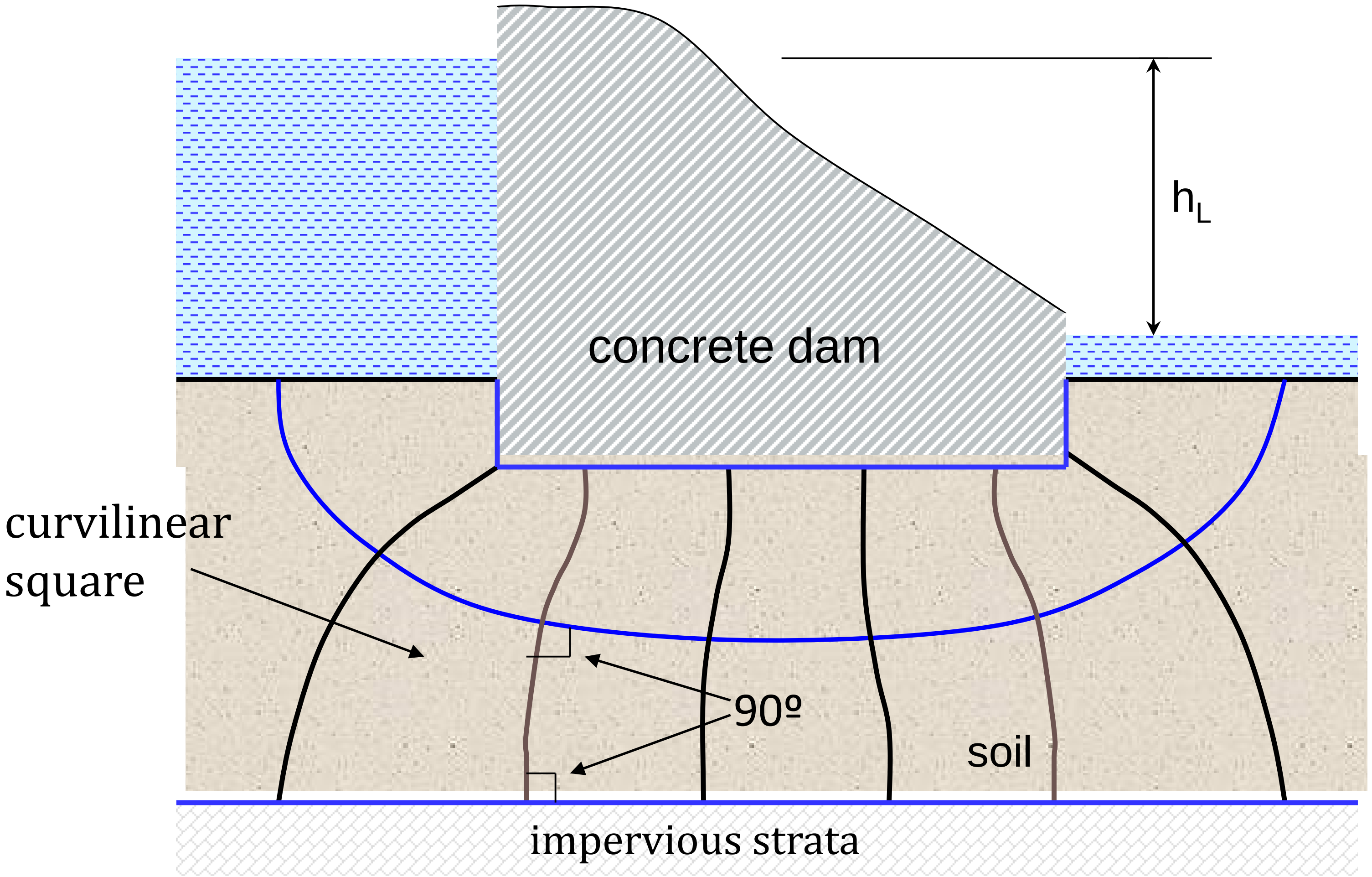
SEEPAGE – FLOW NETS

<u>EQUIPOTENTIAL LINE</u> is simply a contour of constant total head.



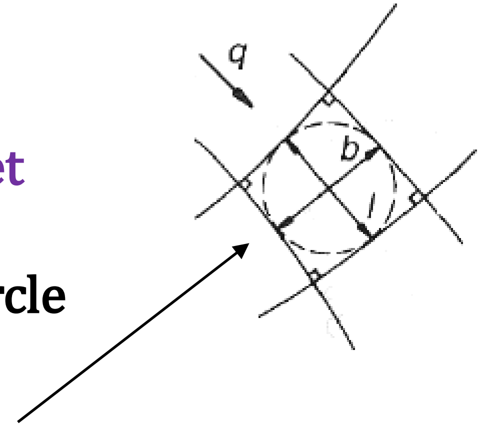
FLOWNET - a network of selected stream lines and equipotential lines.

A **flownet** is a grid obtained by drawing a series of streamlines and equipotential lines

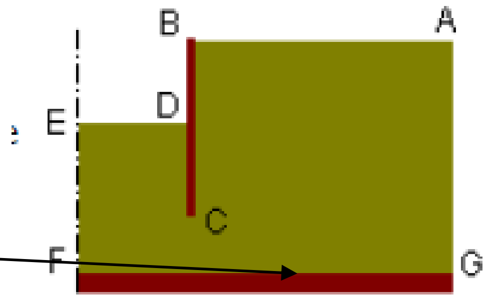


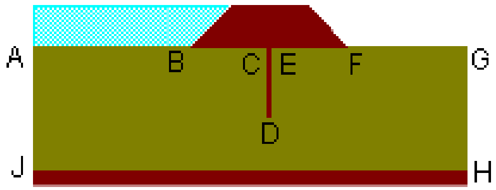
SKETCHING RULES

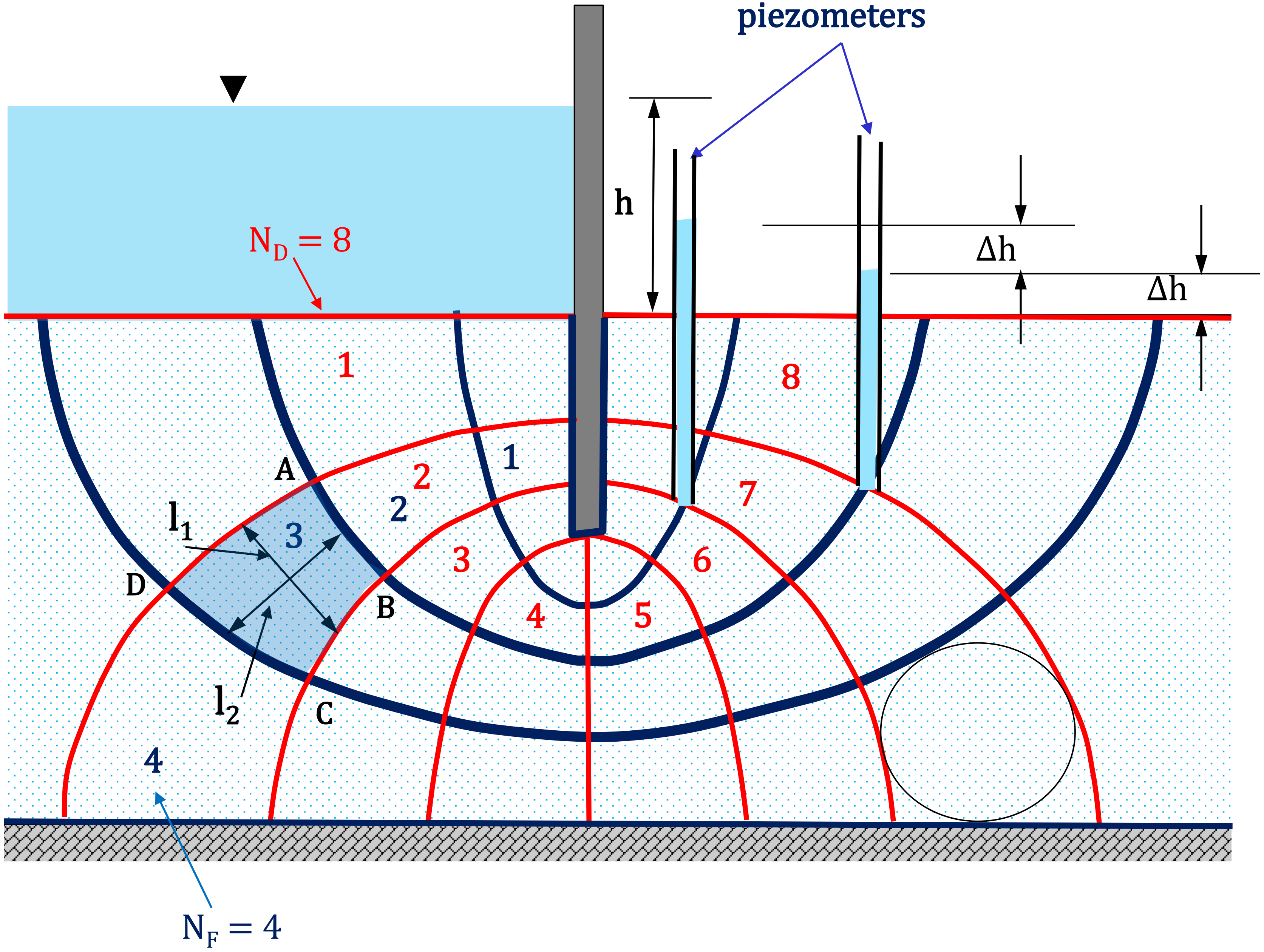
- Flow lines cross the equipotentials at right angles (by definition, there is no flow along an equipotentials and therehore, all of flow must be at 90° to it
- An equipotentials cannot cross other equipotentials (one point cannot have two different values of total head)
- Although an infinite number of flow lines could be sketched, the flow net must be constructed so that each element is as curvilinear square (sides may be curved…. **Curvilinear square** is as broad as it is long, so that a **circle** may be **inscribed** within it that touches all four of its sides



- Impermeable boudaries and lines of symetry are flow lines (**FG** is flow line)
- Bodies of water (such as reservoir) behind a dam, are equipotentials (**AB**)







Each interval between two equipotential corresponds to a head loss **Δh** equal to 1/$\mathbf{N_D}$

$$\Delta h = \frac{h}{N_D}$$

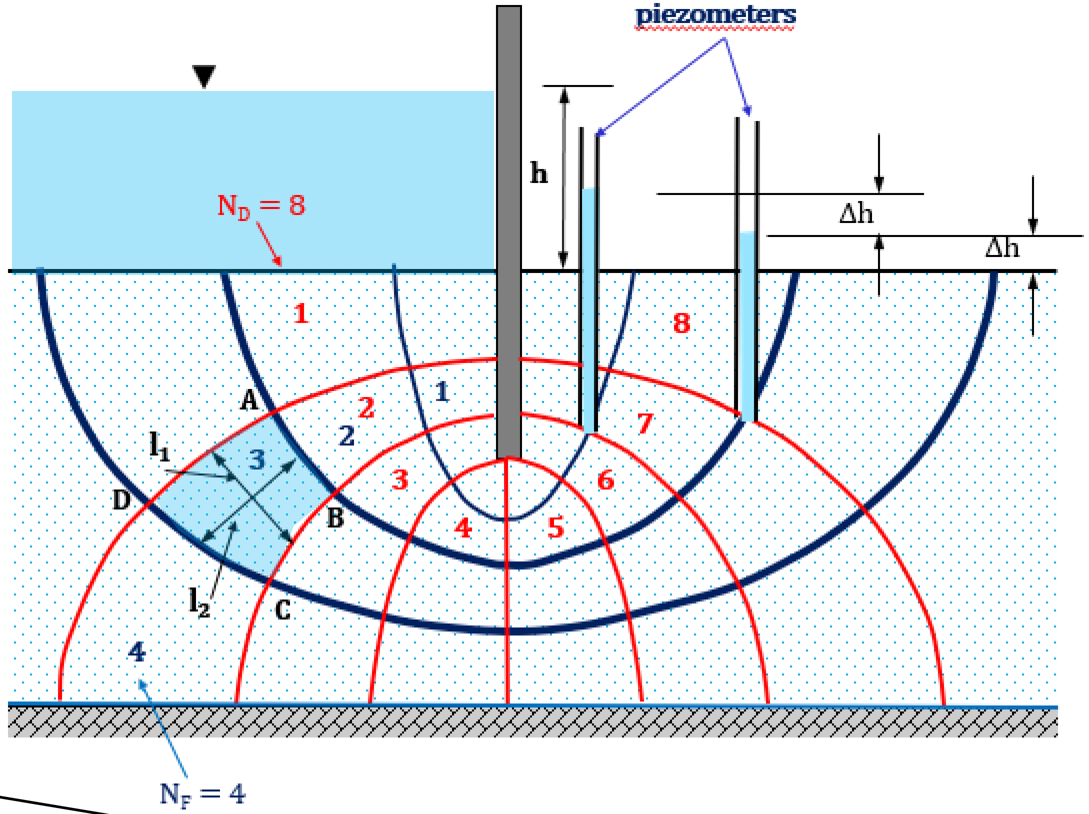
$\mathbf{N_D}$ – total number of **equipotentials drops**

N_f – number of flow tubes

CONSIDER A PAIR OF FLOW LINES

Let us consider the flow through abcd delimited by two flow lines and two equipotentials

$$\Delta h = \frac{h}{N_D}$$



The hydraulic gradient is :

$$i = \frac{\Delta h}{l_1} = \frac{h}{(N_D . l_1)}$$

where l_1 – distance between the two equipotentials;
l_2 – distance between the two flow lines

Applying Darcy´s law, the velocity in the tube ABCD is

$$v = K.i = K\frac{h}{(N_D . l_1)}$$

The flow passing into ABCD, per 1 m width of soil is

$$\Delta q_{ABCD} = Area.velocity = l_2 .1 .K.\frac{h}{N_D l_1}$$

If the FLOW NET $l_1 = l_2$ („squared mesh)

$$\Delta q_{ABCD} = K.\frac{h}{N_D}$$

Flow rate per unit width: $q = N_f \, \Delta q = K.\frac{h.N_f}{N_D}$

N_f – number of flow tubes

The total flow rate is: $Q = q.\mathrm{b}$

b is width perpendicular to the 2D seepage plane:

QUANTITY OF SEEPAGE (Q)

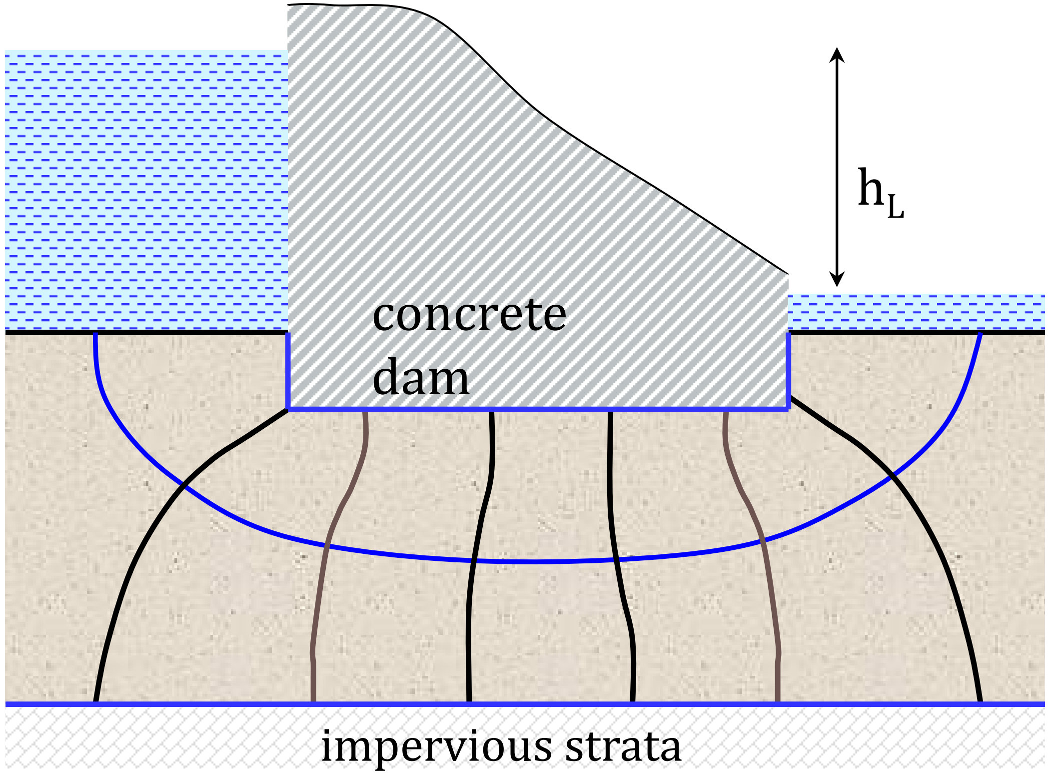
$$Q = kh_L \frac{N_f}{N_d}$$

number of flow channels

….per unit length normal to the plane

number of equipotential drops

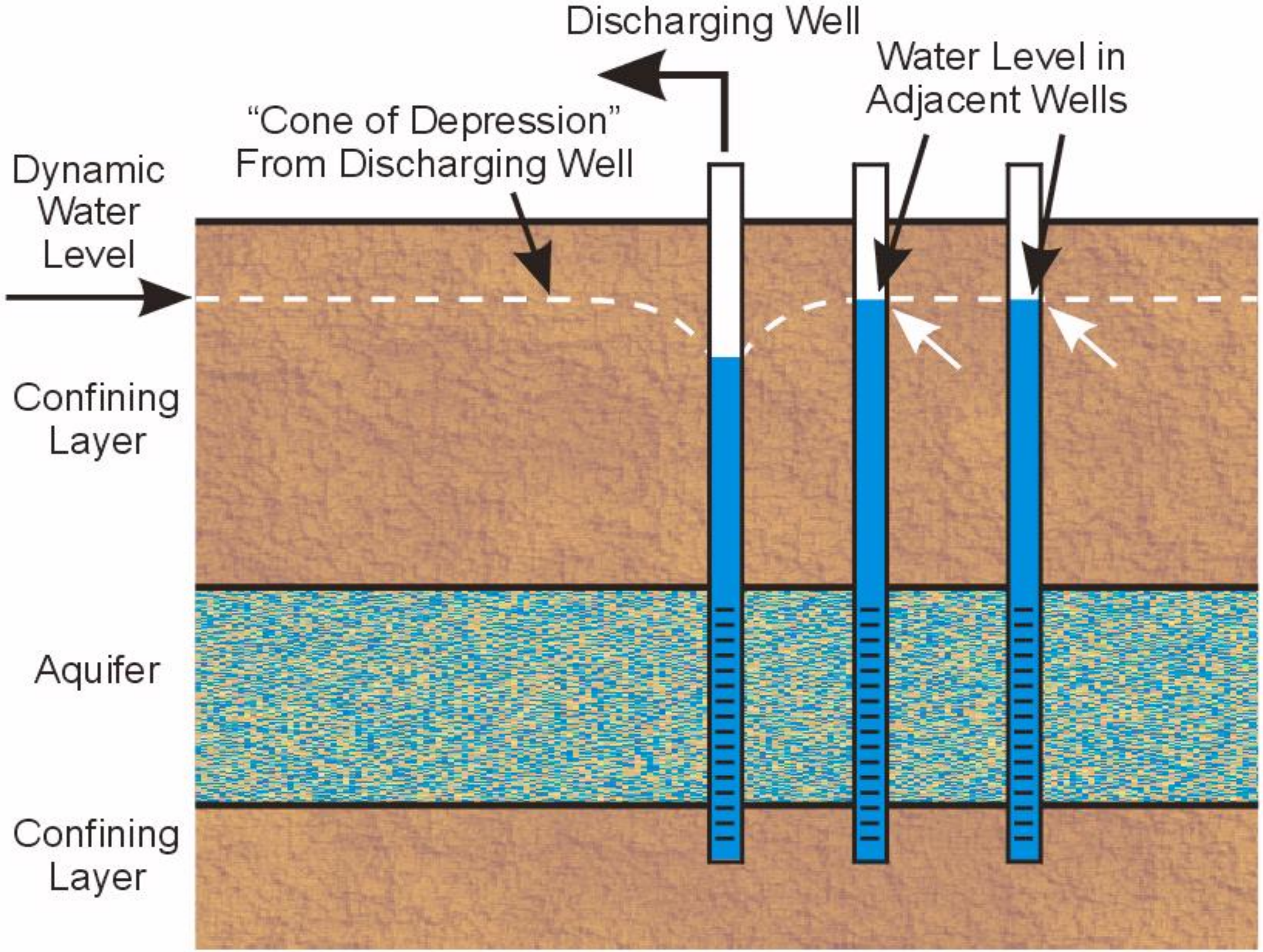
head loss from upstream to downstream



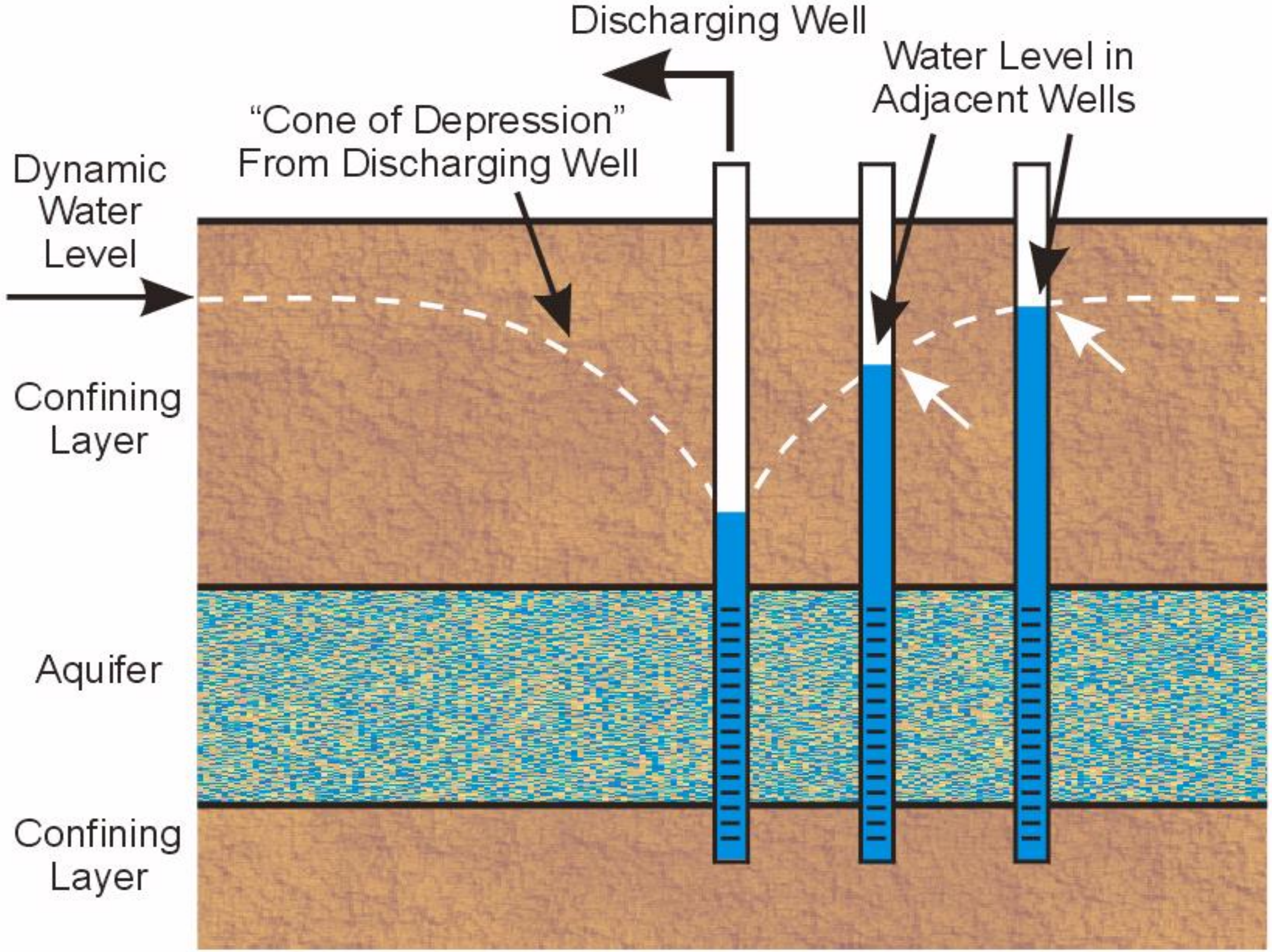
UNSTEADY FLOW TO A WELL IN A CONFINED AQUIFER

Theis method

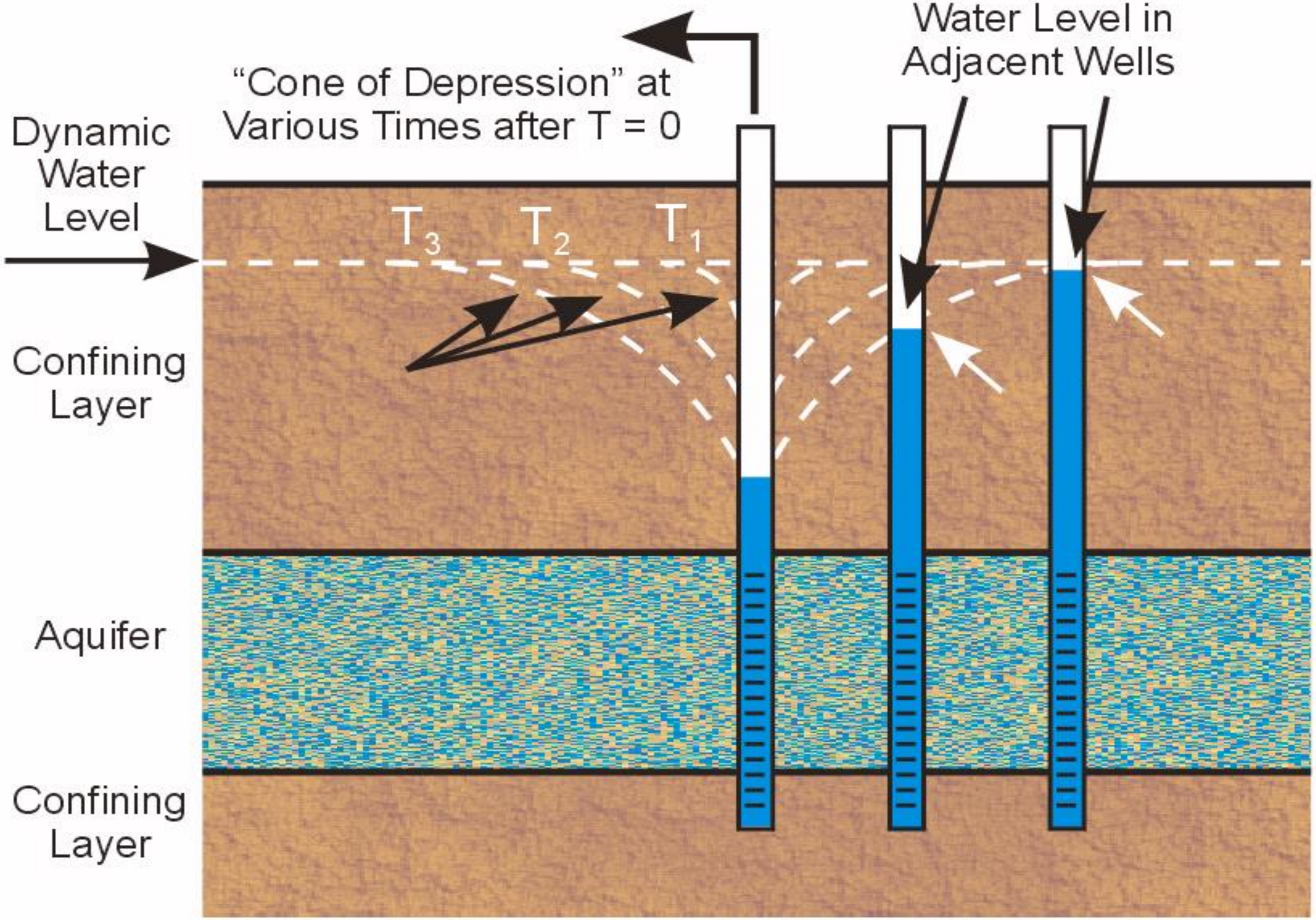
Jacob method



Water Levels in Confined Aquifer Adjacent to Discharging Well

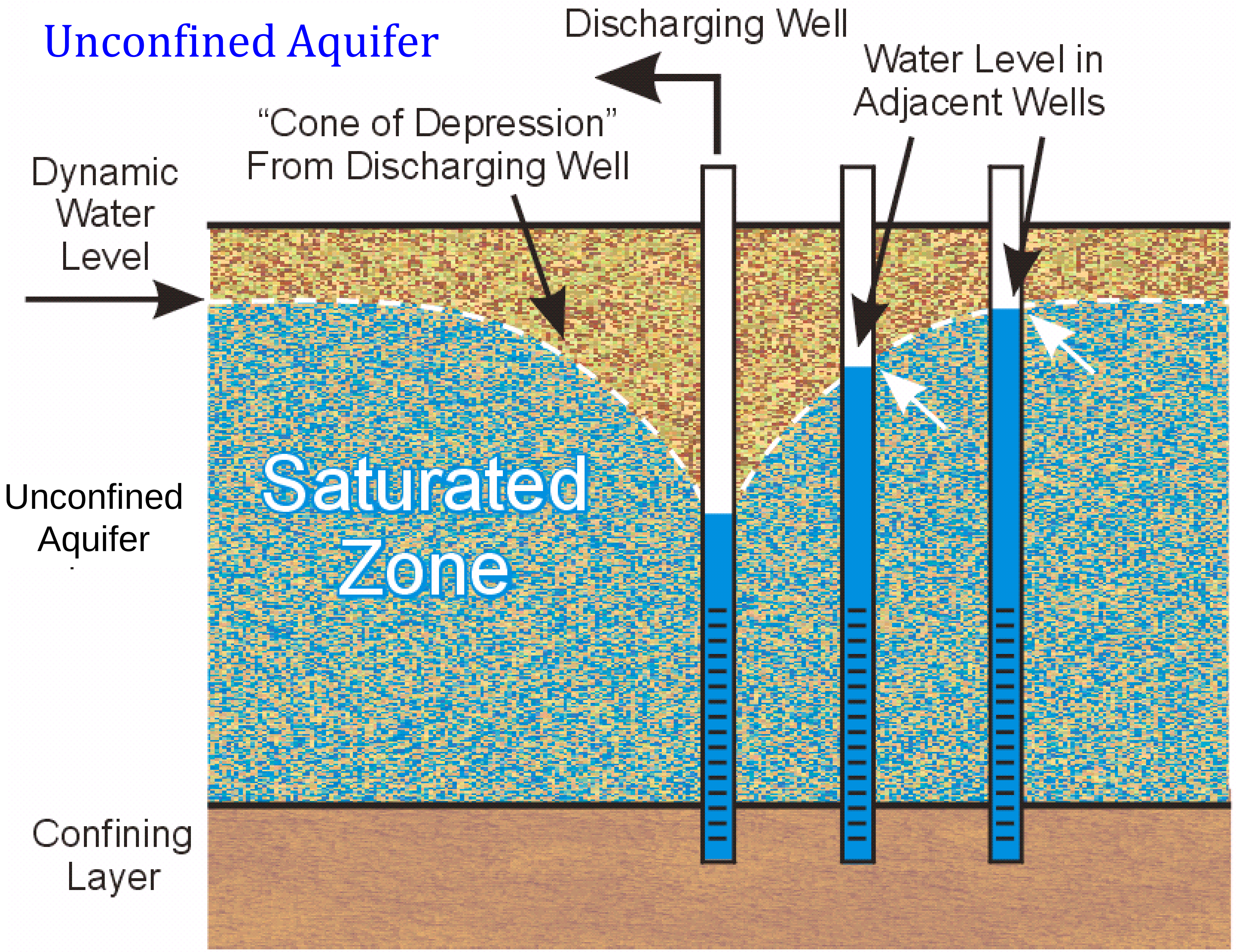


Water Levels in Confined Aquifer
Adjacent to Discharging Well



Water Levels in Confined Aquifer
Adjacent to Discharging Well

Unconfined Aquifer

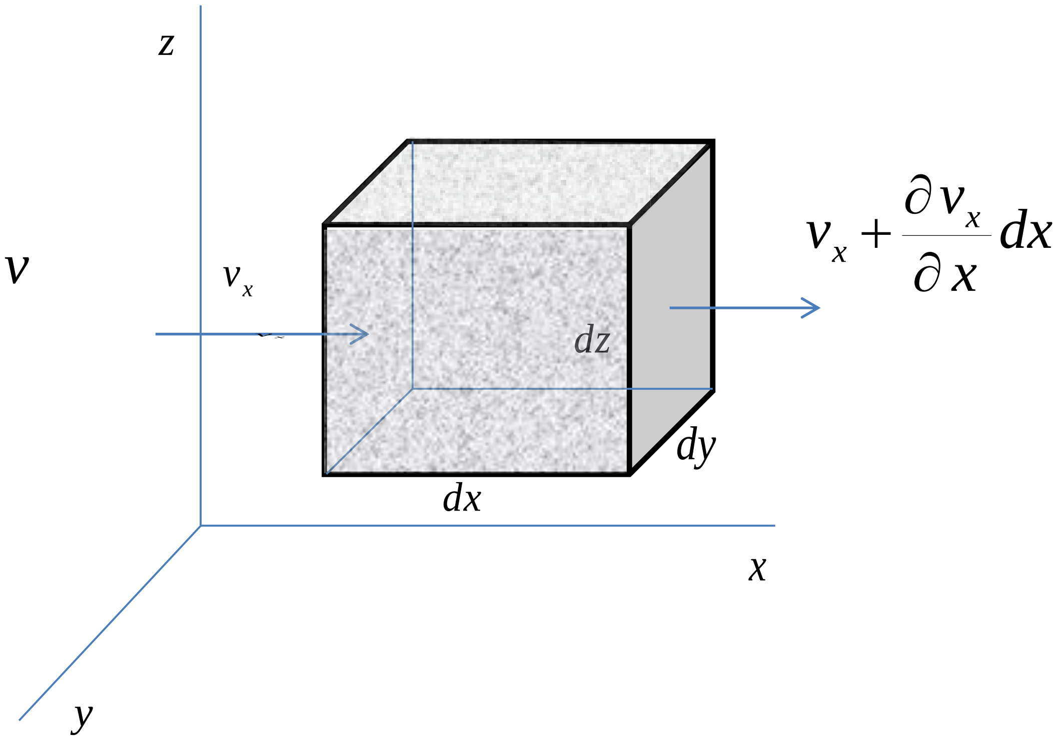


Effect of Discharging Water from a Well in an Unconfined Aquifer on the Adjacent Watertable

BASIC EQ. – UNSTEADY FLOW THROUGH POROUS MEDIA

Assumptions:
- confined aquifer
- Darcy eq.
- balance of mass
- homogeneous, isotropic

Input – output =0   mass ρv



Unsteady flow:

Inflow – outflow = change storage

BASIC EQ. – UNSTEADY FLOW CONFINED AQUIFER

For hydraulic head

$$\boxed{\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = \frac{S}{T}\frac{\partial h}{\partial t}}$$

For drawdown

$$\boxed{\frac{\partial^2 s}{\partial x^2} + \frac{\partial^2 s}{\partial y^2} + \frac{\partial^2 s}{\partial z^2} = \frac{S}{T}\frac{\partial s}{\partial t}}$$

RADIAL FLOW

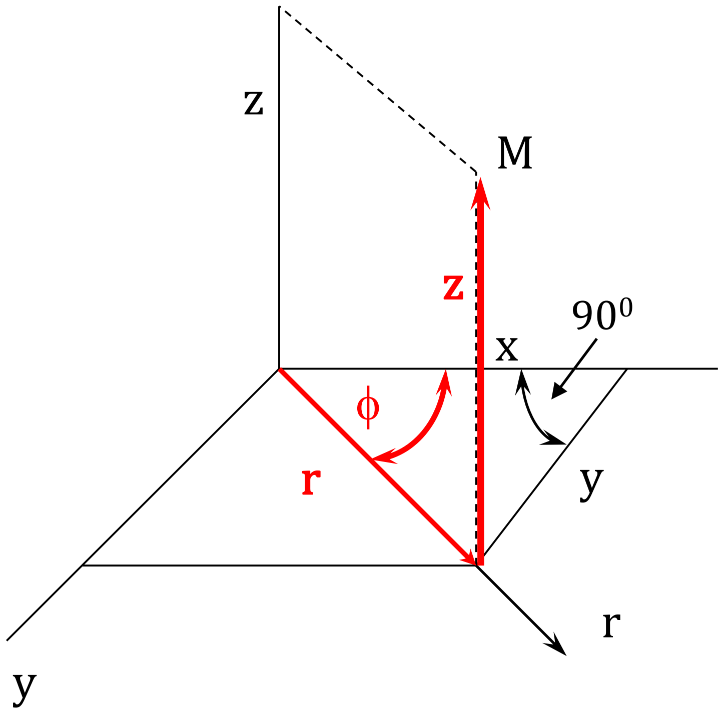
$$\frac{\partial^2 s}{\partial x^2}+\frac{\partial^2 s}{\partial y^2}+\frac{\partial^2 s}{\partial z^2}=\frac{S}{T}\frac{\partial s}{\partial t}$$

Polar coordinate system : r,ϕ,z

cos ϕ= x/r

sin ϕ= y/r

x= r.cos ϕ	$r = (x^2 + y^2)^{1/2}$
y= r.sin ϕ	tg ϕ = (y/x)
z=z	z=z



2D Flow in a confined aquifer

$$\frac{\partial^2 s}{\partial x^2}+\frac{\partial^2 s}{\partial y^2}=\frac{S}{T}\frac{\partial s}{\partial t}$$

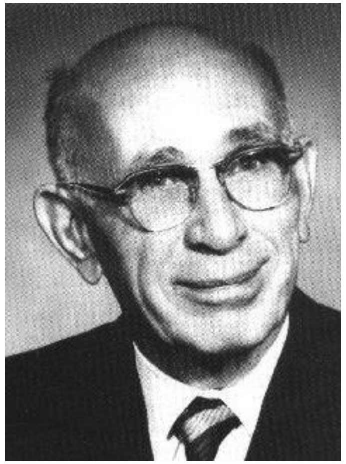
2D Flow in a confined aquifer in radial coordinates

$$\frac{\partial^2 s}{\partial r^2}+\frac{1}{r}\frac{\partial s}{\partial r}=\frac{S}{T}\frac{\partial s}{\partial t}$$

RADIAL FLOW TO A WELL

The solution of the governing equation of unsteady radial flow was solved by C.V. Theis in 1935

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r} = \frac{S}{T}\frac{\partial s}{\partial t}$$



s = drawdown [L]
H = initial head [L]
h = head at r at time t [L]
t = time since pumping began [T]
r = distance from pumping well [L]
Q = discharge rate [L^3/T]
T = transmissivity [L^2/T]
S = Storativity [-]

ASSUMPTIONS – THEIS SOLUTION:

- confined aquifer
- pumping rate Q = const.
- Darcy´s law is valid
- all flow is radial to well
- well is fully penetrating aquifer
- flow is horizontal
- piezometric heads -surface steady prior to pumping
- homogeneous, isotropic, infinite areal extent aquifer
- pumping well receives water from the entire thickness of the aquifer
- transmissivity is constant in space and time
- storativity is constant in space and time
- additional resistances at a well =0 (ideal well)
- well has infinitesimal diameter
- water removed from storage is discharged instantaneously

Basic equation:

$$\frac{\partial^2 s}{\partial r^2}+\frac{1}{r}\frac{\partial s}{\partial r}=\frac{S}{T}\frac{\partial s}{\partial t}$$

Theis solution is given as:

$$s(r,t)=\frac{Q}{4\pi T}(-E_i(-u))=\frac{Q}{4\pi T}\int_u^{\infty}\frac{e^{-u}}{u}du=\frac{Q}{4\pi T}W(u)$$

Where (-Ei(-u)) is written as W(u)

W(u) – Theis well function; u – parameter of well function (-)

$$u=\frac{r^2 S}{4Tt} \quad \Longrightarrow \quad \frac{1}{u}=\frac{4Tt}{r^2 S}$$

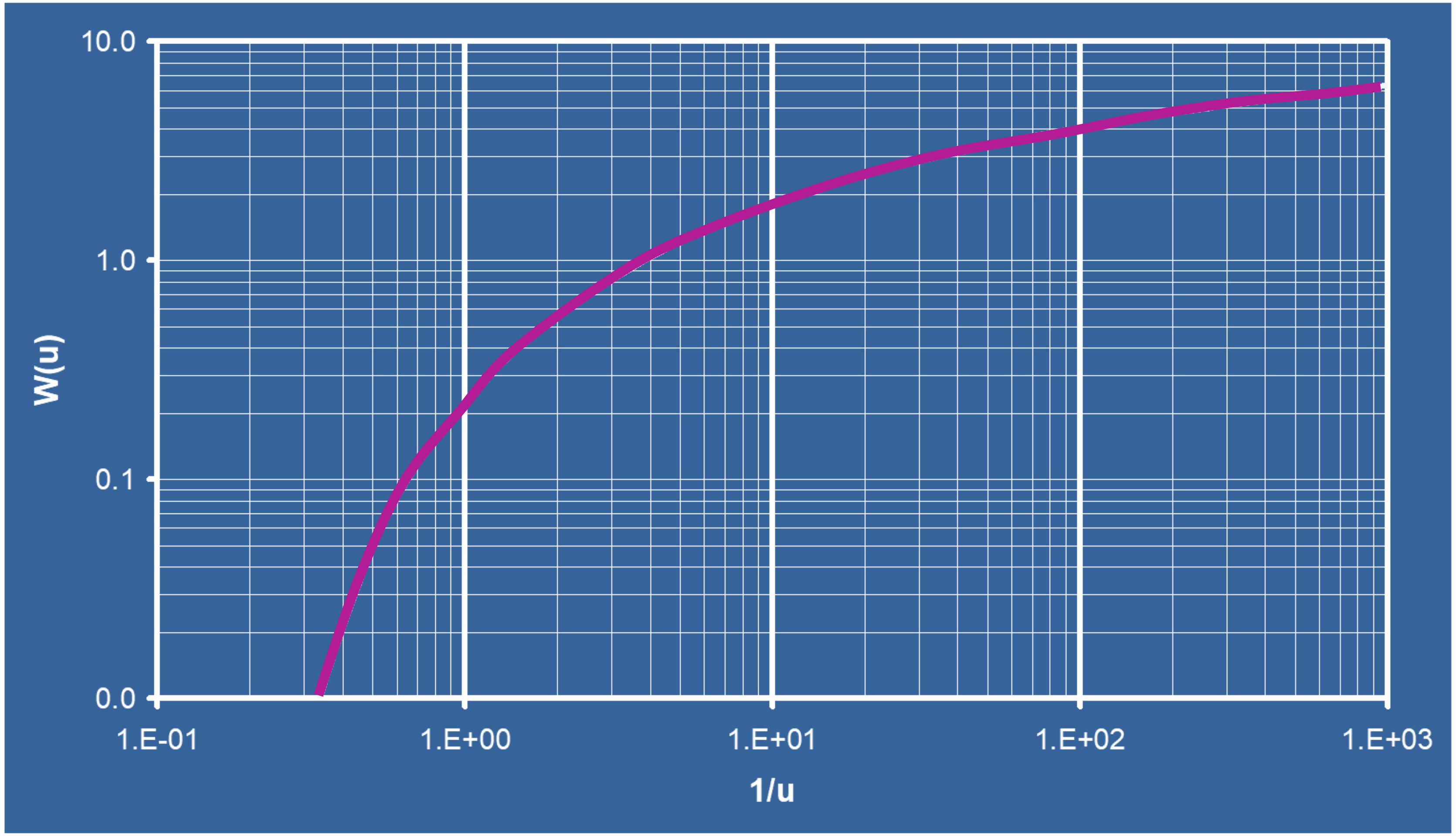
T – time [T]; r – radial distance [L];

$$W(u)=-\gamma-\ln u+u-\frac{u^2}{2.2!}+\frac{u^3}{3.3!}-\frac{u^4}{4.4!}+\ldots\ldots\ldots\ldots$$

γ – Euler´s number ... 0,577216

$$s(r,t)=\frac{Q}{4\pi T}W(u)$$

THEIS CURVE METHOD



W(u) versus 1/u on log-log paper

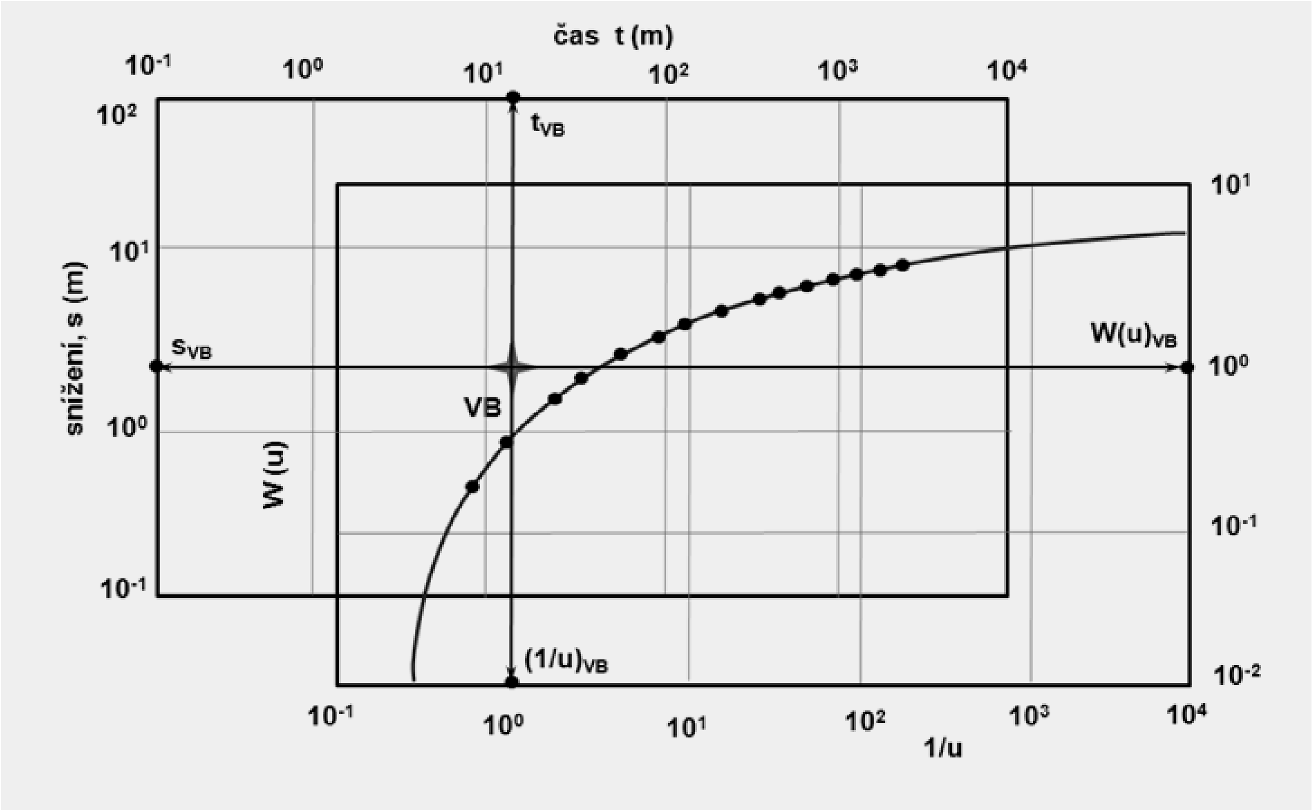
THEIS METHOD – TYPE CURVE

Consequently, we use curve matching techniques

Type curve is W(u) vs 1/u
Plot s vs t for field data
Type curve & field data must be plotted on same log-log paper
Field curve is overlaid on Type curve
Axes must be kept parallel
Best match of curves is found
Pick any convenient point
read corresponding W(u), 1/u, S and t
Use Theis equation for evaluation T & S

THEIS METHOD – TYPE CURVE

Plot drawdown versus time on log-log paper of same scale
Overlay the two plots and match the curves



Match point may be arbitrary point **vb**

Select match point and read W(u), 1/u, drawdown and time
Use these values, plus Q and r from well to solve for T and S

$$T = \frac{Q}{4\pi s_{VB}} W(u)_{VB}$$

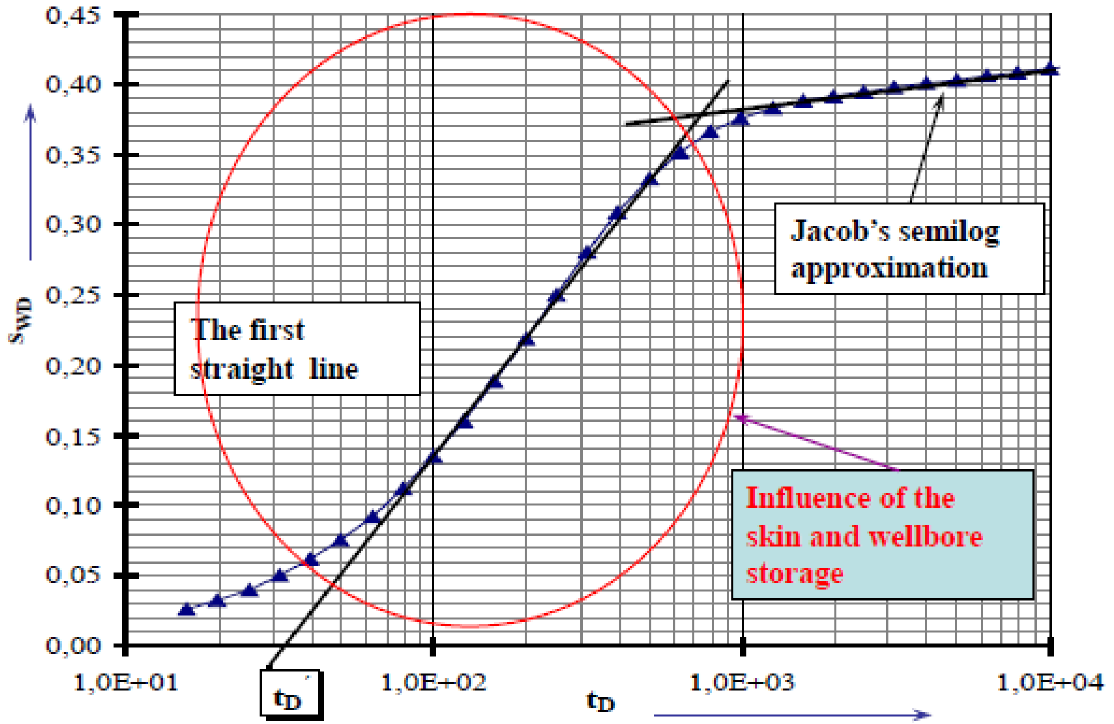
$$S = \frac{4\,T\,u_{VB}\,t_{VB}}{r^2}$$

JACOB´S SEMILOG METHOD

$$W(u) = -\gamma - \ln u \cancel{+ u - \frac{u^2}{2.2!} + \frac{u^3}{3.3!} - \frac{u^4}{4.4!}} + \ldots\ldots\ldots$$

$$s(r,t) = \frac{Q}{4\pi T} \ln \frac{2{,}246\,T\,t}{r^2\,S}$$

$$s(r,t) = \frac{2.3Q}{4\pi T} \log_{10}\left(\frac{2.25Tt}{r^2 S}\right) = \frac{0.183Q}{T} \log_{10}\left(\frac{2.25Tt}{r^2 S}\right)$$



JACOB APPROXIMATION

- Drawdown, *s*

$$s(u)=\frac{Q}{4\pi T}W(u) \qquad u=\frac{r^2S}{4Tt}$$

- Well Function, *W*(*u*)

$$W(u)=\int_u^{\infty}\frac{e^{-\eta}}{\eta}d\eta \approx -0.5772-\ln(u)+u-\frac{u^2}{2!}+\ldots$$

- Series approximation of *W*(*u*)

$$W(u)\approx -0.5772-\ln(u) \qquad \text{for small } u<0.01$$

- Approximation of *s*

$$s(r,t)\approx\frac{Q}{4\pi T}\left[-0.5772-\ln\left(\frac{r^2S}{4Tt}\right)\right]$$

$$s(r,t)=\frac{Q}{4\pi T}\ln\left(\frac{2.25Tt}{r^2S}\right) \qquad s(r,t)=\frac{2.3Q}{4\pi T}\log_{10}\left(\frac{2.25Tt}{r^2S}\right)$$

JACOB APPROXIMATION – TRANSMISIVITY, T

$$\Delta s = s_2 - s_1 = \frac{Q}{4\pi T}\left[ln(t_2) - ln(t_1)\right]$$

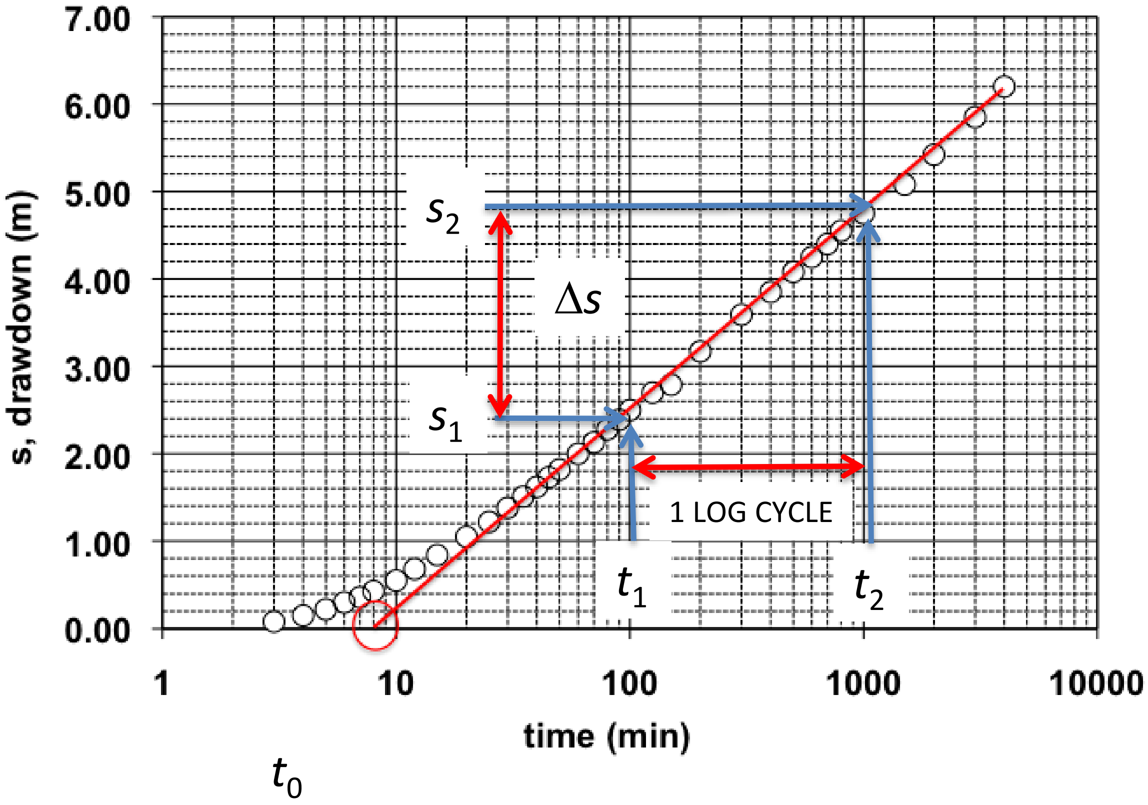
1 LOG CYCLE

$$\log\left(\frac{t_2}{t_1}\right) = \log\left(\frac{10 * t_1}{t_1}\right) = 1$$

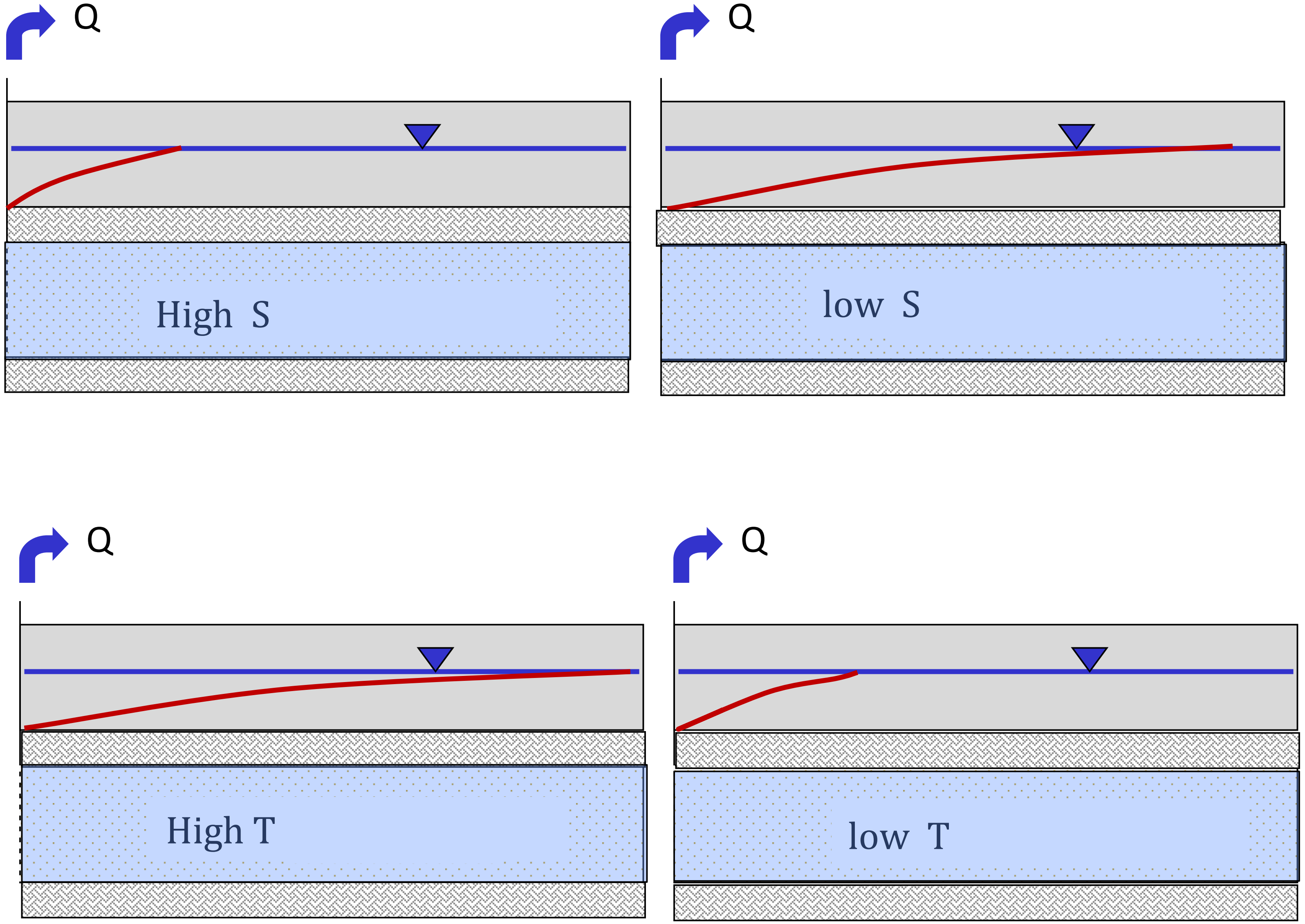
$$\Delta s = \frac{2.3Q}{4\pi T}\left[\log\left(\frac{t_2}{t_1}\right)\right]$$

$$= \frac{2.3Q}{4\pi T}(1)$$

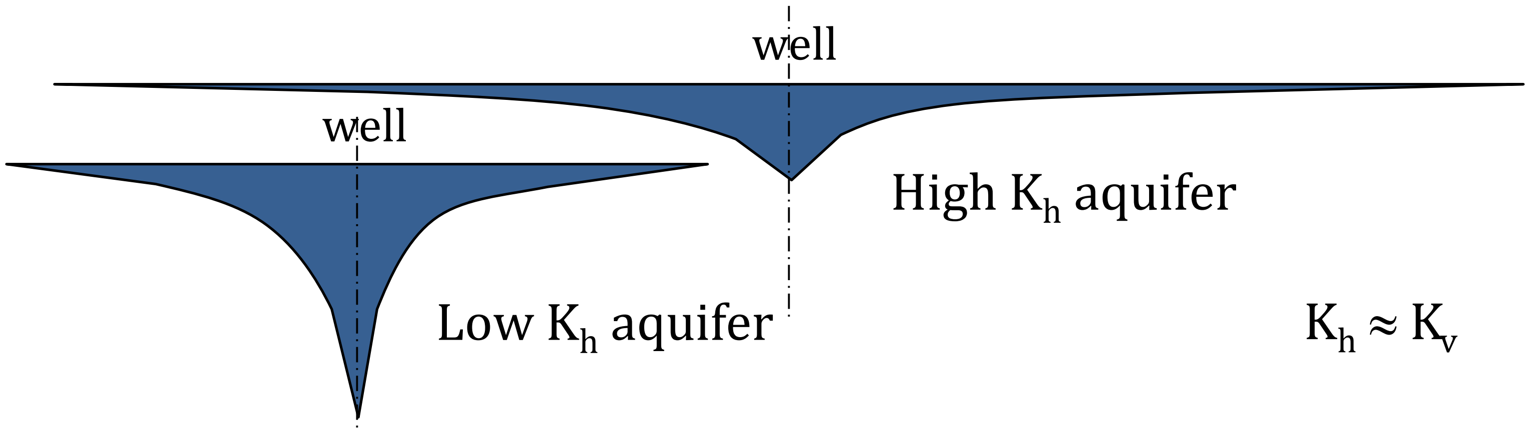
$$\boxed{T = \frac{2.3Q}{4\pi \Delta s}}$$



DIFFERENT T, S YIELDS DIFFERENT GRADIENT AT WELL BORE



CONE OF DEPRESSION



- A zone of low pressure is created centred on the pumping well
- Drawdown is a maximum at the well and reduces radially
- Head gradient decreases away from the well and the pattern resembles an inverted cone called the **cone of depression**
- The cone expands over time until the inflows (from various boundaries) match the well extraction
- The shape of the equilibrium cone is controlled by hydraulic conductivity

steeper gradients occur in low K material

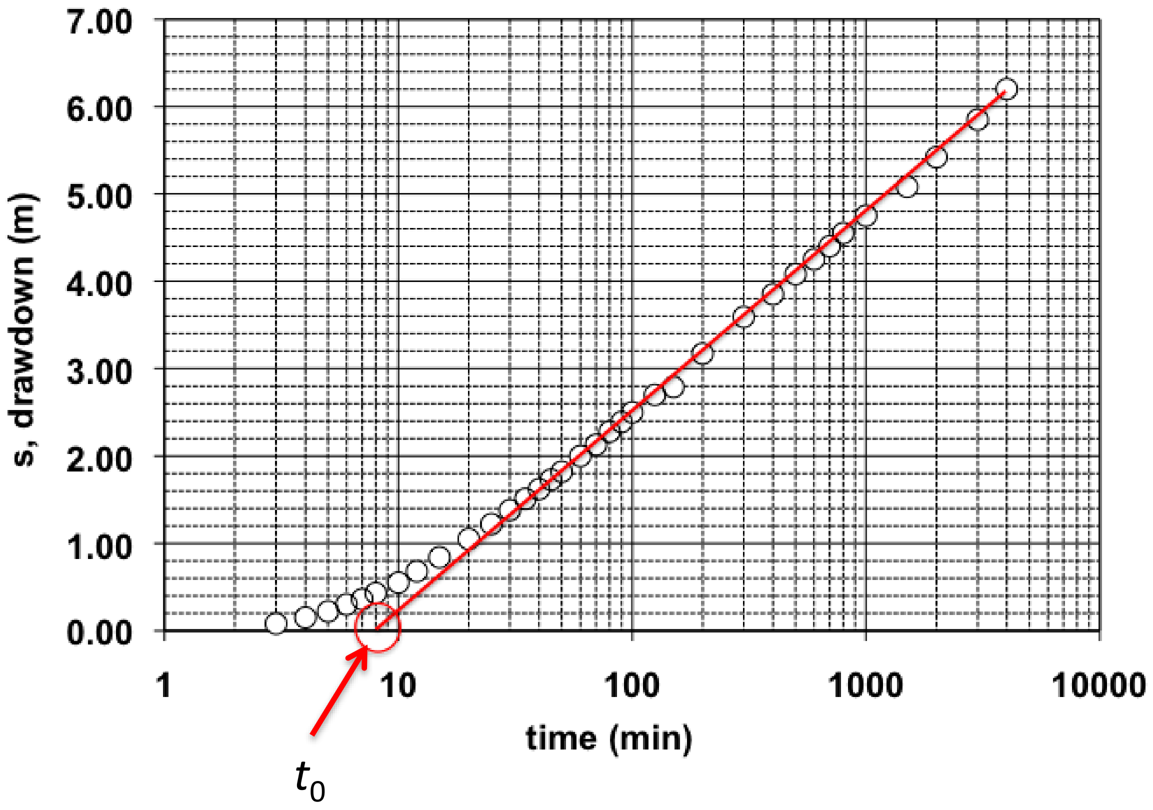
JACOB APPROXIMATION – STORATIVITY, S

$$s = \frac{Q}{4\pi T}\ln\left(\frac{2.25Tt}{r^2 S}\right)$$

$$0 = \frac{Q}{4\pi T}\ln\left(\frac{2.25Tt_0}{r^2 S}\right)$$

$$1 = \frac{2.25Tt_0}{r^2 S}$$

$$\boxed{S = \frac{2.25Tt_0}{r^2}}$$



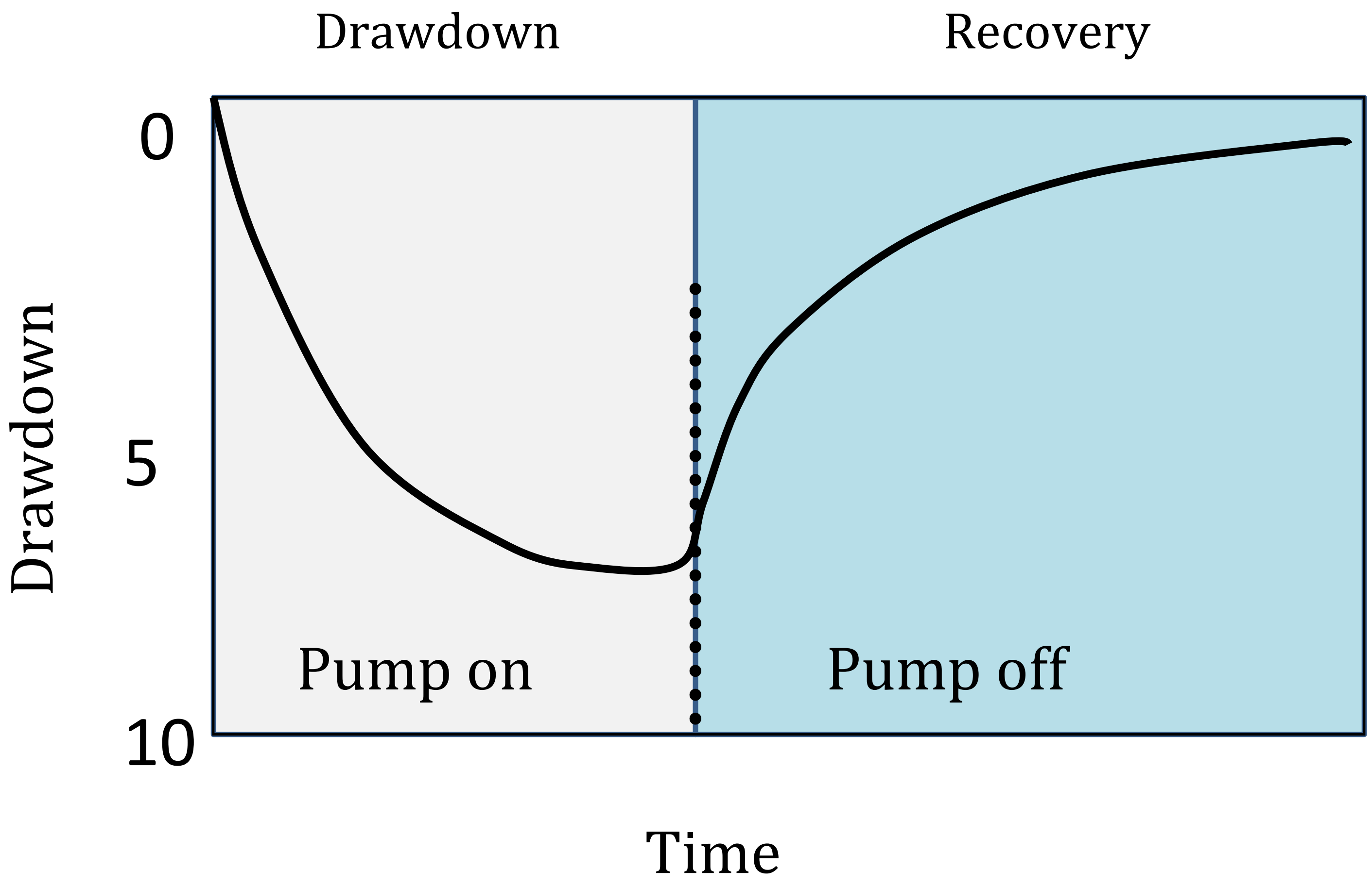
RECOVERY (BUILD-UP) TEST

RECOVERY DATA

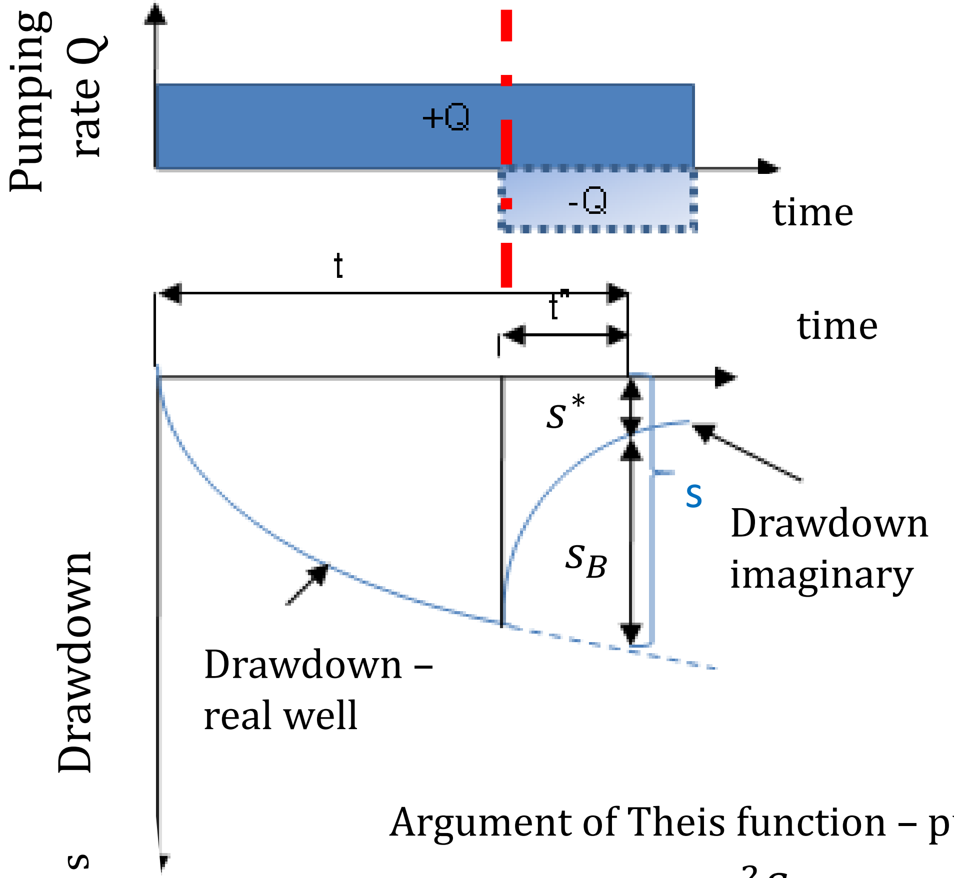
- When **pumping is halted**, water levels rise towards their pre-pumping levels.
- The rate of recovery provides a second method for **calculating aquifer characteristics**.
- **Monitoring recovery heads** is an important part of the well-testing process.
- **Observation well data** (from multiple wells) is preferable to that gathered from pumped wells.
- Pumped well recovery records are less useful but can be used in a more limited way to provide information on aquifer properties.

RECOVERY DATA

(after pumping ceases)



RECOVERY (BUILD-UP CURVE)



Drawdown for build-up $\Rightarrow$ $s^* = s + s_B$

$$s^* = \frac{+Q}{4\,\pi T} W(u) + \frac{-Q}{4\,\pi T} W(u_B)$$

Drawdown for pumping

$$s = \frac{+Q}{4\,\pi T} W(u)$$

Argument of Theis function – pumping

$$u = \frac{r^2 S}{4\,T\,t}$$

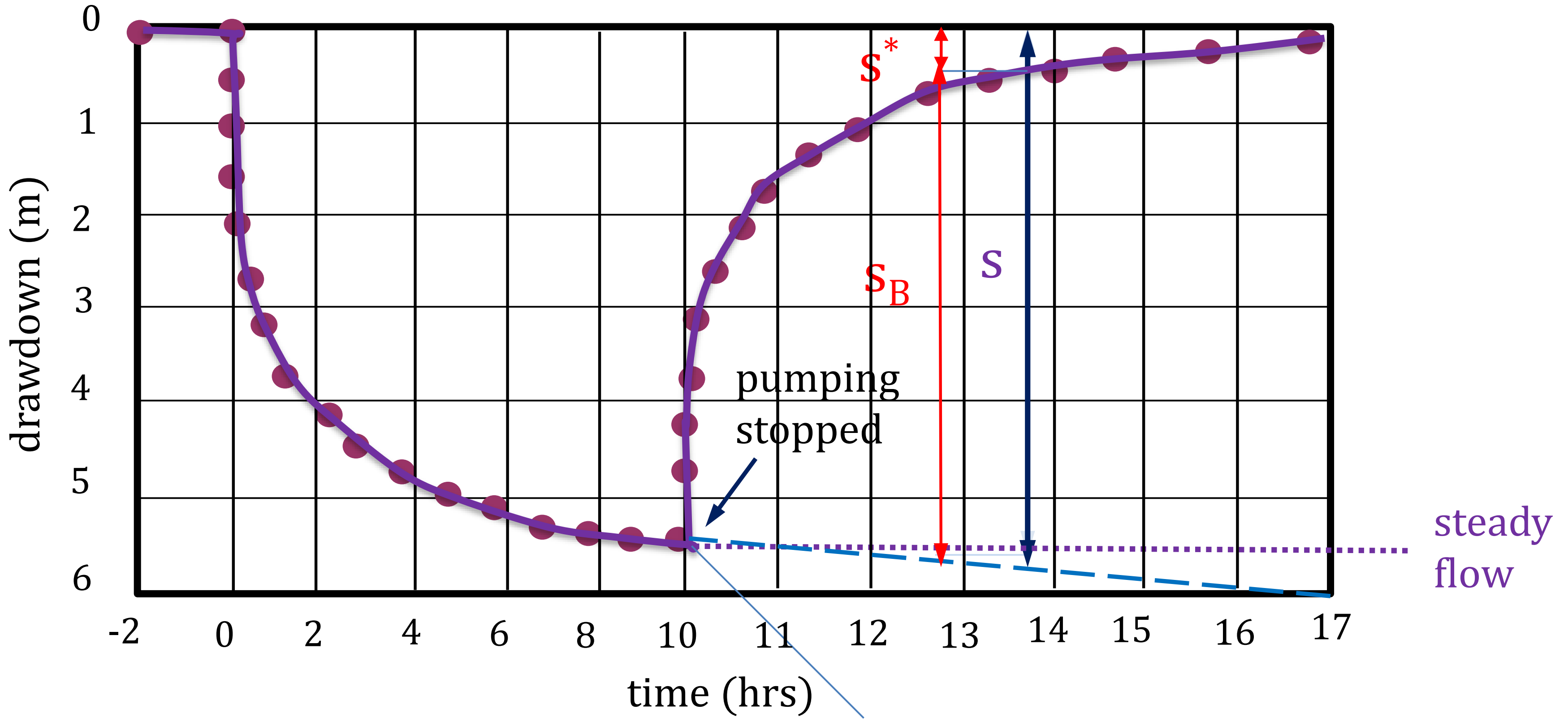
Drawdown for imaginary – build-up

$$s_B = \frac{-Q}{4\,\pi T} W(u_B)$$

Argument of Theis function – build-up

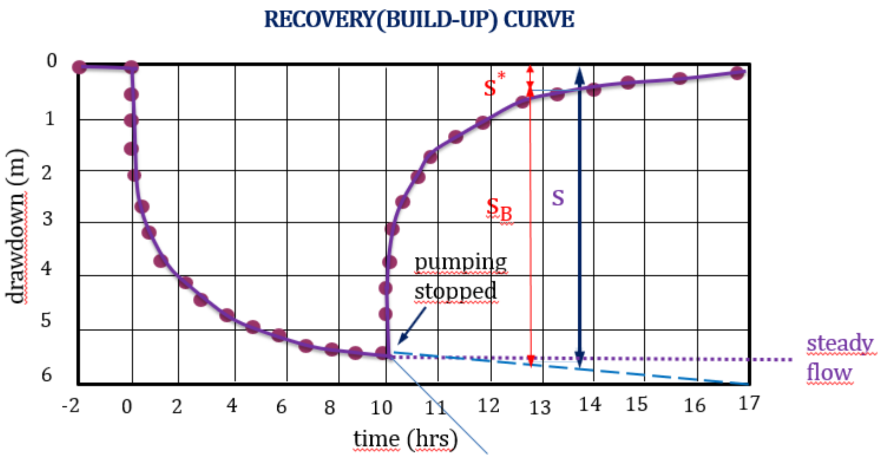
$$u_B = \frac{r^2 S}{4\,T\,t^*}$$

RECOVERY(BUILD-UP) CURVE



RESIDUAL DRAWDOWN AND RECOVERY

SUPERPOSITION



- The total drawdown for $t > t_r$ is:

$$s^* = s - s_B = \frac{Q}{4\pi T}(W(u) - W(u^*))$$

- The Jacob approximation can be applied giving:

$$s^* = s - s_B = \frac{Q}{4\pi T}(ln\frac{2.25Tt}{r^2 S} - ln\frac{2.25Tt^*}{r^2 S})$$

- Simplification gives the residual drawdown equation:

$$s^* = s - s_B = \frac{Q}{4\pi T}(ln\frac{t}{t^*})$$

- The equation predicting the recovery is:

$$s_B = \frac{-Q}{4\pi T}(ln\frac{2.25Tt^*}{r^2 S})$$

For $t > t_r$, the recovery s_r is the difference between the observed drawdown s* and the extrapolated pumping drawdown (s).

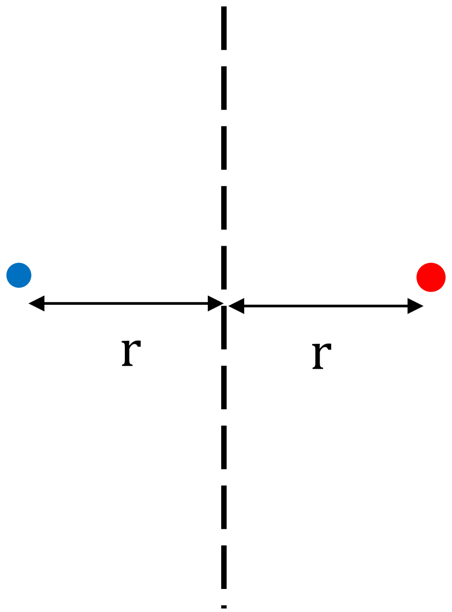
BOUNDED AQUIFERS

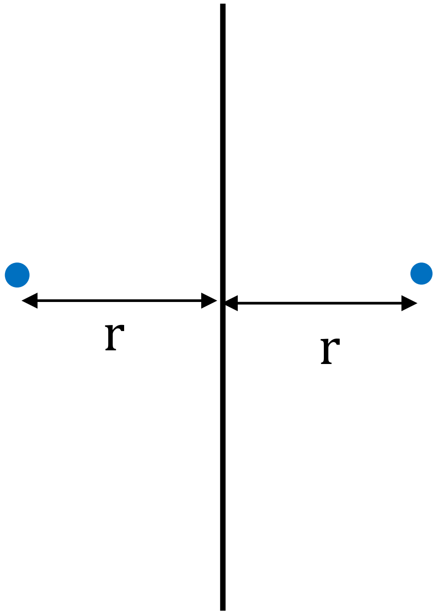
- Superposition was used to calculate well recovery by adding the effects of a pumping and recharge well starting at different times.
- Superposition can also be used to simulate the effects of aquifer boundaries by adding wells at different positions.
- For boundaries, the wells that create the same effect as a boundary are called image wells.
- This relatively simple application of superposition for analysis of aquifer boundaries was for described by Ferris (1959)

IMAGE WELLS

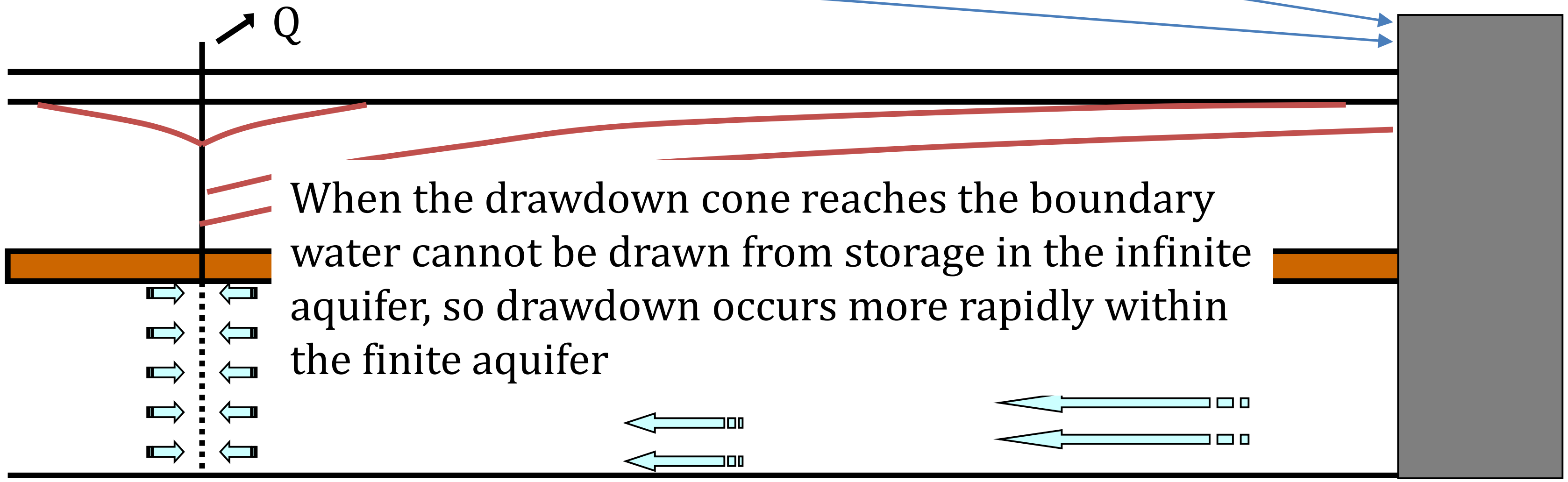
- **RECHARGE BOUNDARIES** at distance (r) are simulated by a recharge image well at an equal distance (r) across the boundary.

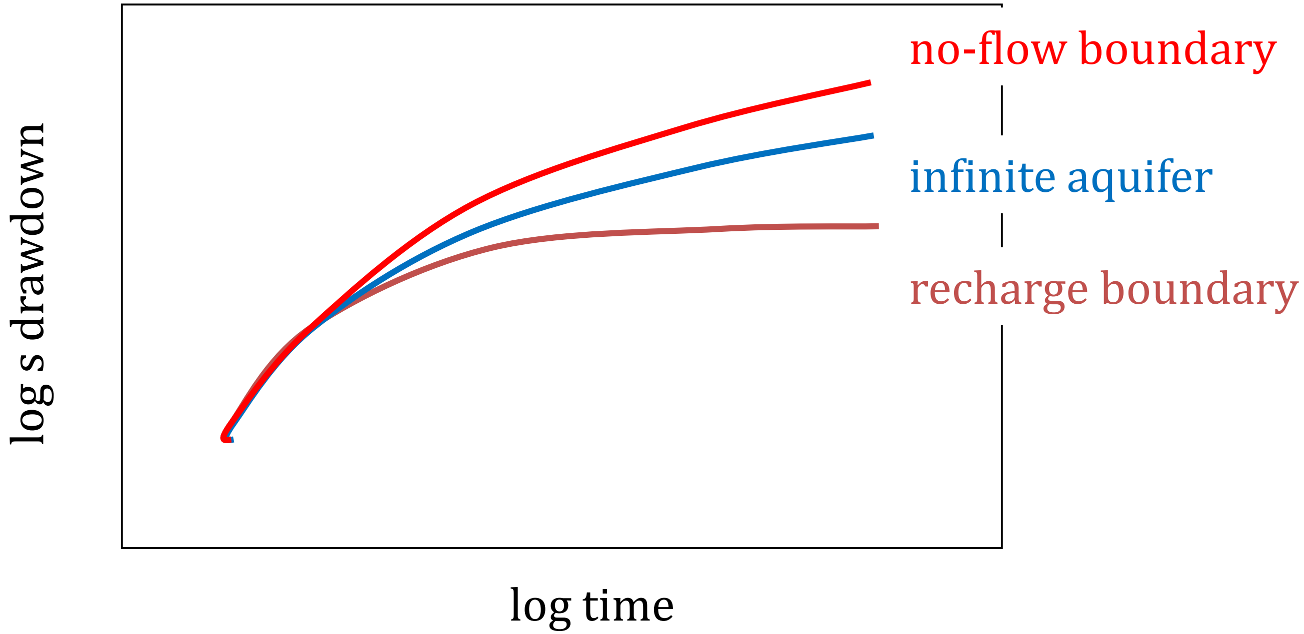
- **BARRIER BOUNDARIES** at distance (r) are simulated by a pumping image well at an equal distance (r) across the boundary.



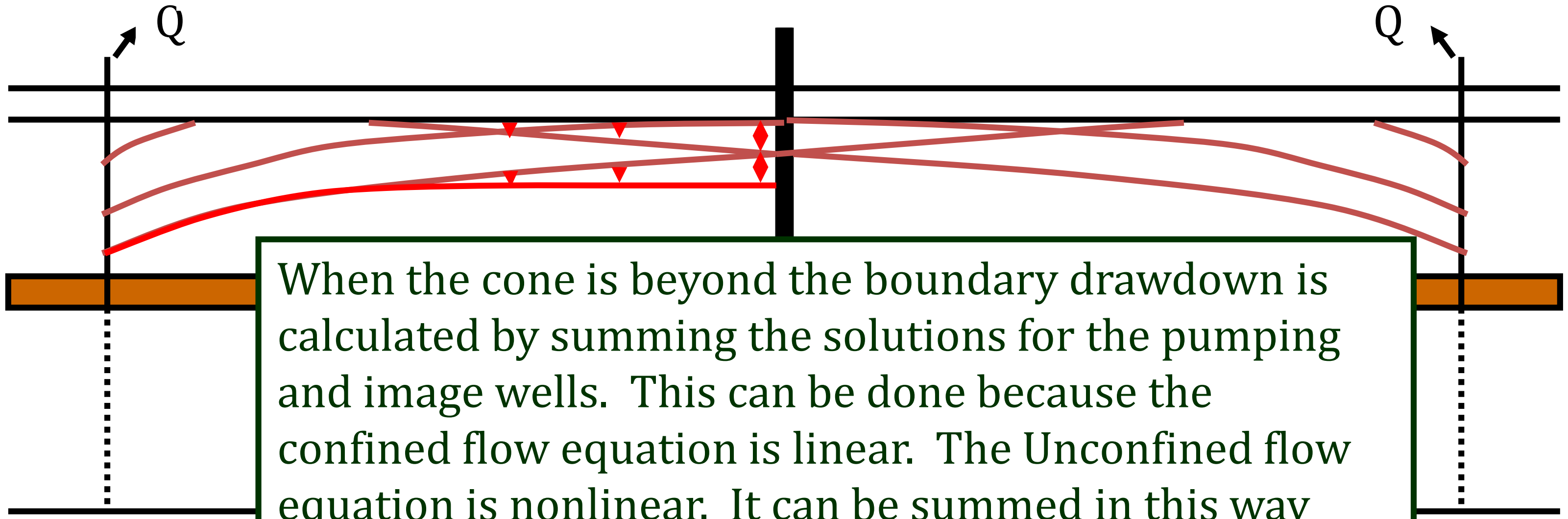


IMPERMEABLE or NO-FLOW BOUNDARY





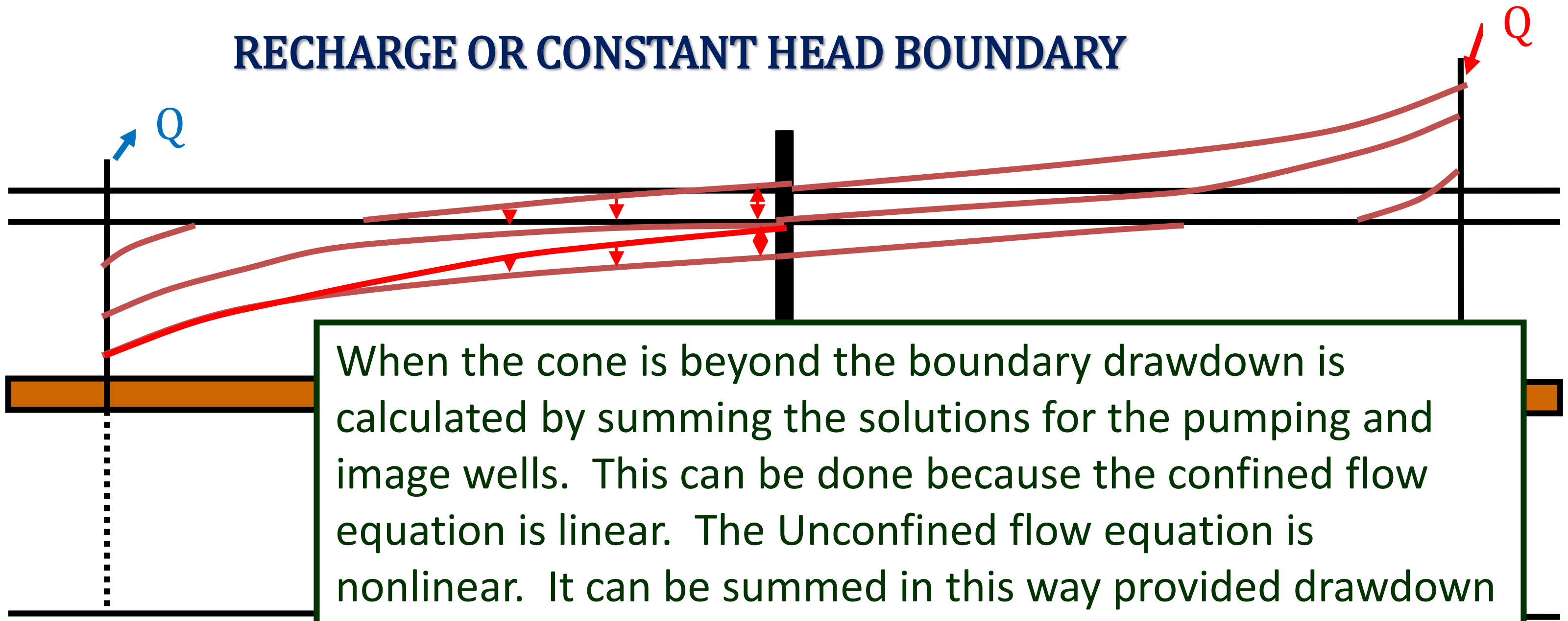
IMPERMEABLE OR NO-FLOW BOUNDARY



When the cone is beyond the boundary drawdown is calculated by summing the solutions for the pumping and image wells. This can be done because the confined flow equation is linear. The Unconfined flow equation is nonlinear. It can be summed in this way provided drawdown is relatively small.

Method of Images - can be used to predict drawdown
by creating a mathematical no-flow boundary
NO-FLOW = NO GRADIENT
So if we place an imaginary well
of equal strength
at equal distance across the boundary
And superpose the solutions
We will have
equal drawdown, therefore equal head at the boundary, hence NO GRADIENT
Let’s look at it

RECHARGE OR CONSTANT HEAD BOUNDARY

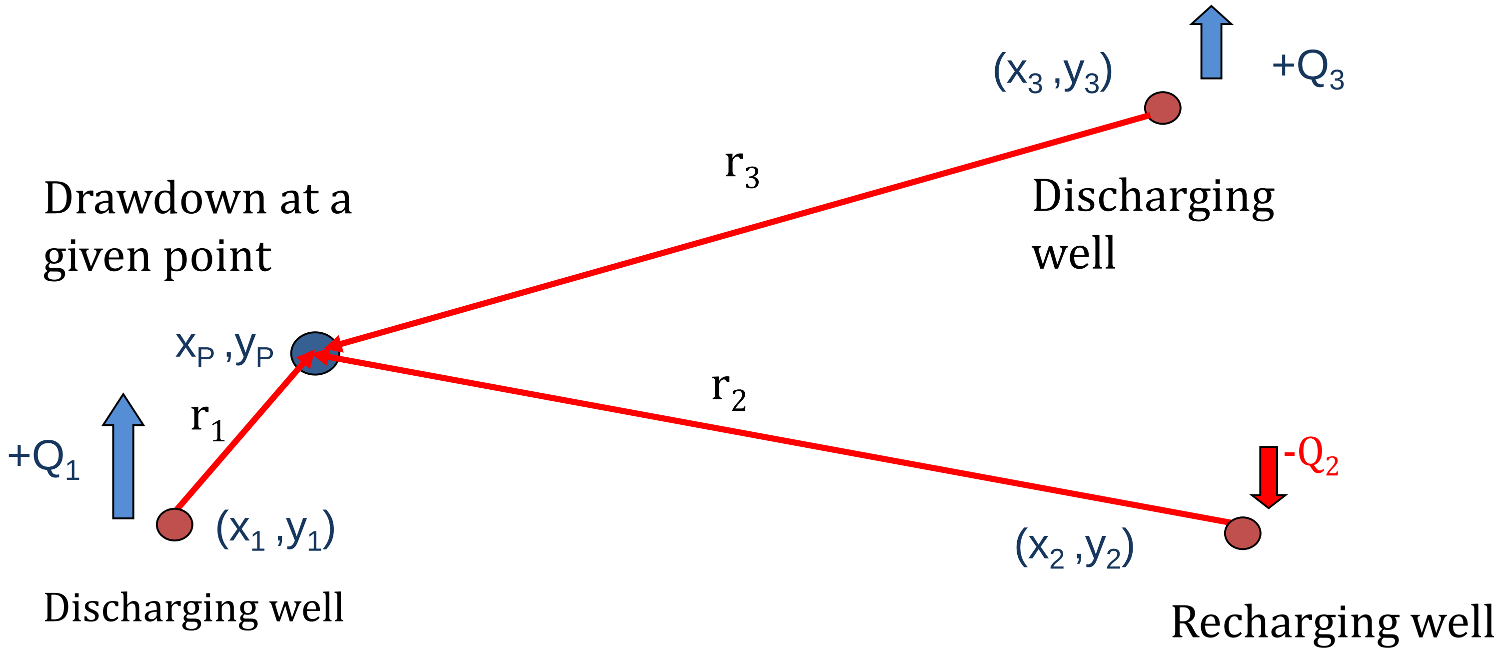


When the cone is beyond the boundary drawdown is calculated by summing the solutions for the pumping and image wells. This can be done because the confined flow equation is linear. The Unconfined flow equation is nonlinear. It can be summed in this way provided drawdown is relatively small.

Method of Images - can be used to predict drawdown
by creating a mathematical constant head boundary
CONSTANT HEAD = NO CHANGE IN HEAD
So if we place an imaginary well
of equal strength but opposite sign
at equal distance across the boundary
And superpose the solutions
We will have
equal but opposite drawdown, therefore NO HEAD CHANGE
Let’s look at it

MULTIPLE WELLS

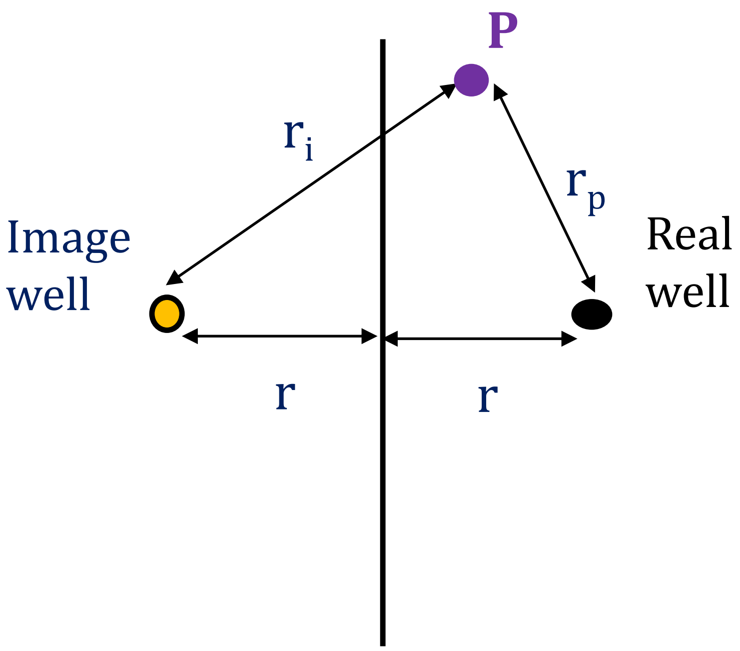
$$s = \frac{Q_1}{4\pi T}W(u_1) - \frac{Q_2}{4\pi T}W(u_2) + \frac{Q_3}{4\pi T}W(u_3) \quad \text{where} \quad u_i = \frac{r_i^{\,2} S}{4T\,t_i} \qquad i = 1,2,....$$



$$r_1 = \sqrt{(x_p - x_1)^2 + (y_p - y_1)^2}$$

Sum of s_1 (Q_1, r_1) s_2 (Q_2, r_2) (Q_2 is) s_3 (Q_3, r_3).

GENERAL SOLUTION -



The general solution for adding image wells to a real pumping well can be written:

$$s_P = s_R \pm s_i = \frac{Q}{4\pi T}\left(W(u_R) \pm W(u_i)\right)$$

where

$$u_R = \frac{r_P^2 S}{4Tt} \qquad u_i = \frac{r_i^2 S}{4Tt}$$

and r_p,r_i are the distances from the pumping and image wells respectively.

- For a barrier boundary, for all points on the boundary $r_p = r_i$ and the drawdown is doubled.
- For a recharge boundary, for all points on the boundary $r_p = r_i$ and the drawdown is zero.

“REAL WELL“ – SKIN EFFECT

- **Skin, W**, refers to a region near the wellbore of improved or reduced permeability compared to the bulk formation permeability.

REASON FOR POSITIVE SKIN

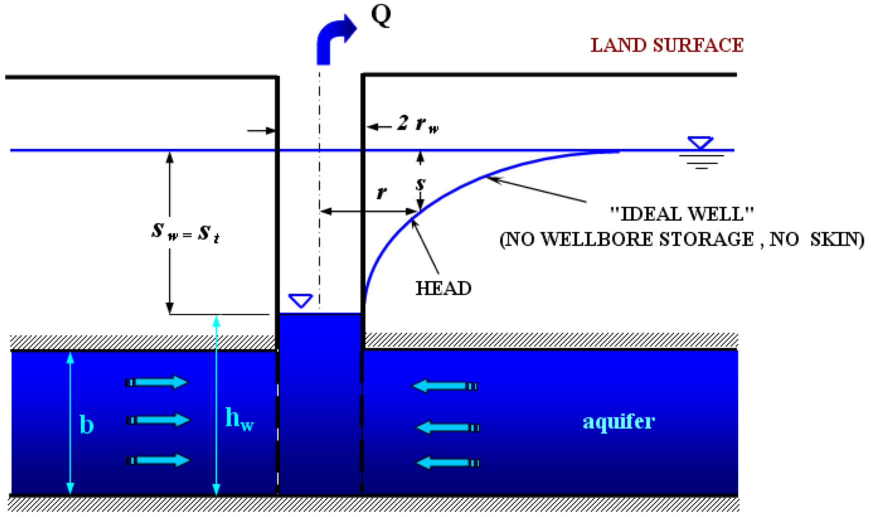
- **Overbalanced drilling** (filtrate loss)
- **Damaged perforations**
- **Gravel pack**
- Unfiltered completion fluid
- **Partial completion**
- **Fines migration** after long term production
- **Non-darcy flow**
- Condensate banking (acts like **turbulence**)

"IDEAL WELL"

- no additional resistance at a well
- the well radius, r_w is infinitesimally small

***The partial differential* equation describing radial flow to a well fully penetrating confined aquifer is (in cylindrical coordinates)**

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r} = \frac{S}{T}\frac{\partial s}{\partial t}$$



Drawdown around a production well (ideal well)

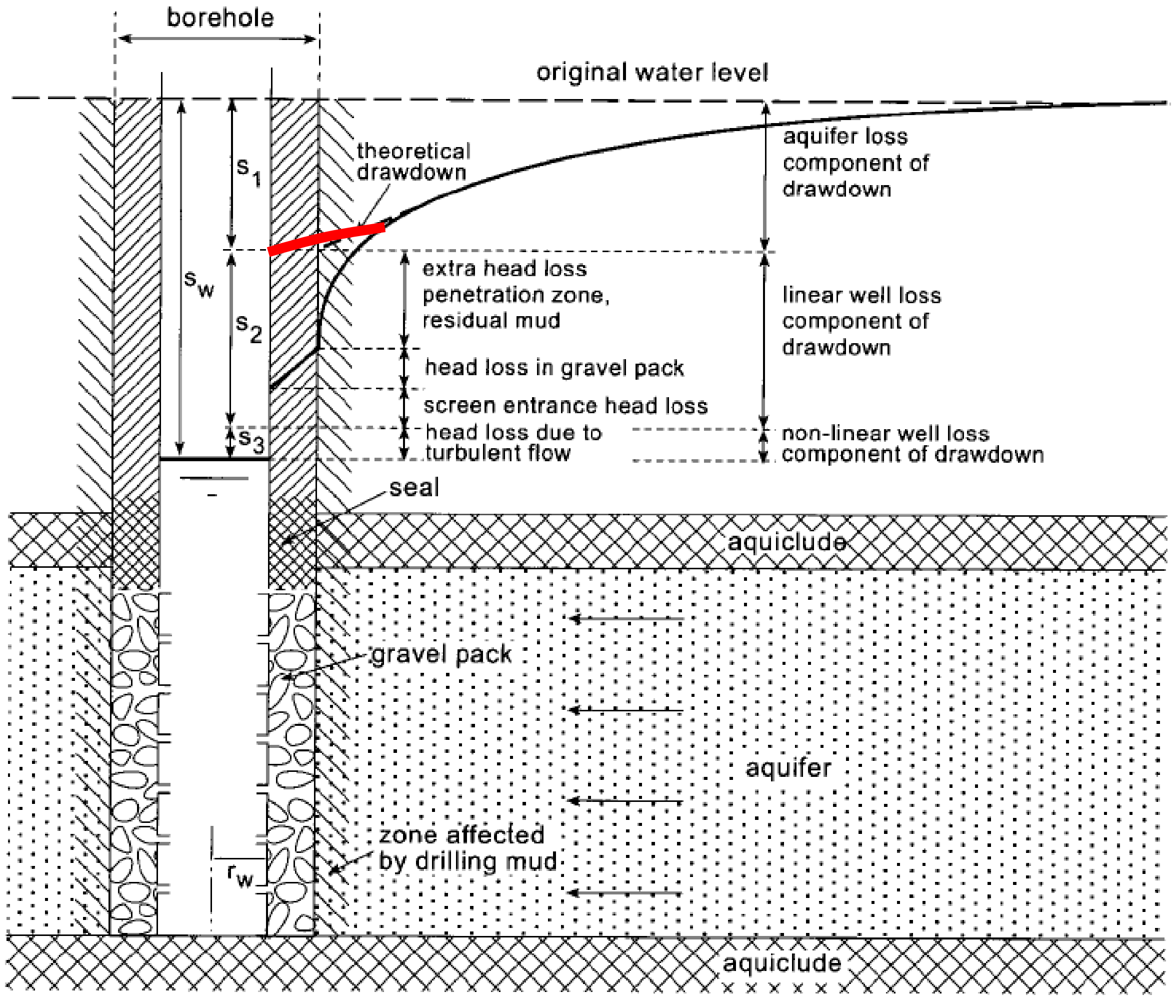
Where **s** is drawdown; ***r*** is radial distance from well; ***S*** is storativity; ***T*** is transmisivity

DRAWDOWN AT THE REAL WELL

- Drawdown in a pumped well consists of two components:

- Aquifer losses
  - Head losses that occur in the aquifer where the flow is laminar
  - Time-dependent
  - Vary linearly with the well discharge

- Well losses
  - Aquifer damage during drilling and completion
  - Turbulent friction losses adjacent to well, in the well and pipe



REAL WELL (skin effect)

As a water well ages, the rate at which water may be pumped (commonly referred to as the well yield, flow or performance) tends to decrease,

More often, reduced well yield over time can be related to changes in the water well itself including:

- Incrustation from mineral deposits (Fe, Mn)
- Bio-fouling by the growth of microorganisms
- Physical plugging of "aquifer" (the saturated layer of sand, gravel, or rock through which water is transmitted by sediment
- Sand pumping
- Well screen or casing corrosion
- Pump damage

A submersible pump being pulled from a well exhibiting iron oxide, iron bacteria and biofilm.

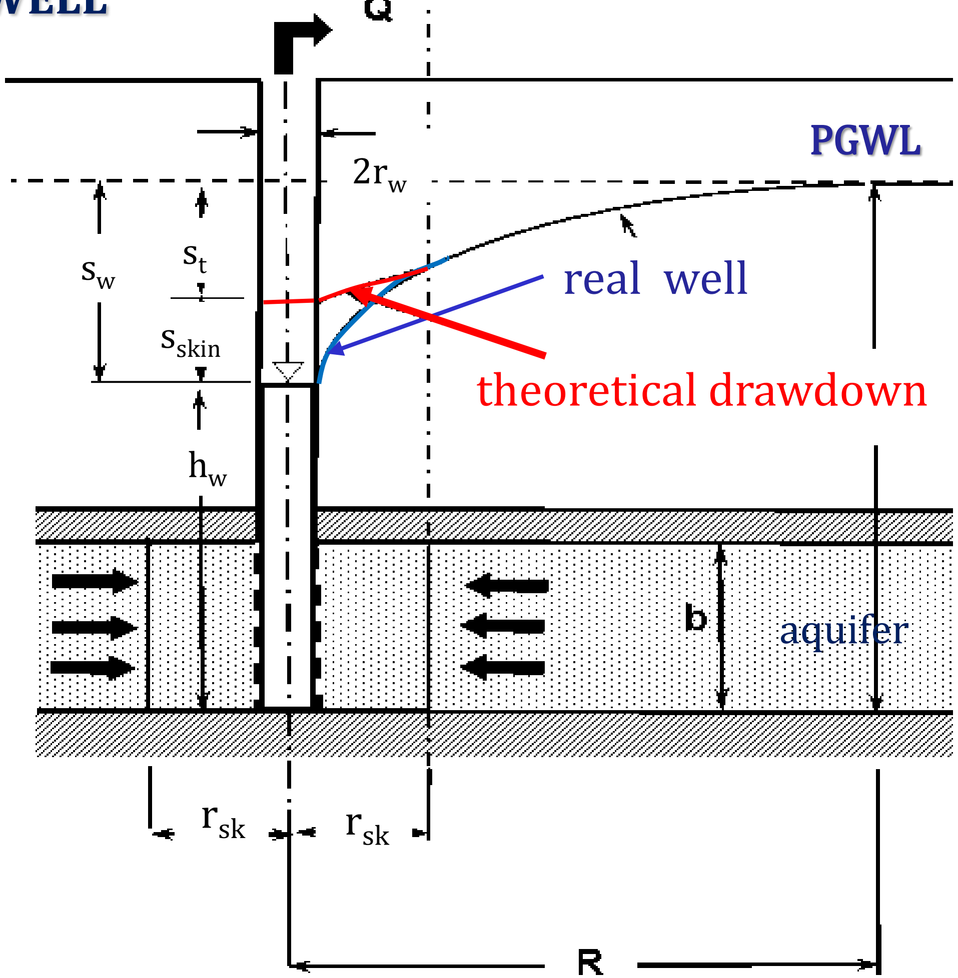
Major changes in any of the following well characteristics is an indication that your well or pump is in need of attention:

- Decreased pumping rate
- Decreased water level
- Decreased specific capacity
- Increased sand or sediment content in the water
- Decreased total well depth

The two most common methods to rehabilitate a water well are:

- chemicals to dissolve the incrusting materials from the well
- physically cleaning the well

"REAL WELL"



Drawdown around a production well with skin effect and wellbore storage (real well)

$$s_{skin} = s_d + s_{pp} + s_{inc} + s_{turb} + s_O$$

$$s_w = s_t + s_{skin}$$

TOTAL SKIN (s_{skin})

Total skin is a summation of the following skin components:

- Skin due to damage (s_d)
- Skin due to partial penetration (s_{pp}) for a partially penetrated well only
- Skin due to inclination (s_{inc})
- Skin due to turbulence (s_{turb}) or non-Darcy flow (for gas wells only)

The value of (s_{skin})

“REAL WELL“

A) THE SKIN EFFECT

The additional resistance is due to hydromechanical, chemical, and biological factors that occur during drilling or completion operations, and during the exploitation of a well. This additional resistance causes an additional“ drawdown at a “real” well (s_{skin}). The drawdown at the “real” well (with skin and wellbore storage

$$s_w = s_t + s_{skin}$$ ← van Everdingen, 1953

s_t is drawdown at an “ideal” well , and s_{skin} is additional drawdown at a well caused by additional resistance.

Equation (1) indicates that the drawdown at a “real“ well differs from drawdown at an “ideal“ one by an additive amount

$$s_{skin} = \frac{Q}{2\pi T} W$$

where Q is pumping rate, T is transmissivity, and W is skin factor.

ASSUMPTIONS

- confined aquifer
- pumping rate Q = const.
- Darcy´s law is valid
- all flow is radial to well
- well is fully penetrating
- flow is horizontal
- potentiometric surface steady prior to pumping
- homogeneous, isotropic, infinite areal extent
- pumping well fully penetrates and receives water from the entire thickness of the aquifer
- transmissivity is constant in space and time
- storativity is constant in space and time
- well has finite diameter, d
- water removed from storage is discharged instantaneously
- Additional resistances (skin effect) $\neq 0$

SKIN FACTOR-W

- Steady flow

$$s_w = \frac{Q}{2\pi T}\left(ln\frac{R}{r_W} + W\right)$$

- Unsteady flow:

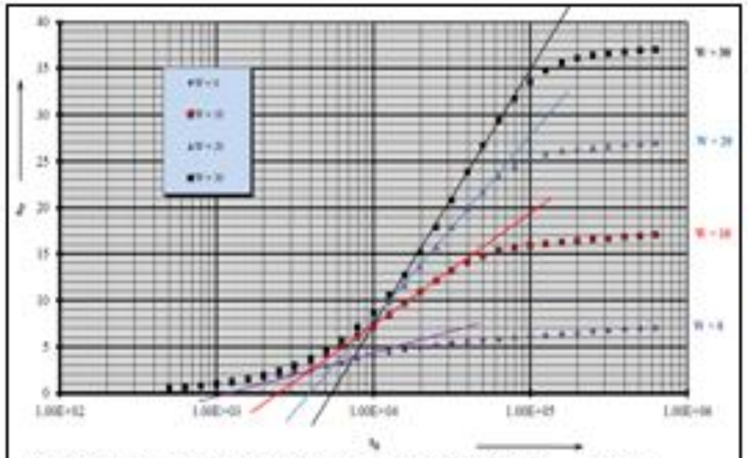


Fig. Plot of s_{wD} vs. t_D for various values of W and C_D = const.

a) Theis solution:

$$s_w = \frac{Q}{4\pi T}(W(u) + 2W)$$

b) For $t_D > 25$ (Jacob semilog. method)

$$s_w = \frac{Q}{4\pi T}\left(ln\frac{2.246Tt}{r_w^2 S} + 2W\right) \Rightarrow W=\frac{2\pi T s_V}{Q}-\frac{1}{2}\left(\ln t+\ln\frac{T}{r_V^2 S}+0{,}8091\right)$$

For drawdown s_1 (time t_1) and s_2 (time t_2)

$$s_2 - s_1 = \Delta s = \frac{0.183Q}{T}\left(log\frac{2.246T}{r_w^2\ S} + \log t_2 + 2W - log\frac{2.246T}{r_w^2\ S} - \log t_1 - 2W\right)$$

$$\Delta s = \frac{Q}{4\pi\ T}\left(ln\frac{t_2}{t_1}\right)$$

and

$$\Delta s = \frac{0.183Q}{T}\left(log\frac{t_2}{t_1}\right) \Rightarrow \text{Transmisivity, T}$$

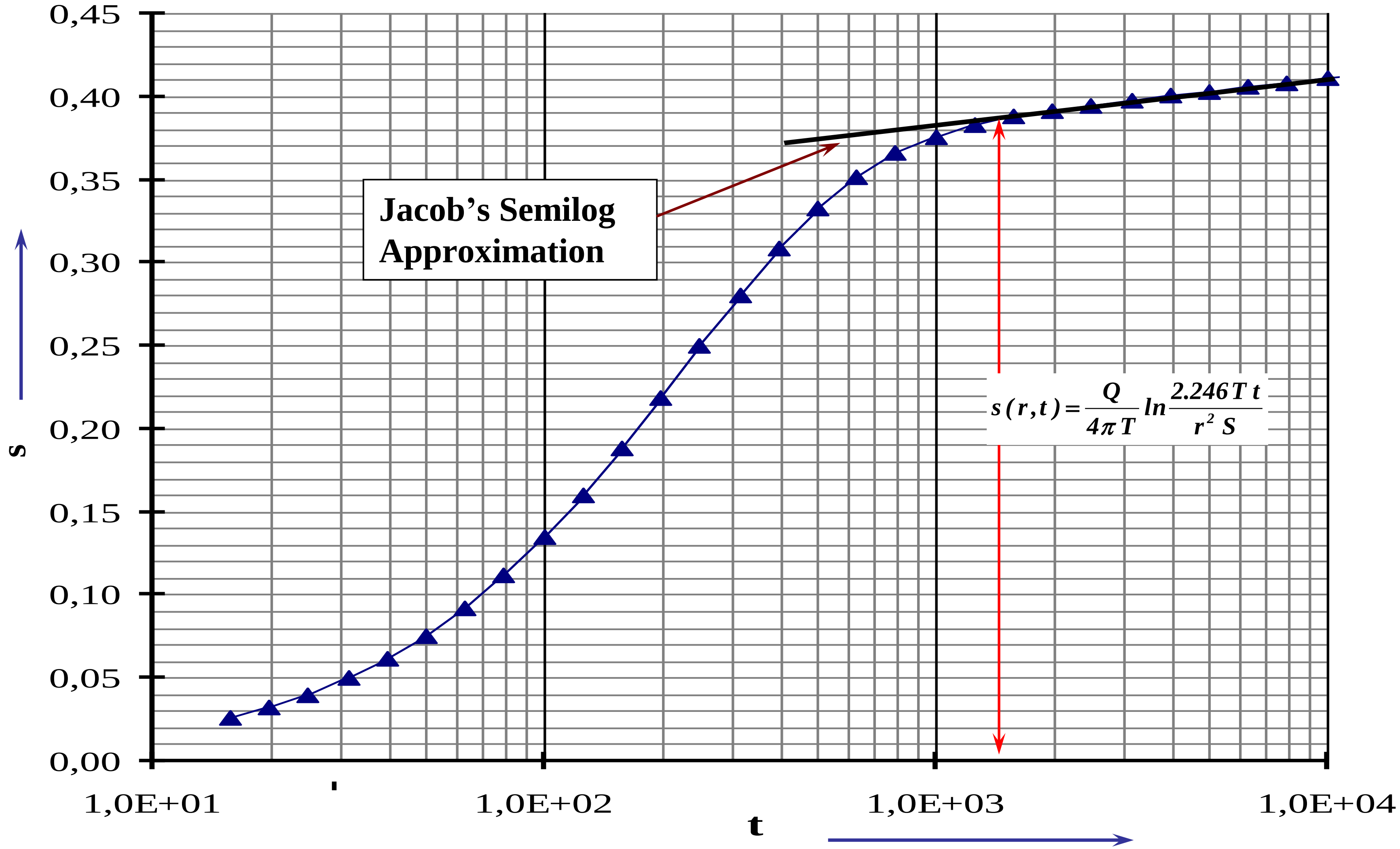


Fig. Pumping test at a well (Jacob's semilogarithmic approximation)- drawdown at the distance r

back

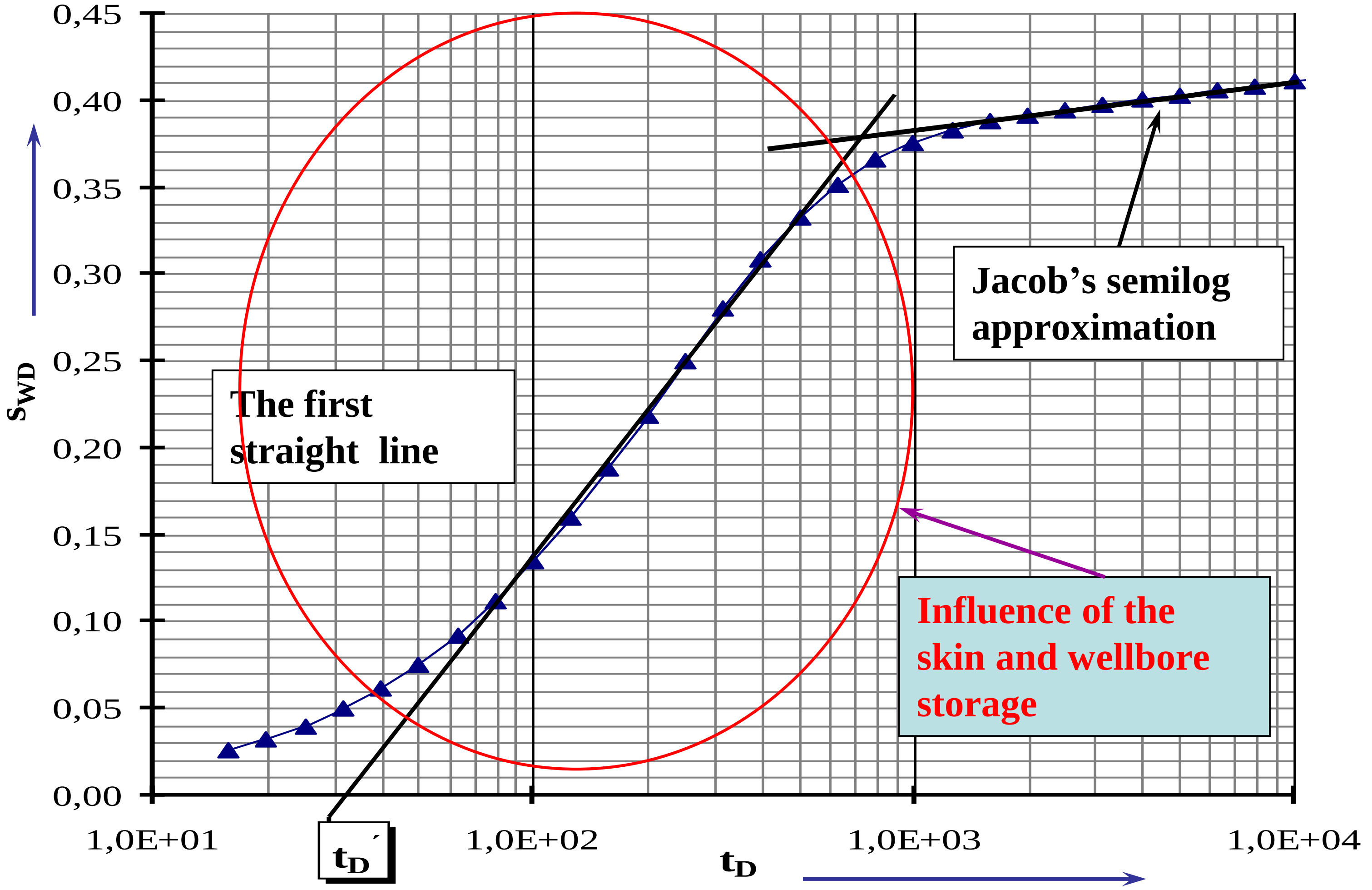


Fig. Graph s_{WD} *vs. log* t_D for a well with wellbore storage and skin (C_D = 100; W = 10)

EVALUATION OF THE WELL REHABILITATION

WELL – **HV** at Veselí nad Lužnicí – **mechanical rehabilitation** (figures)

Well radius .... $r_W = 1.5\ m$
Well depth............................ $h = 9\ m$

Jacob's semilogarithmic approximation :

Transmissivity .... $T = 0{,}0109\ m^2\ s^{-1}$
Storativity .... $S = 0{,}13$
Dimensionless Wellbore Storage $C_D = 5.8$

	Before rehabilitation	After rehabilitation	1 year after rehabilitation
$Q(m^3.s^{-1})$	$3.35x10^{-3}$	$3.7x10^{-3}$	$3.52x10^{-3}$
Length of the aquifer test (min)	180	240	75

Parameters -HV:

Q = 0,0037 m^3s^{-1}

r_W = 1,5 m

T = 0,0109 m^2s^{-1}

S = 0,13

C_D = 5,8

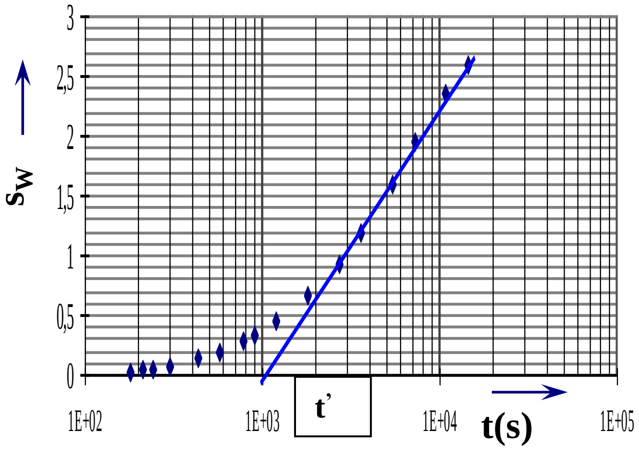


Fig. Graph of s_w vs. log t – before rehabilitation

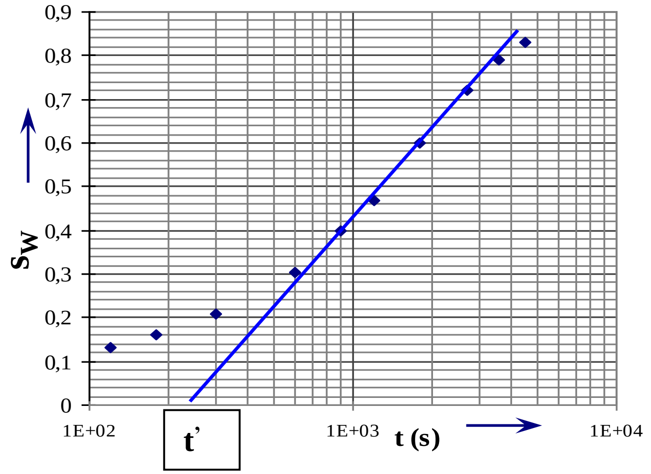


Fig. Graph of s_w vs. log t – after rehabilitation

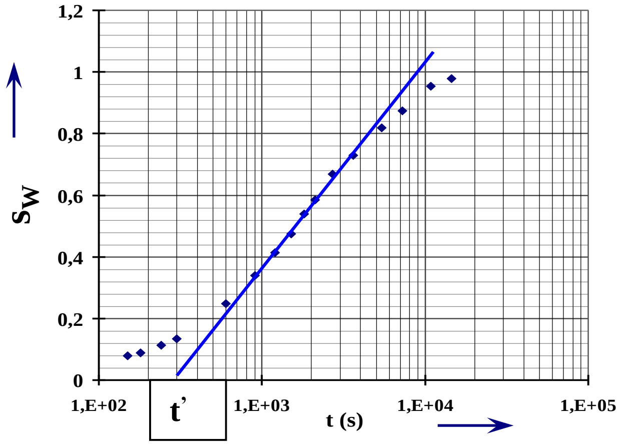
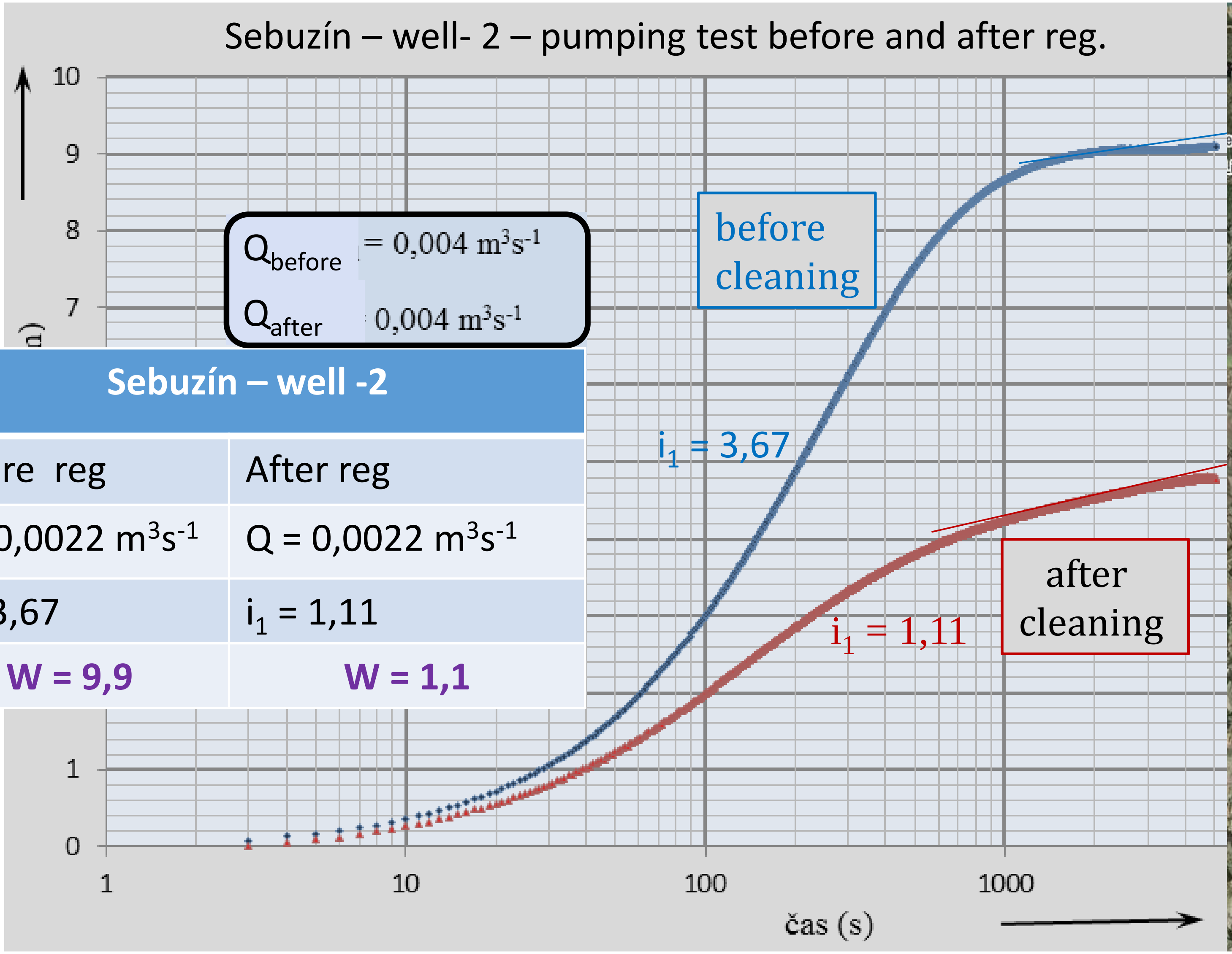


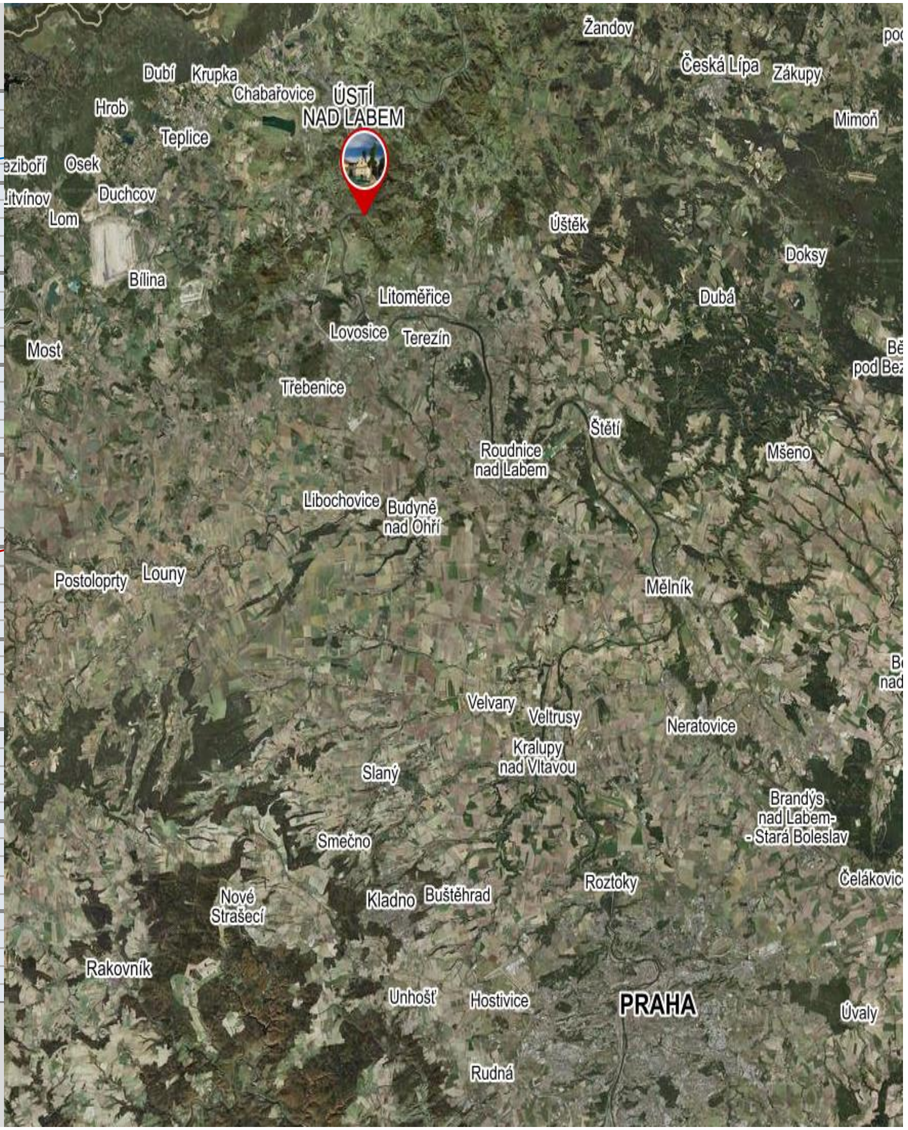
Fig. Graph of s_w vs. log t – 1 year after rehabilitation

Evaluation

	Before rehabilitation	*After rehabilitation*	*1 year after rehabilitation*
Skin factor, W	*43*	*15*	*19*
s_{skin} (m)	*1.02*	*0.36*	*0.45*



Sebuzín – well -2	
Before reg	After reg
$Q = 0{,}0022\ m^3s^{-1}$	$Q = 0{,}0022\ m^3s^{-1}$
$i_1 = 3{,}67$	$i_1 = 1{,}11$
W = 9,9	**W = 1,1**

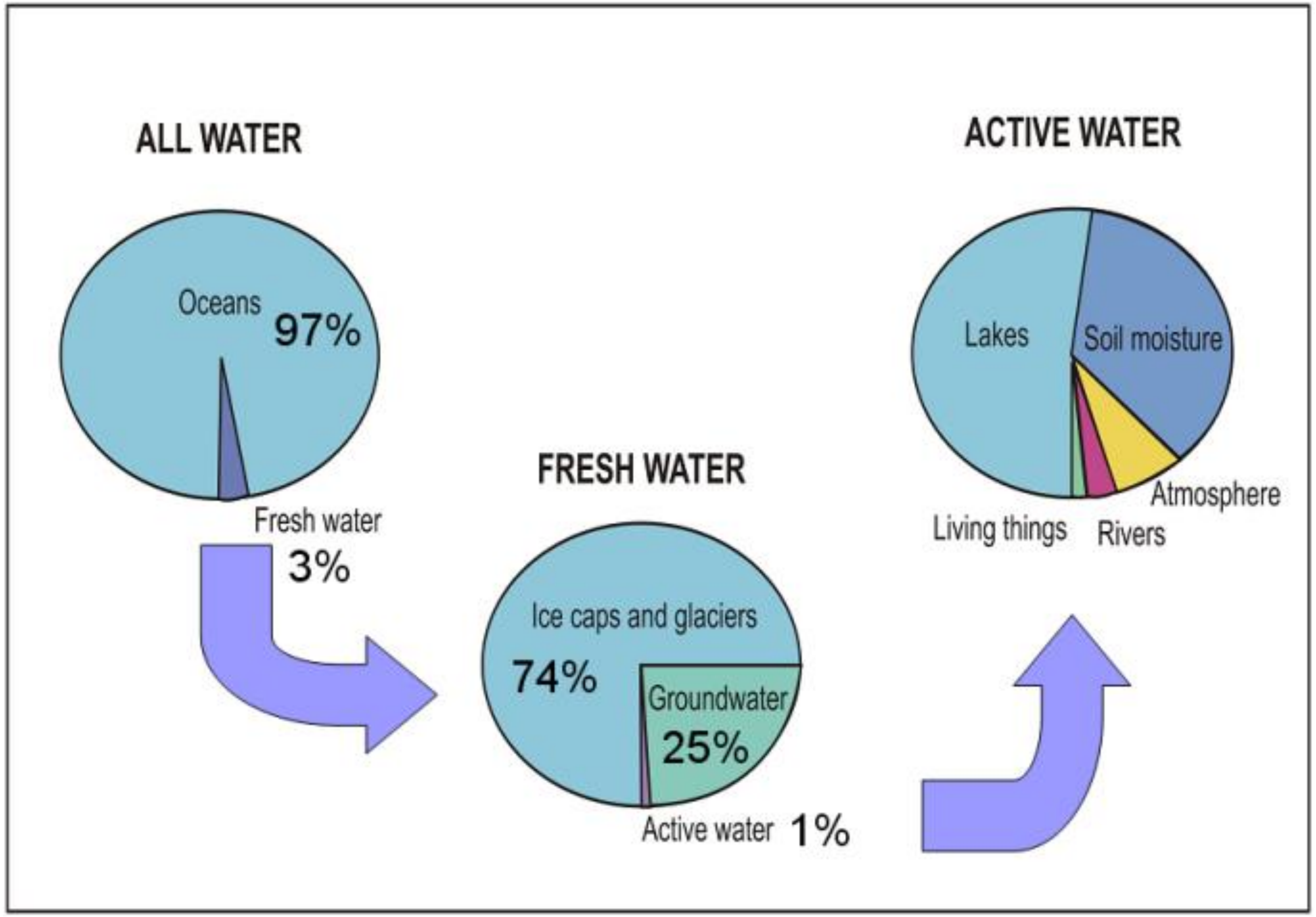


End

- ground water: the water that lies beneath the ground surface, filling the pore space between grains in bodies of sediment and clastic sedimentary rock, and filling cracks and crevices in all types of rock
- ground water is a major economic resource, particularly in the dry western areas of the US and Canada
- source of ground water is rain and snow that falls to the ground a portion of which percolates down into the ground to become ground water

The Water Table

- saturated zone: the subsurface zone in which all rock openings are filled with water
- water table: the upper surface of the zone of saturation
- vadose zone: a subsurface zone in which rock openings are generally unsaturated and filled partly with air and partly with water; above the saturated zone
- capillary fringe: a transition zone with higher moisture content at the base of the vadose zone just above the water table


ALL WATER
Oceans 97%
Fresh water 3%
FRESH WATER
Ice caps and glaciers
74%
Groundwater
25%
Active water 1%
ACTIVE WATER
Lakes
Soil moisture
Atmosphere
Living things
Rivers

Groundwater includes:

- Vadose zone or zone of aeration:
  - Partially saturated with water
  - Capillary effects interact with gravity
- Capillary Fringe
  - Zone of saturation above water table where water is drawn up by capillary suction (negative pore pressure)
- Water Table
  - Except for capillary fringe, zone of saturation. Surface where pore pressure is atmospheric
- Perched Water Table
  - Zone of saturation above water table where hydraulic conductivity is less than infiltration and water “ponds”

- Recharge
  - Water entering groundwater system through infiltration or from surface water

- Discharge
  - Water leaving groundwater system usually to surface water or other flow system boundaries

- Storage
  - Water in the the groundwater system

End