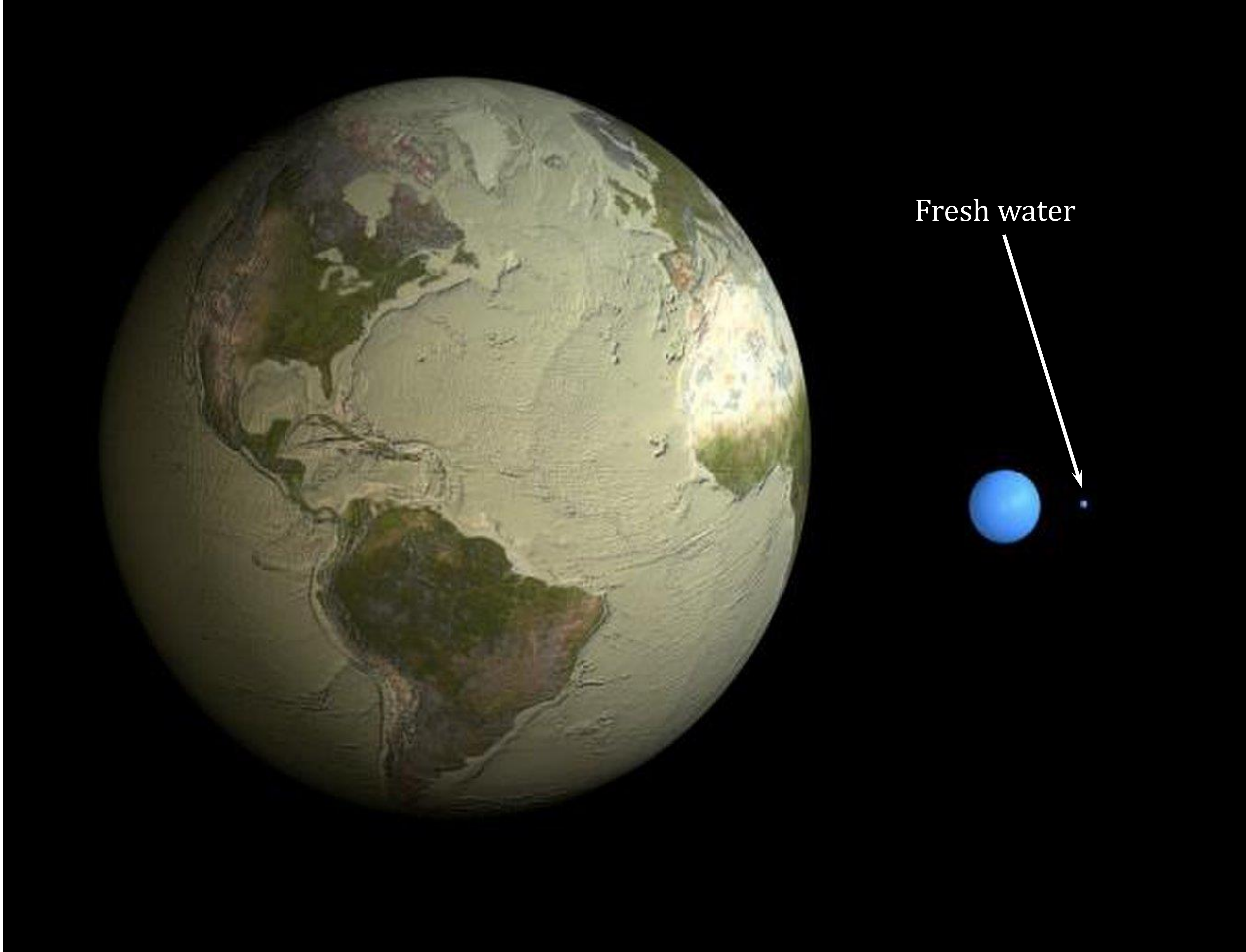
Groundwater hydraulics

Summer semester
ECTS – 6 cr

2020_2021



ENVA + EGS + ERASMUS

Czech University of Life Sciences Prague
Faculty of Environmental Sciences

Administrative Details

3 hours per week (lectures – 2 hours per week
tutorials – 1 hours per week)

Lecturer : **Pavel Pech** (room MCEV 2 – D432)
Department of Water Resources and Environmental Modeling
E-mail: pech@fzp.czu.cz
Tel.: 22438 2132
www: http://home.czu.cz/pech/

- Consultation hours: Mon. 8.00-10.00

The grading will be perform according to the following point distribution:
Thus, the maximum points available are 100 points.

- Homeworks – 4 examples - 20 points
- Final exam 80 points

50-65 good 66-80 very good 81-100 excellent

LITERATURE

KRESIC, N., 2007: Hydrogeology and Groundwater Modeling. CRC Press. pp. 807.

SCHWARTZ, F.W., ZHANG, H., 2003: Fundamentals of Ground Water. John Wiley & Sons. INC.

CHARBENEAU, R., J., 2006: Groundwater Hydraulics and Pollutant Transport. Waveland Pres, INC.

FREEZE, R. ALLAN, CHERY, JOHN A. : Groundwater. Englewood Cliffs, NJ: Prentice Hall Inc. 1979.

HÁLEK, V., ŠVEC, J.: Groundwater Hydraulics. Elsevier. 1979.

BEAR, J., VERRUIJT, A., 1992: Modeling Groundwater Flow and Pollutin.D. Reidel Publ. Comp.Dordrecht.

BATU, V., 1998: Aquifer Hydraulics. John Wiley and Sons. N.Y.

BEAR, J., 1979: Hydraulics of Groundwater. McGraw-Hill, Inc., New York.

ZHENG, C., BENNETT, G.D, 2002: Applied Contaminant Transport Modeling, John Wiley and Sons. N.Y.

FREE BOOKS:

HYDRAULICS:

Han, D. 2008. **Concise Hydraulics.** ISBN 978-87-7681-396-3

Al-Shemmeri, T., T. 2012. **Engineering Fluid Mechanics**. ISBN 978-87-403-0114-4

GROUNDWATER HYDRAULICS:

Jelmert, T.,A. 2013. **Introductory Well Testing**. ISBN 978-87-403-0445-9

+ lectures 2018_2019

http://home.czu.cz/pech/

password (heslo) Hydra2019

prof . Ercan Kahya (Fluid Mechanics)

Classroom rules:

In the classroom:

- do not sleep
- have paper and pencil
- turn off mobile phones
- do nor enter after the lecture has begun
- do not leave until the lecture is complete
- do not eat during lecture
- ask questions when needed

Groundwater hydraulics: Lectures 2018_2019

Z 115

X =	Date	Groundwater Hydraulics (2/1-zk)
1.	12.2.	
2.	19.2	Introduction. Properties of fluids
3.	26.2.	Hydrostatics. Pressure and hydrostatic forces.
4.	5.3.	Hydrodynamics. Flow regimes. Basic equations.
5.	12.3.	Introduction to groundwater. Fundamentals of aquifer hydraulics, effective stress, compressibility and elasticity.
6.	19.3.	Basic equations. Darcy´s law. Dupuits assumptions. Limitations of the Darcian approach.
7.	26.3.	Multi-layered aquifer system. Seepage. Flow nets.
8.	2.4.	Steady and unsteady flow to wells – confined and unconfined aquifers. Pumping and recovery tests – evaluation.
9.	9.4.	Image well theory. Well flow near aquifer boundaries, multiple well problems.
10.	16.4.	Real wells. Wellbore storage, skin effect. Evaluation of well cleanning. Modelling.
11.	23.4.	External lecturer
12.	30.4	**Test**

STUDENTS – ENV. MOD. 2018_2019

Repetition is the mother of all skills..

- Unknown

PHYSICS ...MECHANICS ...FLUID MECHANICS ... HYDRAULICS... GROUNDWATER HYDRAULICS

FLUID MECHANICS – aimed at solving of technical tasks of balance and motion of fluids and mutual effect of fluid and solids

In civil engineering – fluid is „**WATER**“

HYDRAULICS AND GROUNDWATER HYDRAULICS solves:

- under what external conditions
- with what losses
- under which discharge
- under what level and pressure
- in what form
- with what force effect

water moves through pipes, river channels, hydraulic structures or earth environment (porous media)

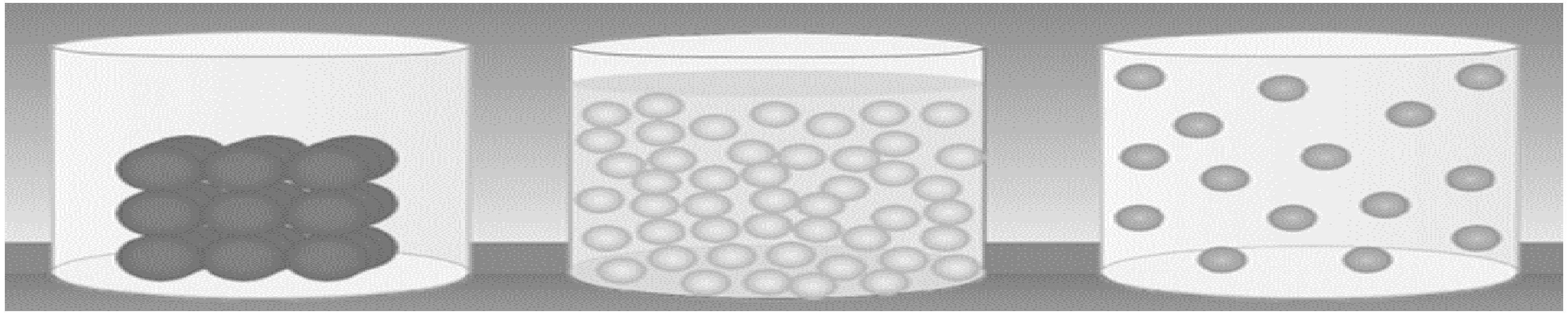
MECHANICS: The oldest physical science that deals with both stationary and moving bodies under the influence of forces.

FLUID MECHANICS: The science that deals with the behavior of fluids at rest (*fluid statics*) or in motion (*fluid dynamics*), and the interaction of fluids with solids or other fluids at the boundaries.

STATICS (HYDROSTATICS): The branch of mechanics that deals with water at rest.

DYNAMICS (HYDRODYNAMICS): The branch that deals with fluid (water) in motion.

STATES OF MATTER



Solid Liquid Gas

SOLID

- The molecules are held together with strong bonds. They don´t move very easily so solids can keep their own shape and size

- ❑ Rigid
- ❑ Fixed shape
- ❑ Fixed volume

LIQUID

- The molecules have weaker bonds. They can move around slightly so liquids can flow. They can´t keep their shape unless they are in a container

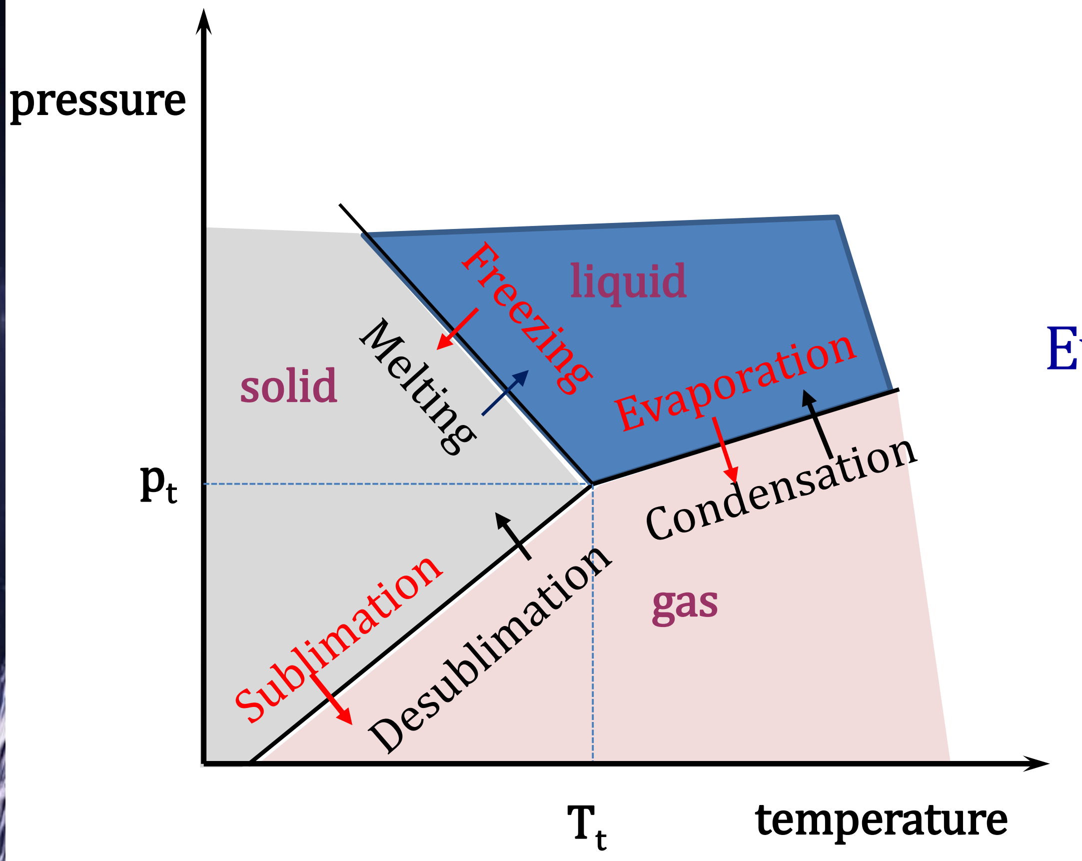
- ❑ Not rigid
- ❑ No fix shape
- ❑ Fixed volume

GAS

- The molecules are free to move around

They can spread around an open space freely and quickly. Gases can´t keep their shape unless they are kept in sealed container

- ❑ Not rigid
- ❑ No fixed shape
- ❑ No fixed volume



3 - PHASES OF WATER: solid x liquid x gas (function of T and p)

Evaporation: Liquid $\rightarrow$ Gas

Condensation: Gas $\rightarrow$ Liquid

Melting: Solid $\rightarrow$ Liquid

Freezing: Liquid $\rightarrow$ Solid

Sublimation: Solid $\rightarrow$ Gas

Desublimation: Gas $\rightarrow$ Solid

For water: $T_t = 0{,}01$ °C; $p_t = 612$ Pa

- A **DIMENSION** is the measure by which a physical variable is expressed qualitatively

		International	SI-units
➢ Basic dimensions: (or primary quantities)	Length	L	m
	Time	T	s
	Mass	M	kg

➢ We can derive any **SECONDARY QUANTITY** from the primary quantities

i.e. Force = (mass) x (acceleration) : $F = M\,L\,T^{-2}$

- A unit is a particular way of attaching a number to the qualitative dimension:

DIMENSIONS AND UNITS

International units	SI Unit	British Gravitational (BG) Unit	English Engineering (EE) Unit
Mass [M]	Kilogram (kg)	Slug	Pound-mass (lb_m)
Length [L]	Meter (m)	Foot (ft)	Foot (ft)
Time [T]	Second (s)	Second (s)	Second (s)
Temperature [Θ]	Kelvin (K)	Rankine (°R)	Rankine (°R)
Force [F]	Newton ($1N=1\ kg \cdot m/s^2$)	Pound (lb)	Pound-force (lb_f)

NON – SI UNITS

Quantity	Unit	Symbol	Derivation
Time	minute	min	60 s
Time	hour	h	3 600 s
Temperature	degree Celsius	°C	K-273.15
Angle	degree	°	π/180 rad
Speed	kilometre per hour	km/h	-
Volume	litre	l	10^{-3} m^3
Pressure	bar	b	10^2 kN m^{-2}

MULTIPLES OF UNITS

Name	Symbol	Factor	Number
giga	G	10^9	1 000 000 000
mega	M	10^6	1 000 000
kilo	K	10^3	1 000
milli	m	10^{-3}	0. 001
micro	μ	10^{-6}	0. 000 001

CONVERSION FACTORS

Item	Conversion
Length	1 in = 25.4 mm 1 ft = 0.3048 m 1 yd = 0.9144 m 1 mile = 1.609 km
Mass	1 lb = 0.4536 kg
Volume	1 in^3 = 16.39 cm^3 1 UK gallon = 4.546 litre 1 US gallon = 3.785 litre
Velocity	1 km/h = 0.2778 m/s
Pressure	1000 Pa = 1000 N m^{-2} = 0.01 bar
power	1 horsepower = 745.7 W

DERIVED UNITS WITH SPECIAL NAMES

Quantity	Unit	Symbol	Derivation
Force [F]	Newton	N	kg m s^{-2}
Work, Energy [E]	Joule	J	N m
Power [P]	Watt	W	J s^{-1}
Pressure [p]	Pascal	Pa	N m^{-2}

DIMENSIONS AND UNITS		
Quantity	**Symbol**	**Dimensions**
Velocity	v	LT^{-1}
Acceleration	a	LT^{-2}
Area	A	L^2
Volume	V	L^3
Discharge	Q	L^3T^{-1}
Force	F, G	$M\ L\ T^{-2}$
Pressure	p	$ML^{-1}T^{-2}$
Gravity acceleration	g	LT^{-2}
Temperature	T	Θ
Mass concentration	C	ML^{-3}

FORCES IN LIQUID

INTERNAL FORCES – molecular, electromagnetic phenomena, thermal motion of molecules they are not taken into account (exception – surface tension and capilarity)

EXTERNAL FORCES – consequence of force field

A. **Body** (mass, volume) forces - inertia force, gravity force

From Newton´s law:

$$F = m \cdot a$$

m- mass

a – acceleration

B. **Surface** forces– pressure force, tension force

$$F_\sigma = \sigma A$$

σ –tension

A - area

$$F = p.A$$

p - pressure

FLUID PROPERTIES

- **FUNDAMENTAL APPROACH**: Study the behavior of individual molecules when trying to describe the behavior of fluids
- **ENGINEERING APPROACH**: Characterization of the behavior by considering the average, or **macroscopic**, value of the quantity of interest, where the average is evaluated over a small volume containing a large number of molecules

  ∴ Treat the fluid as a **CONTINUUM**: Assume that all the fluid characteristics vary continuously throughout the fluid

FLUID PROPERTIES

- **DENSITY,** ρ (kg/m^3) (H_2O approx.1000) The density of a fluid is defined as mass per unit volume
  - SALINITY
  - TEMPERATURE

$$\rho = \frac{dm}{dV} \text{.........} \rho = \frac{m}{V}$$

- **SPECIFIC WEIGHT,** γ (N/m^3) (H_2O approx. 9810)
  - $\gamma = \rho \cdot g$ (H_2O approx. 9810)

Water

Temperature (^oC)	Density (kg/m^3)
0	999.87
+4	1000
+10	999.73
+20	998.23
+100	958.4

The specific weight of fluid is its weight per unit volume.

- **SPECIFIC VOLUME:** $\upsilon = \frac{1}{\rho}$

Volume occupied by unit mass of fluid.

Specific volume is the **reciprocal of density.**

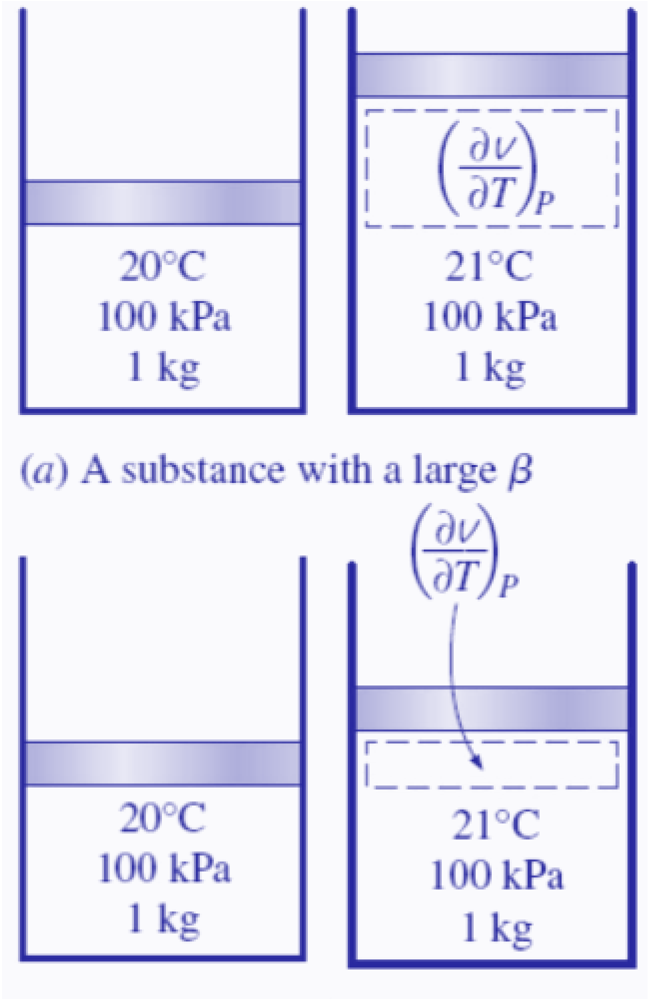
Densities of Some Common Substances

Material	Density (kg/m^3)*	Material	Density (kg/m^3)*
Air (1 atm, 20°C)	1.20	Iron, steel	7.8×10^3
Ethanol	0.81×10^3	Brass	8.6×10^3
Benzene	0.90×10^3	Copper	8.9×10^3
Ice	0.92×10^3	Silver	10.5×10^3
Water	1.00×10^3	Lead	11.3×10^3
Seawater	1.03×10^3	Mercury	13.6×10^3
Blood	1.06×10^3	Gold	19.3×10^3
Glycerine	1.26×10^3	Platinum	21.4×10^3
Concrete	2×10^3	White dwarf star	10^{10}
Aluminum	2.7×10^3	Neutron star	10^{18}

- **VOLUME COMPRESSIBILITY** (p)
- It is defined as:

  Change in volume due to change in pressure.”

$$\frac{\Delta V}{V_0} = -\beta_p \cdot \Delta p \qquad \beta_p = \frac{\Delta V}{V_0 \cdot \Delta p} \qquad [\mathrm{Pa}^{-1}]$$



(*a*) A substance with a large β

(*b*) A substance with a small β

- **VOLUME EXPANSIVITY** (T)

  Change in volume due to change in temperature.”

$$V = V_0(1+\beta\Delta T) \qquad \beta = \frac{\Delta V}{V_0 \cdot \Delta T} \qquad [\mathrm{K}^{-1}]$$

• SURFACE TENSION - CAPILLARITY

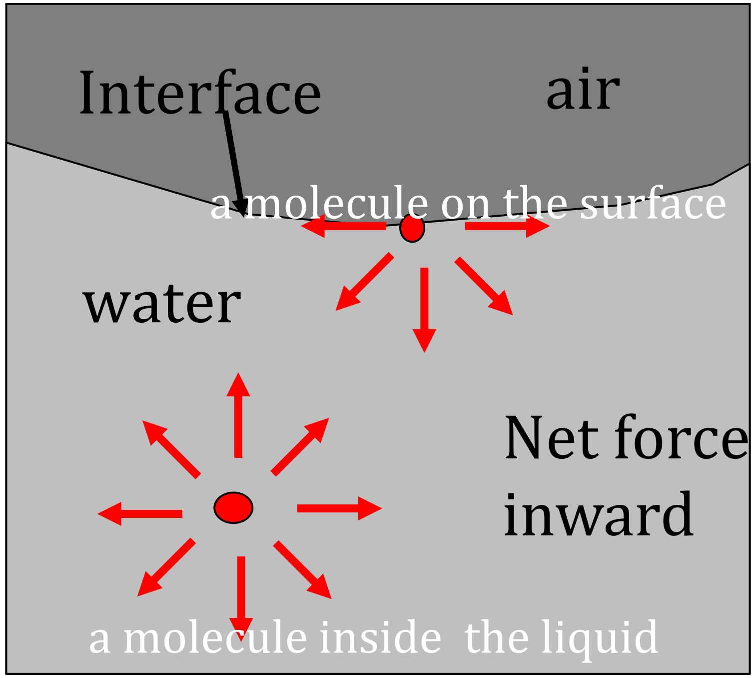
Below surface, forces act equally in all directions

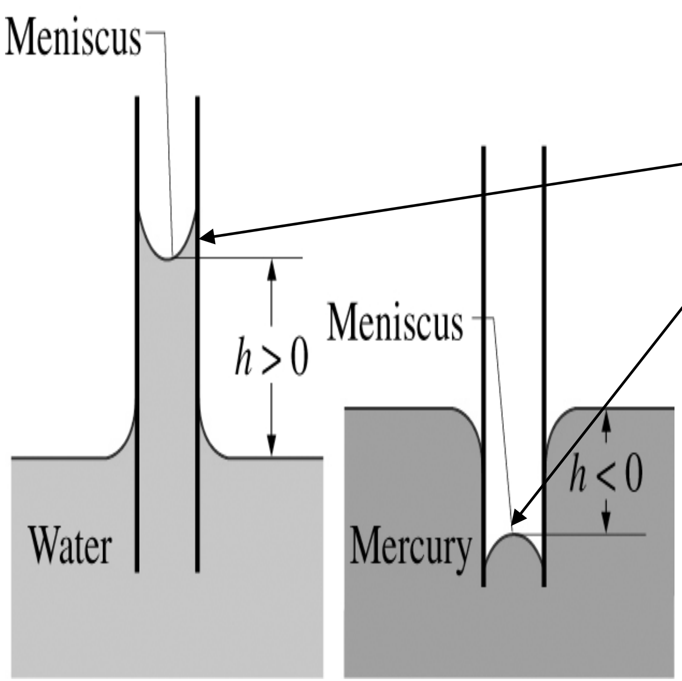
At surface, some forces are missing, pulls molecules down and together, like membrane exerting *tension* on the *surface*

If interface is curved, higher pressure will exist on concave side

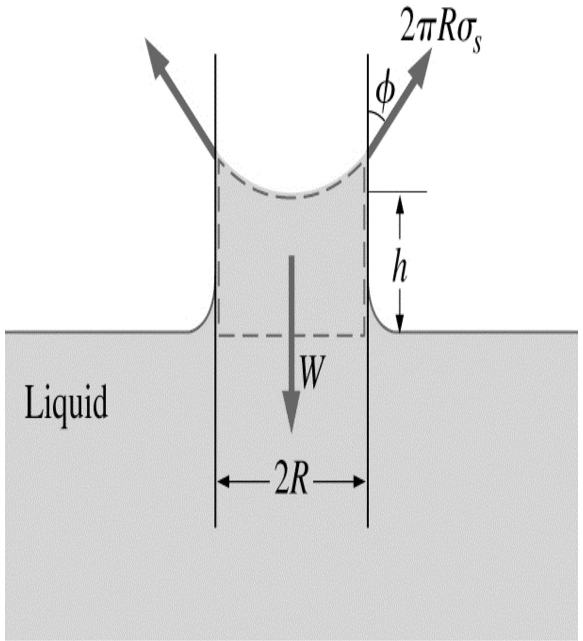
Pressure increase is balanced by surface tension, σ

σ **= 0.073 N/m (@ 20°C)**





- **CAPILLARY EFFECT** is the rise or fall of a liquid in a small-diameter tube.
- The curved free surface in the tube is call the **meniscus**.
- **Water** meniscus curves up because water is a ***wetting fluid.***
- **Mercury** meniscus curves down because mercury is a ***nonwetting fluid.***



Equilibrium of surface tension force and gravitional pull on the water cylinder of height produces:

$$2\pi\sigma R\cos\phi = \pi R^2 h\gamma \quad \Rightarrow \quad h = \frac{2\sigma\cos\phi}{\gamma R}$$

σ	surface tension
ϕ	angle - liguid x solid
γ	specific weight of liquid
R	radius of tube

VISCOSITY

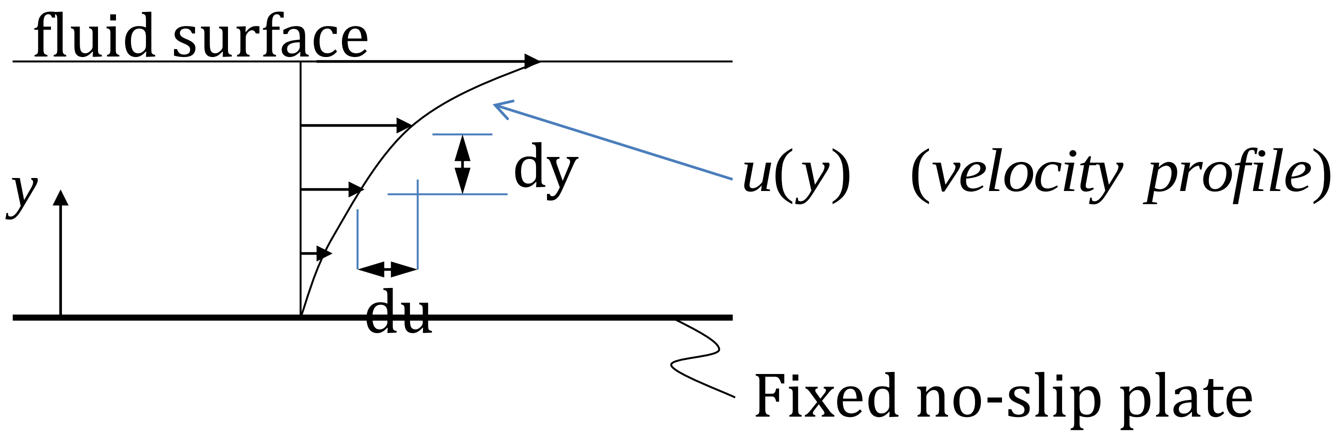
NEWTON'S EQUATION OF VISCOSITY

Viscosity is a measure of the resistance of a fluid to deform under shear stress.

Shear stress due to viscosity between layers: $\tau = \mu \dfrac{du}{dy}$

μ - **dynamic viscosity** (coeff. of viscosity)

$\nu = \dfrac{\mu}{\rho}$ - kinematic viscosity



Use definition of shear force:

$$F = \tau A = \mu A \frac{du}{dy}$$

IDEAL FLUID

- An *ideal* fluid may be defined as:

  “A fluid in which there is *no friction* *i.e* zero viscosity.”

- Although such a fluid does not exist in reality, many fluids approximate frictionless flow at sufficient distances, and so their behaviors can often be conveniently analyzed by assuming an ideal fluid.

REAL FLUID

- friction forces give rise to a fluid property called *viscosity*.

STANDARDS IN HYDRAULICS

Acceleration of gravity $g = 9.81 \text{ m s}^{-2}$

Atmospheric pressure $(p_{at}) – 1.013 \,.\, 10^{5}$ Pa

Properties of water (T = 15 °C (39 °F) and p = 1 atm)

Density of water $\rho = 999 \text{ kg m}^{-3}$

Density of air at 4° C : 1.20 kg/m^3

Specific weight $\gamma = 9800 \text{ N m}^{-3}$

Surface tension $\sigma = 0.073 \text{ N m}^{-1}$

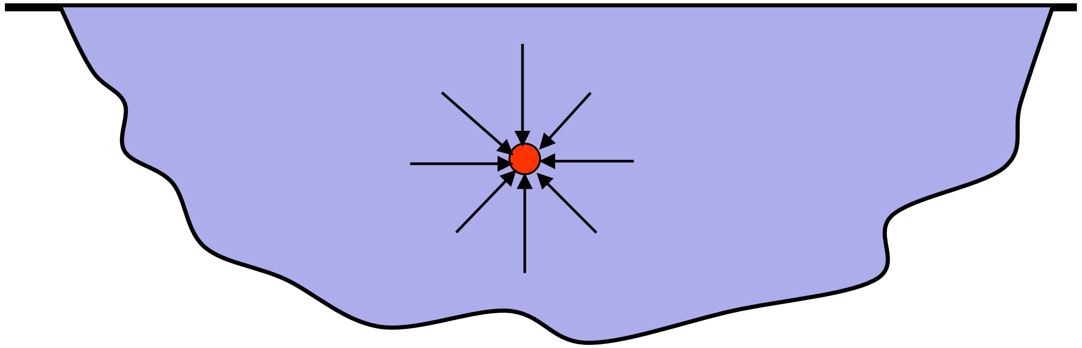
Viscosity $\mu = 1.14 \,.\, 10^{-3} \text{ N.s m}^{-2}$

Kinematic viscosity $\nu = 1.14 \,.\, 10^{-6} \text{ m}^{2}\text{.s}^{-1}$

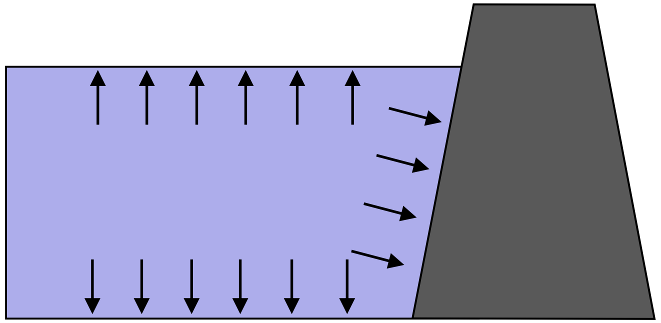
DEFINITION OF PRESSURE

Pressure is defined as the amount of force exerted on a unit area of a substance:

$$P = F / A$$



Pressure is a *Normal Force*
(It acts perpendicular to
the surface)
It is also called a *Surface Force*

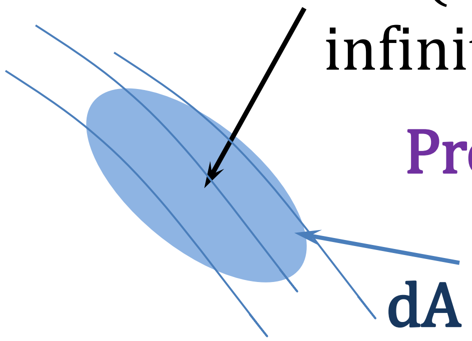


Dam

DEFINITION OF PRESSURE

Pressure is defined as the ratio of normal force to area at a point (force per unit area)

dF (elementary force … that acts on an infinitesimal unite of area dA)



Pressure : $p = dF/dA$ [Pa] - for infinite area

$p = F/A$ [Pa] - for finite area

Pressure force

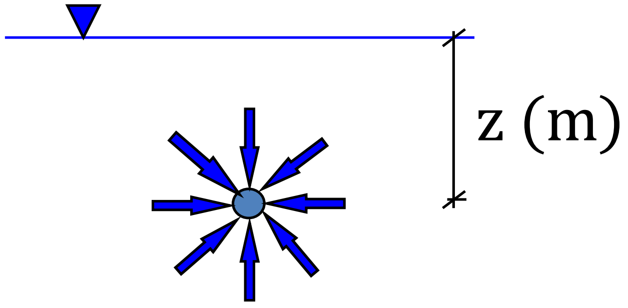
Units: N/m^2(Pa), lbs/ft^2 (psf), lbs/in^2 (psi)

$p = dF/dA \Rightarrow dF = p\,dA$

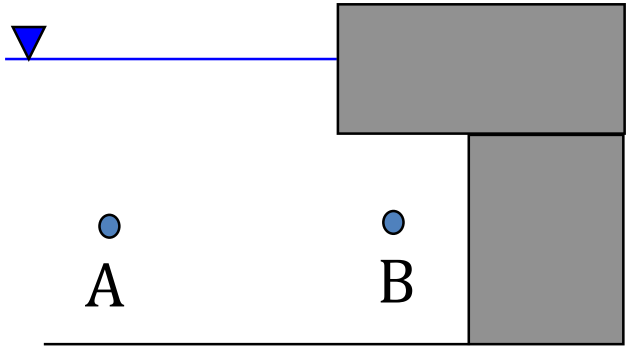
$dF = p\,dA$ …. $\int_A dF = \int_A p\,dA \Rightarrow$ $F = p\,A$

$1\text{ atm} = 1.013 \cdot 10^5\text{ Pa} = 760\text{ torr} = 14.7\text{ psi.}$

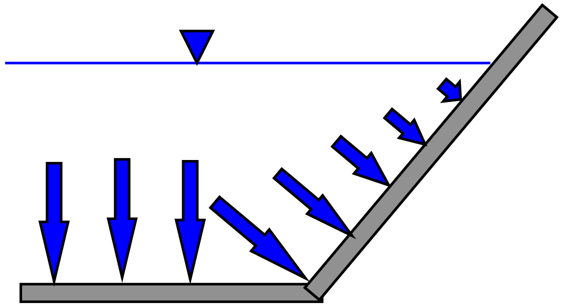
PRESSURE PROPERTIES

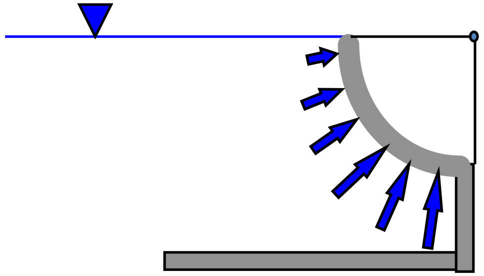


Pressure at any point in a fluid is the same in all directions   p=ρgz



Pressure the same at A and B.





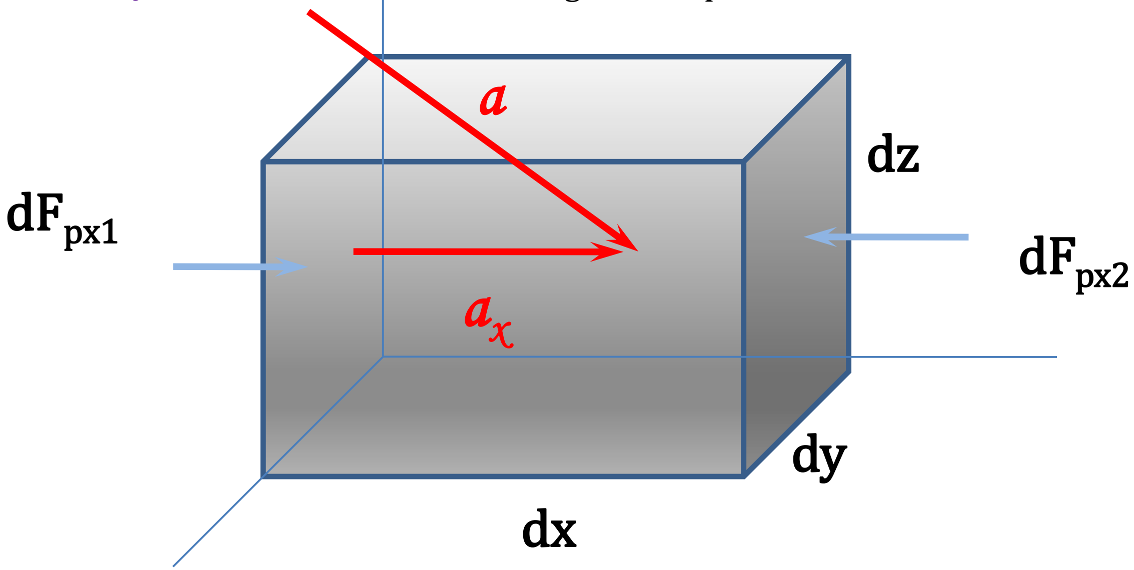
Pressure is always perpendicular to a surface.

HYDROSTATIC DIFFERENTIAL EQUATION (Euler´s eq.)

- is derived by applying force equilibrium to a static body of fluid

$a\ (a_x, a_y, a_z)$

$$F_{weight} + F_{pressure} = 0$$



$$dm = \rho.dV = \rho.dx.dy.dz$$

$$dF_{ox} = a_x\ .\ dm = a_x\ .\ \rho.\ dx.dy.dz$$

$$dF_{px1} = p.dy.dz$$

$$dF_{px2} = (\ p + dp).\ dy.dz$$

- apply force equilibrium in the x direction

$$p.dy.dz - \left(p + \frac{\partial p}{\partial x} dx \right).dy.dz + \rho.a_x.dx.dy.dz = 0$$

$$a_x - \frac{1}{\rho}.\frac{\partial p}{\partial x} = 0 \Rightarrow \boxed{\rho.a_x = \frac{\partial p}{\partial x}}$$

For y and z direction :

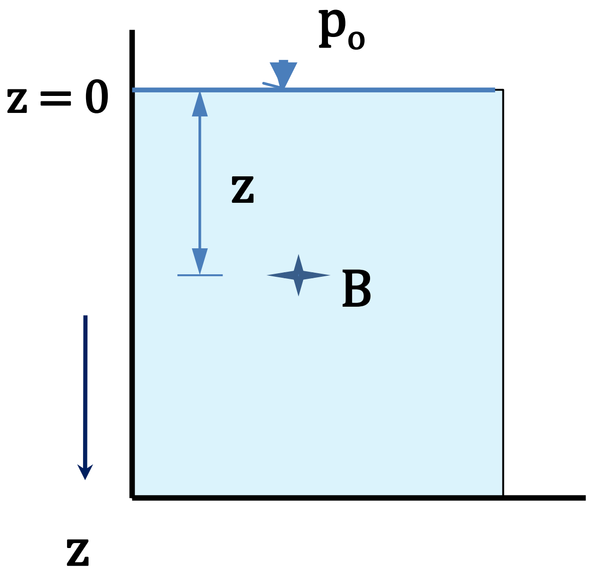
$$\rho.a_y = \frac{\partial p}{\partial y} \qquad \rho.a_z = \frac{\partial p}{\partial z}$$

The final result is

$$dp = \rho\left(a_x.dx + a_y.dy + a_z.dz\right) = \left(\frac{\partial p}{\partial x}\right).dx + \left(\frac{\partial p}{\partial y}\right).dy + ..$$

Euler´s hydrostatic equation

HYDROSTATICS : DETERMINATION OF PRESSURE



Mass forces

$$\rho . a_x = \frac{\partial p}{\partial x}$$

- For gravity force .... $a_z = g$

$(a_x = a_y = 0)$

$$dp = \rho g \, dz$$

pro $\rho = \text{const.}$ and $g = \text{const.}$

$$\int dp = \int \rho \, g \, dz$$

$$p = \rho g z + C$$

C - (integral constant) from condition at the free water level

Pressure at point B

$$p_B = p_0 + \rho g z$$

Pressure head $\frac{p}{\rho g}$

It is the pressure expressed in terms of height of fluid.

The term **elevation (head)** means the vertical distance from some reference level to a point of interest.

Piesometric head

$$h = \frac{p}{\rho g} + z = \frac{p}{\gamma} + z$$

ABSOLUTE AND GAGE PRESSURE

- **ABSOLUTE PRESSURE**: The pressure of a fluid is expressed relative to that of vacuum (=0)
- **GAGE PRESSURE**: pressure-measuring devices are calibrated to read zero in the atmosphere, and therefore indicate gage pressure,

➢ Usual pressure gages record gage pressure. To calculate absolute pressure:

$$P_{abs} = P_{atm} + P_{gage}$$

Pressure below atmospheric pressure is called **VACUUM PRESSURE**,

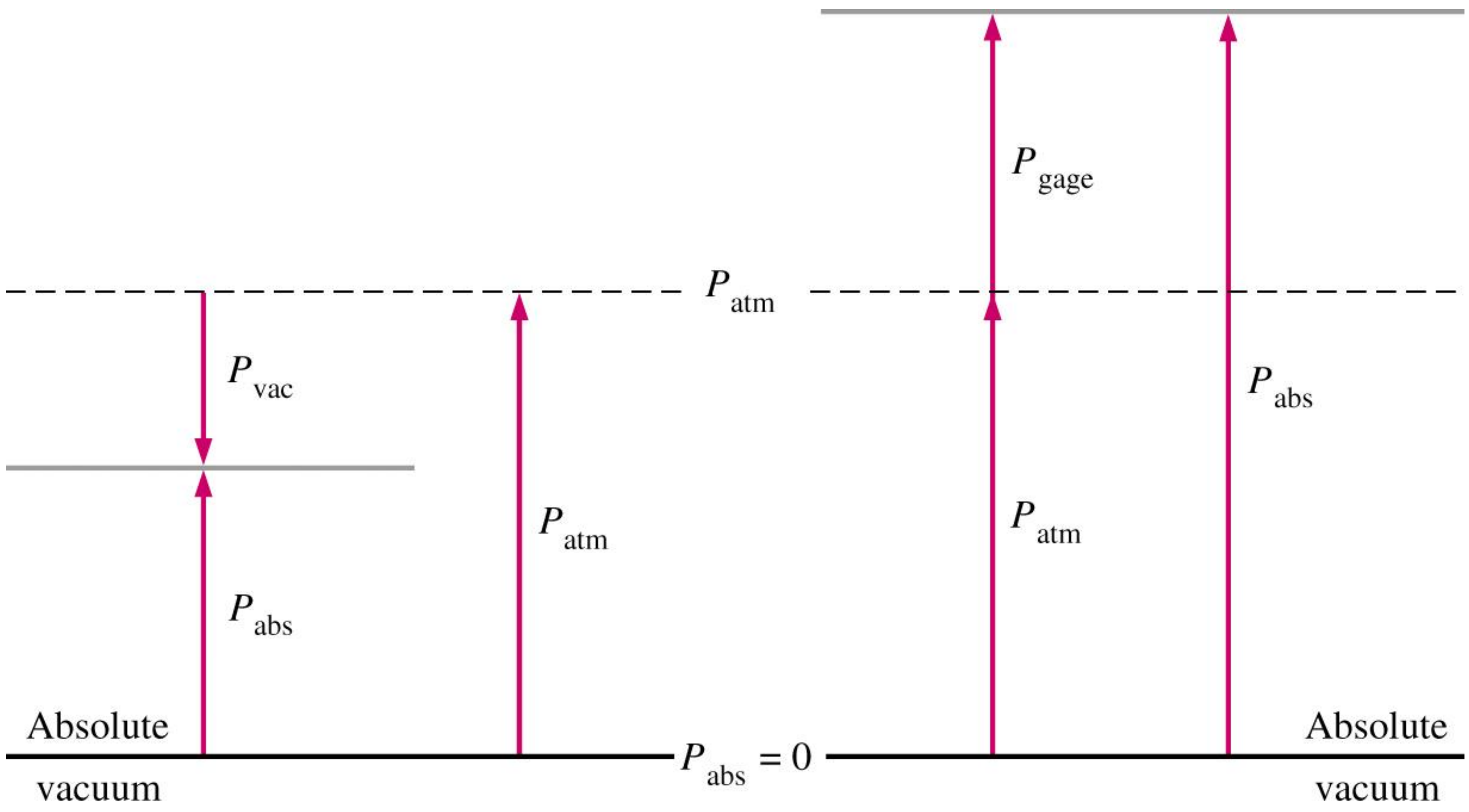
$$P_{vac} = P_{atm} - P_{abs}.$$

PRESSURE

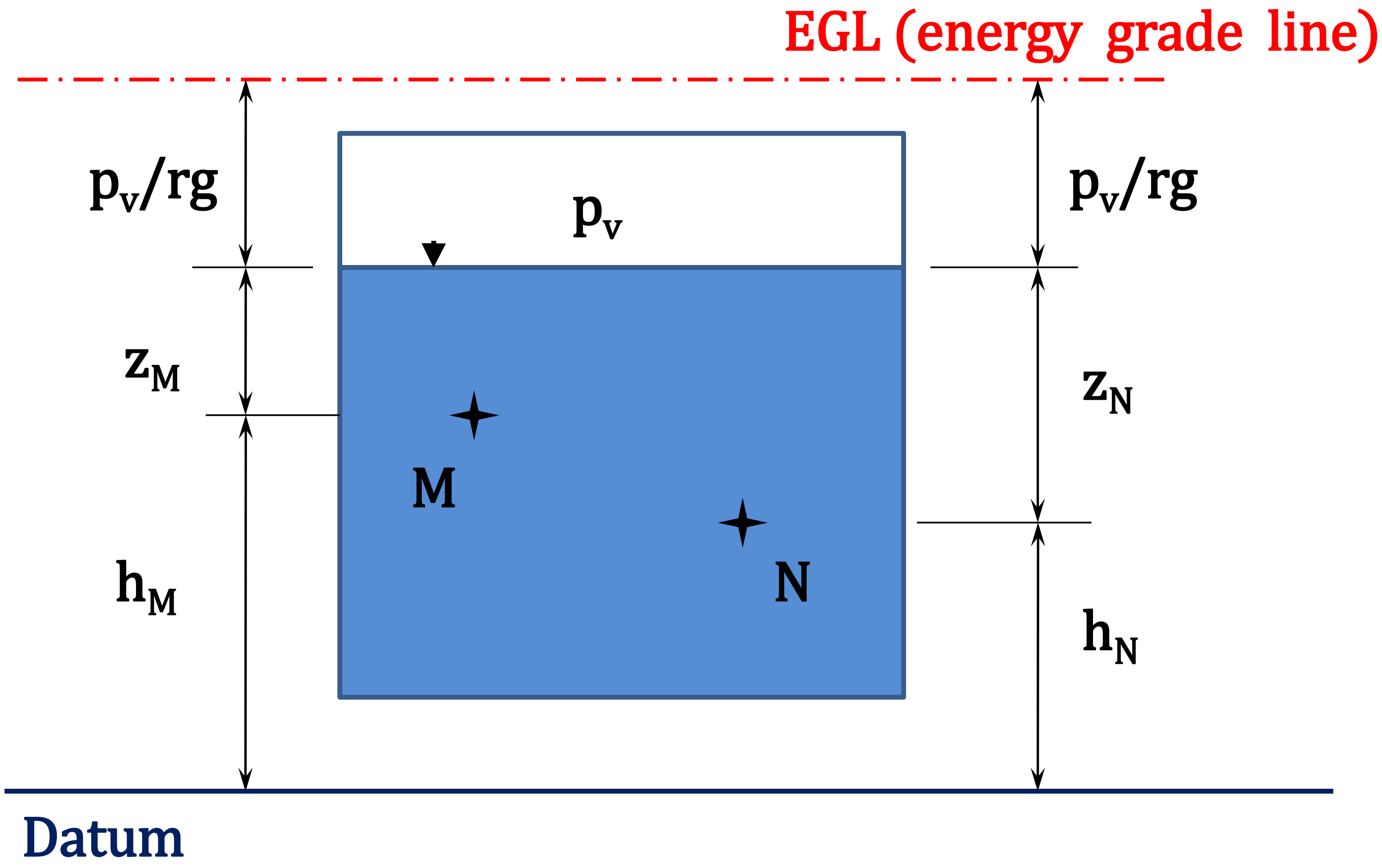
$$p_{abs} = p_{atm} + p_{gage}$$

- **Atmospheric Pressure:** It is the force per unit area exerted by the weight of air above that surface in the atmosphere
- **Gage Pressure: It is the pressure, measured with the help of pressure measuring** instrument in which the atmospheric pressure is taken as Datum
- **Absolute Pressure:** It is the pressure equal to the sum of atmospheric and gauge pressures.
- If we measure pressure relative to absolute zero (perfect Vacuum) we call it **absolute pressure.**
- **Vacuum**: If the pressure is below the atmospheric pressure we call it as vacuum.

Absolute, gage, and vacuum pressures



PRESSURE TANK WITH FLUIDS



$$h_M + z_M + \frac{p_v}{\rho g} = h_N + z_N + \frac{p_v}{\rho g} = konst.$$

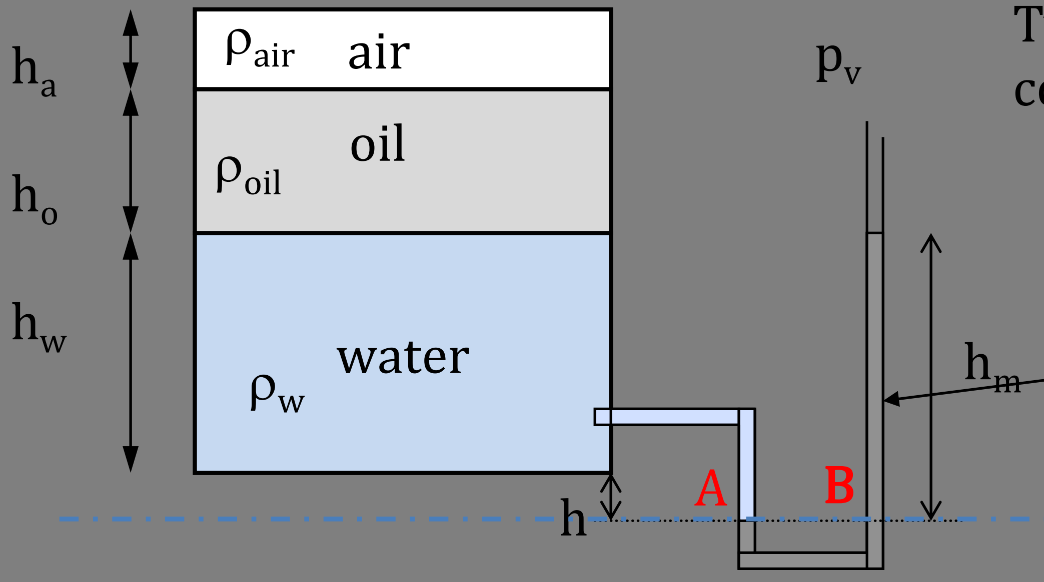
PRESSURE IN TANK WITH THREE FLUIDS

For multi-fluid systems

Pressure change across a fluid column of height *h is* $\Delta P = \rho g h$.

Pressure increases downward, and decreases upward.

Two points at the same elevation in a continuous fluid are at the same pressure.



Manometer liquid: mercury

GGL

Calculate the gage pressure of air.

A : $p_{air} + p_{oil} + p_{water}$ $\quad h_a \rho_{air} g + h_o \rho_{oil} g + (h_w + h) \rho_w g =$

B : $p_v + p_m = p_v + h_m \rho_m g$

Blaise Pascal (1623-1662)

Pressure at a Point: Pascal's Law $F_S \rangle\rangle F_{V(B)}$

F_S – surface force; F_V – volume force

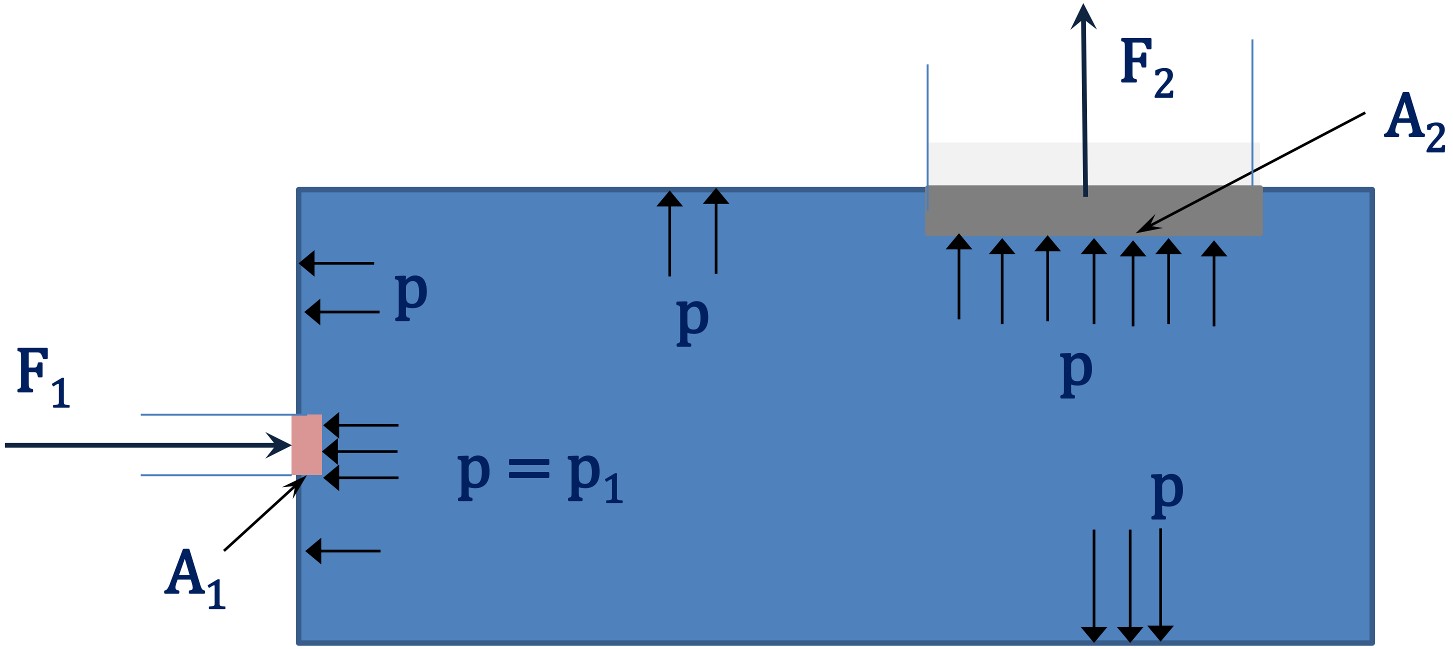
Pressure is the normal force per unit area at a given point acting on a given plane within a fluid mass of interest.

Pressure is independent of direction!

In a closed system, pressures transmitted to a fluid are identical to all parts of the container.

Gradual pressure change dp in small closed volume of liquid is the same in all directions and passes on all points of liquid without any change.

Pascal's Law: the pressure at a point in a fluid at rest, or in motion, is independent of the direction as long as there are no shearing stresses present.

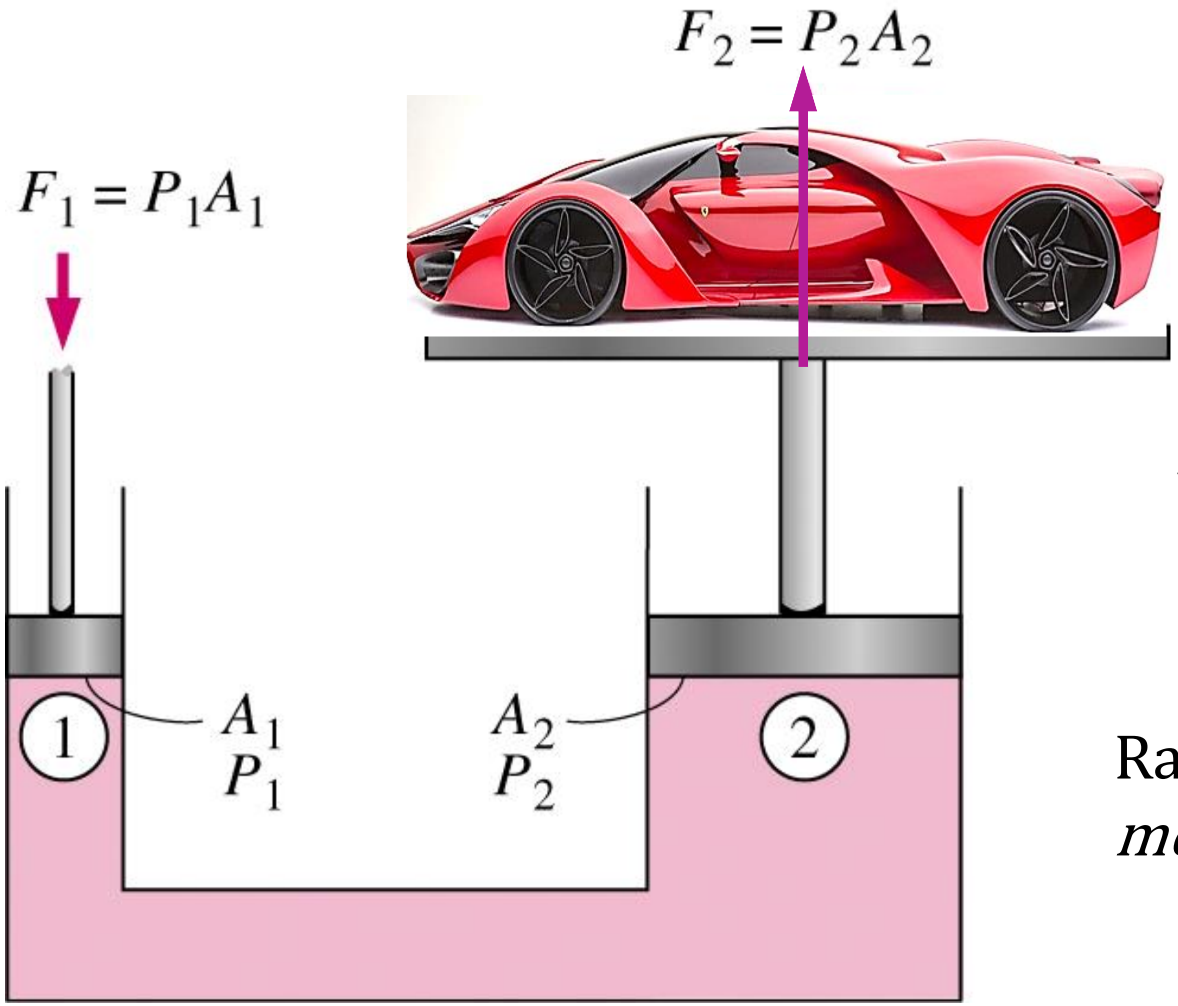


$$p_1 = \frac{F_1}{A_1} = p_2 = \frac{F_2}{A_2} = p \quad \Rightarrow \quad F_2 = p\,A_2 = F_1 \frac{A_2}{A_1}$$

$$F_2 = \eta\, F_1 \frac{A_2}{A_1}$$

η - loss coefficient

p-pressure; A-area; F-force;



In picture, pistons are at same height:

$$P_1 = \frac{F_1}{A_1} = P_2 = \frac{F_2}{S_2} \rightarrow \frac{F_2}{F_1} = \frac{A_2}{A_1}$$

Ratio A_2/A_1 is called *ideal mechanical advantage*

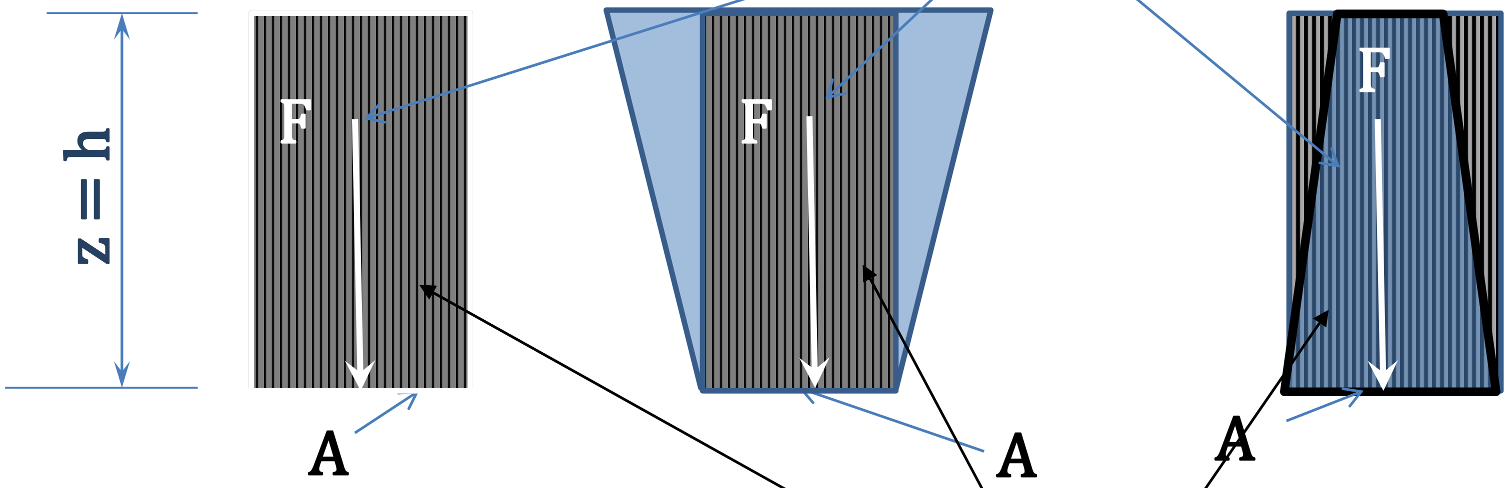
Lifting of a large weight by a small force by the application of Pascal´s law

a) HORIZONTAL BOTTOM

$F = p.A = \rho.g.h.A$ Volume of pressure body

Hydrostatic paradoxon

Pressure prism



$$dF = \rho.g.z.dA \rightarrow \quad F = \int_A dF = \int_A \rho.g.z.dA = \rho.g.z\int_A dA$$

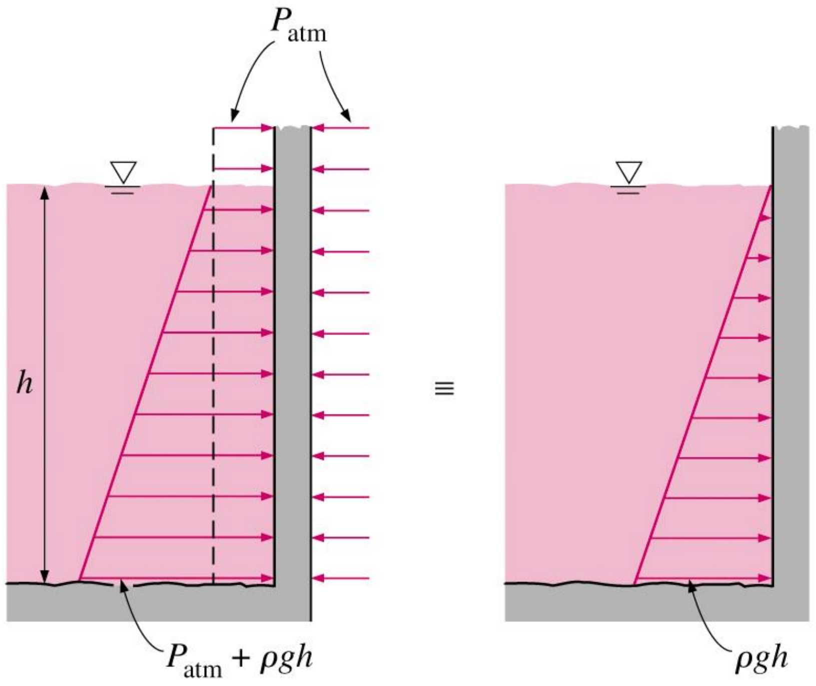
$$\rightarrow \quad F = p.A = \rho.g.h.A$$

V_{PB}=h. A – Volume of pressure body

A_{PD} - area of pressure diagram

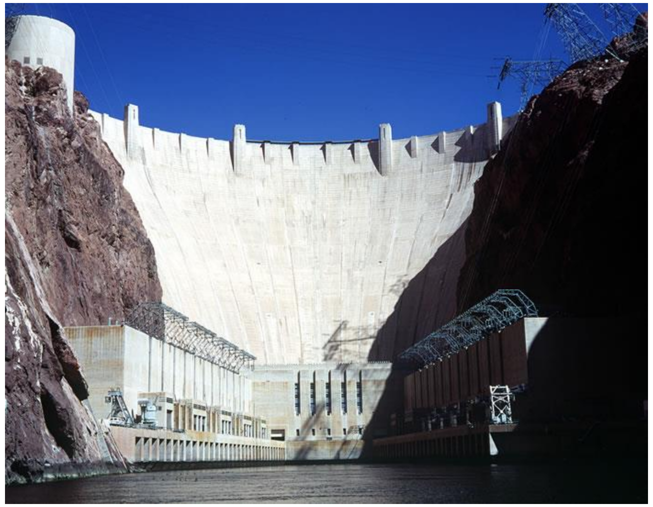
Pressure prism is a geometric representation of *hydrostatic forces*

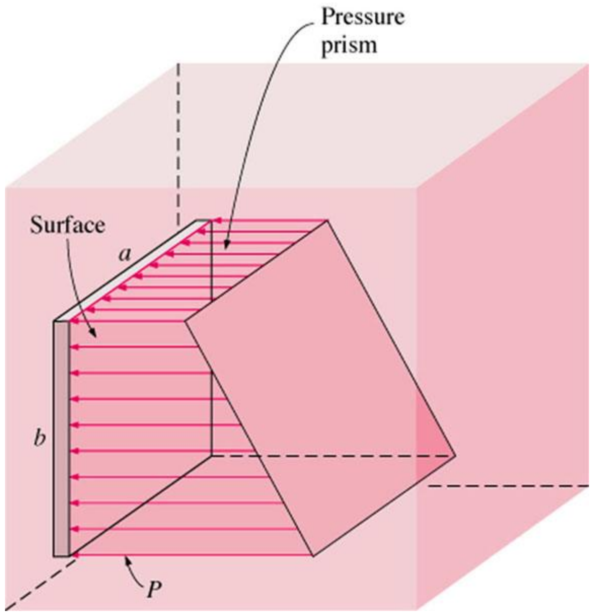
HYDROSTATIC FORCES ON PLANE SURFACES

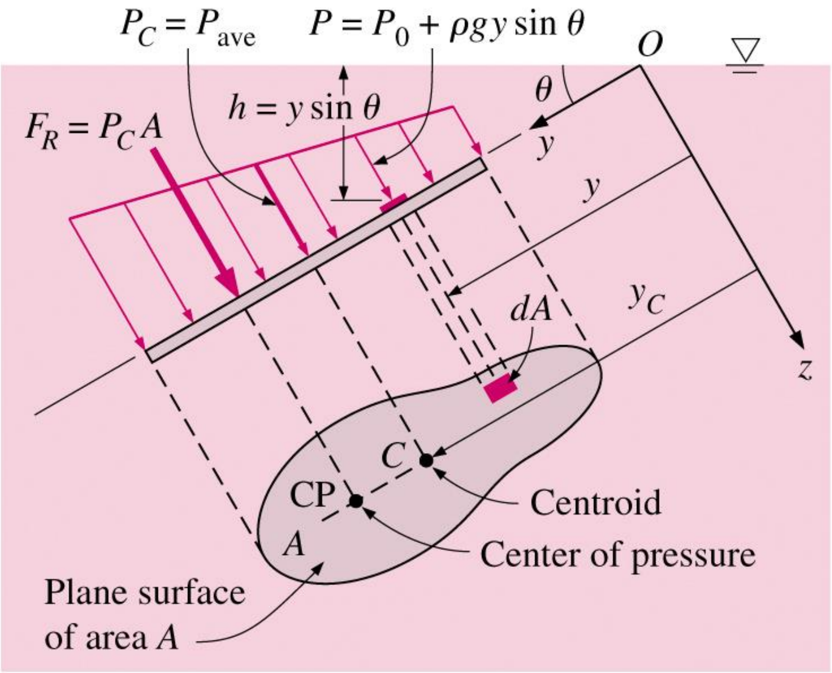


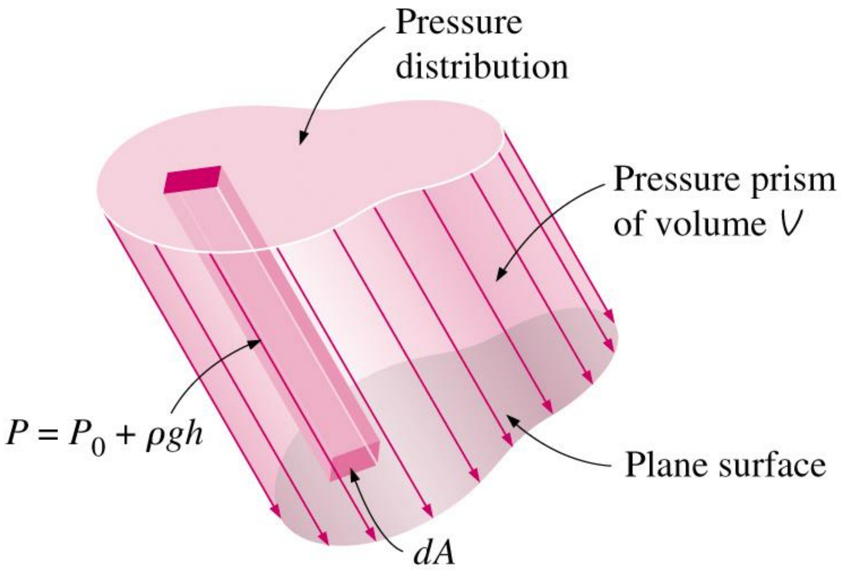
(*a*) P_{atm} considered (*b*) P_{atm} subtracted

- On a *plane* surface, the hydrostatic forces form a system of parallel forces
- Atmospheric pressure P_{atm} can be neglected when it acts on both sides of the surface.



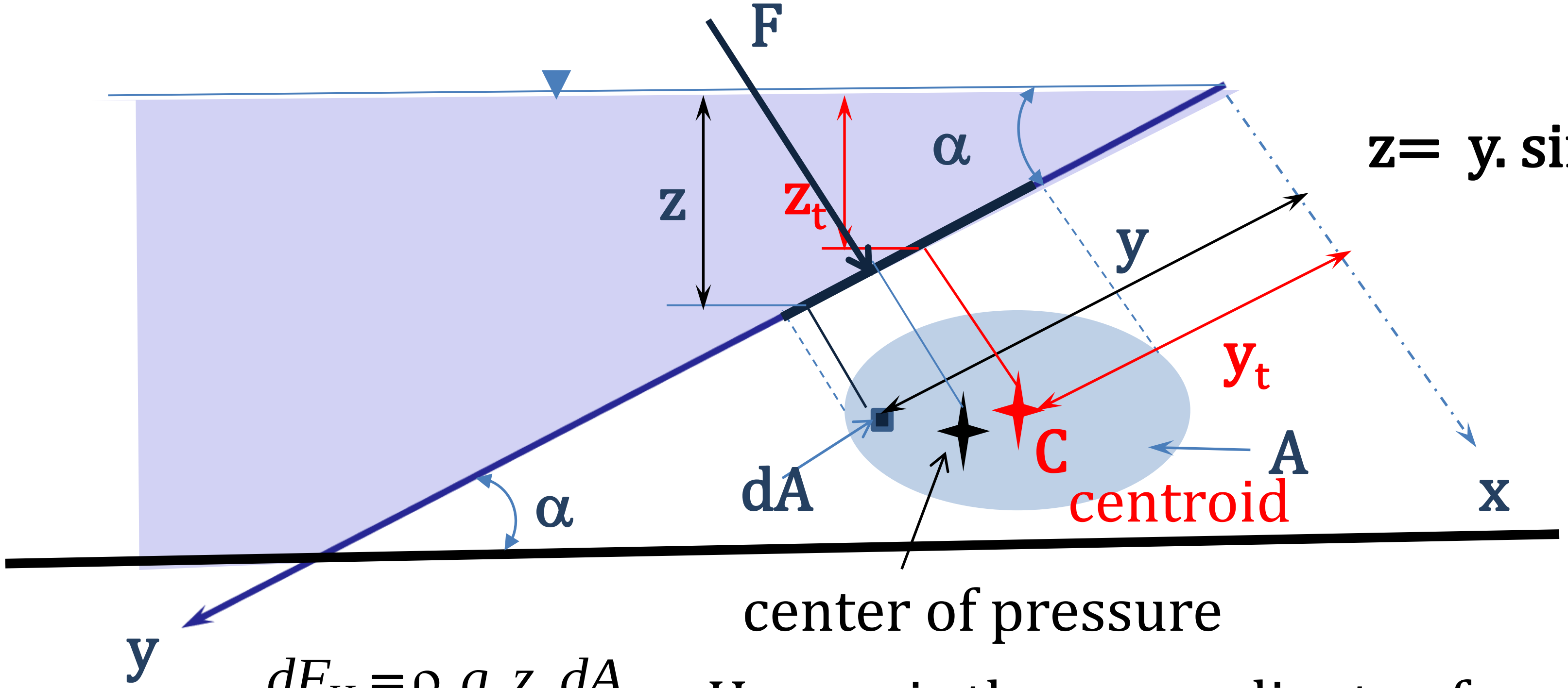






$$V = \int dV = \int P\, dA = F_R$$

METHOD FOR SIMPLE INCLINED PLANE SURFACES



$z = y.\sin\alpha$

$$dF_H = \rho.g.z.dA$$

Here y_t is the y-coordinate of centroid of area

$$F_H = \int_A \rho.g.z.dA = \int_A \rho.g.\,y.\sin\alpha.dA = \rho.g.\sin\alpha \int_A y.dA$$

$\int_A y.dA$ .....Moment of an area A about the x axis

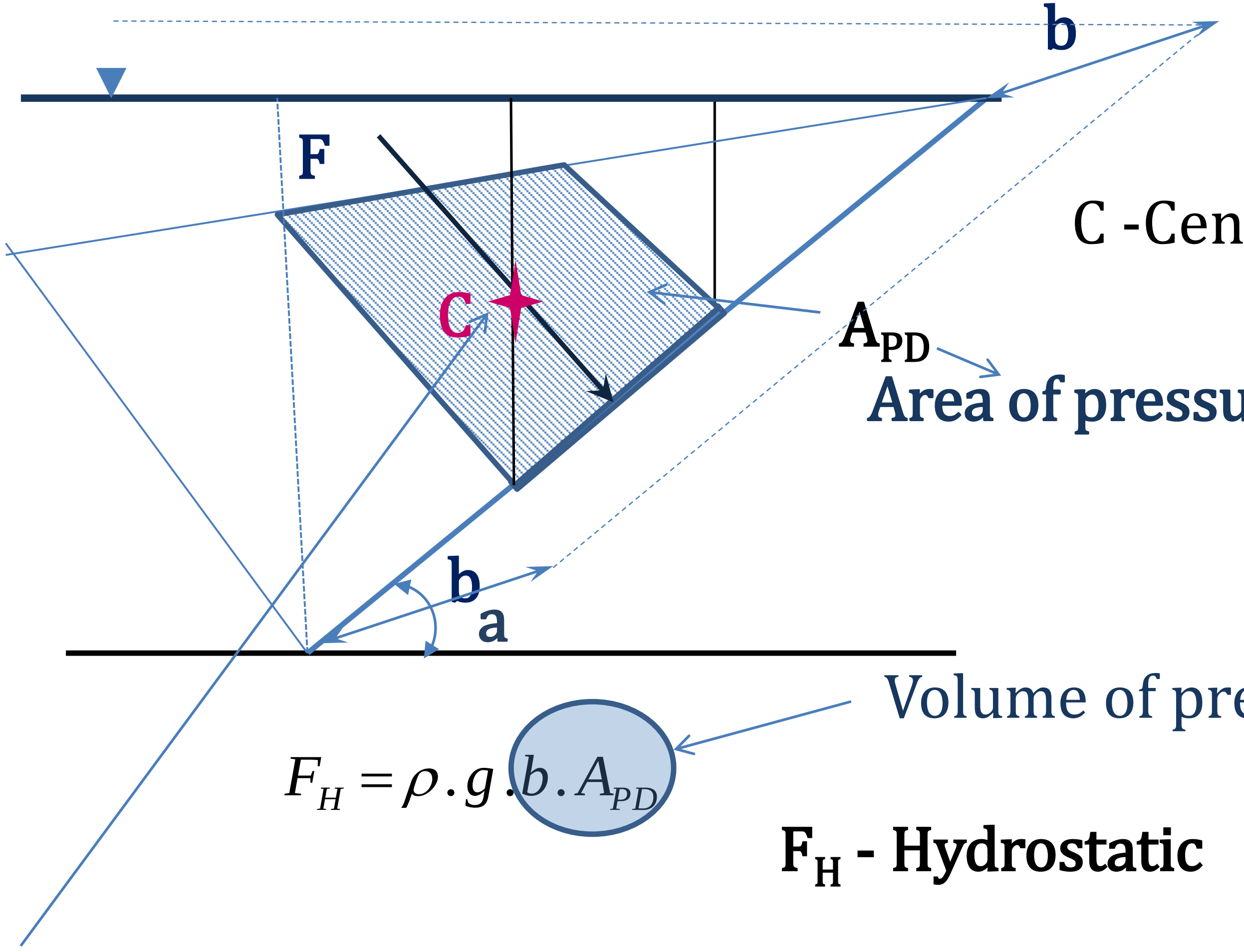
$$M_x = y_T.A$$

$$F_H = \rho.g.\sin\alpha.y_T.A = \rho.g.z_T.A$$

Pressure prism

Pressure Prism method

Direction of force
is normal to the plane



C -Centroid of the area (PD)

A_{PD} Area of pressure diagram (PD) $[m^2]$

Volume of pressure body $[m^3]$

$$F_H = \rho . g . b . A_{PD}$$

F_H - Hydrostatic Force

F passes through the centroid of the area (PD)

HYDRODYNAMICS

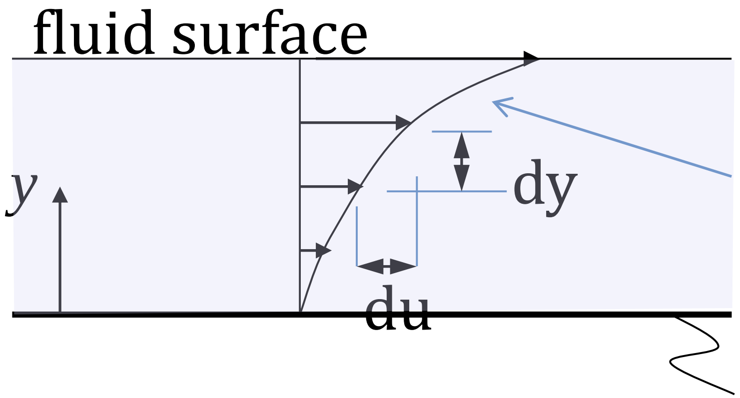
NEWTON'S EQUATION OF VISCOSITY

Viscosity is a **measure** of the resistance of a fluid to **deform under shear stress.**

SHEAR STRESS due to viscosity between layers: $\tau = \mu \dfrac{du}{dy}$

μ - **dynamic viscosity** (coeff. of viscosity)

$\nu = \dfrac{\mu}{\rho}$ - **kinematic viscosity**



$u(y)$ $(velocity\ profile)$

Fixed no-slip plate

Use definition of
SHEAR FORCE:

$$F = \tau A = \mu A \frac{du}{dy}$$

Dynamic viscosities of some fluids at 1 atm and 20°C (unless otherwise stated)

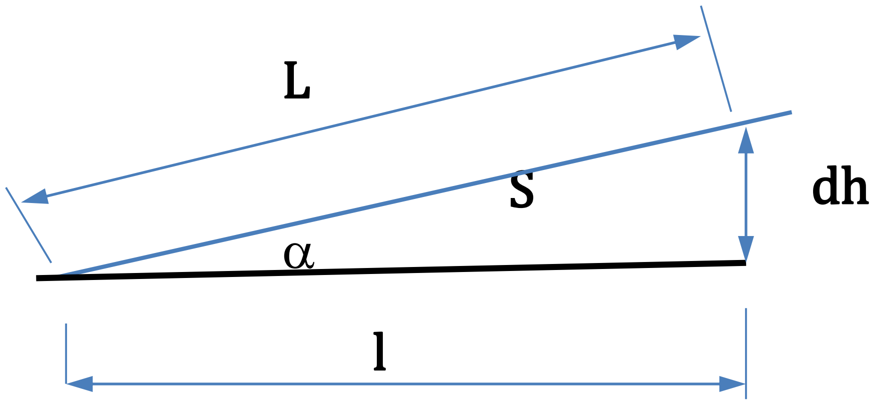
Fluid	Dynamic Viscosity μ, kg/m · s
Glycerin:	
−20°C	134.0
0°C	10.5
20°C	1.52
40°C	0.31
Engine oil:	
SAE 10W	0.10
SAE 10W30	0.17
SAE 30	0.29
SAE 50	0.86
Mercury	0.0015
Ethyl alcohol	0.0012
Water:	
0°C	0.0018
20°C	0.0010
100°C (liquid)	0.00028
100°C (vapor)	0.000012
Blood, 37°C	0.00040
Gasoline	0.00029
Ammonia	0.00015
Air	0.000018
Hydrogen, 0°C	0.0000088

Cengel_Cimbala, 2006

CHARACTERISTICS OF HYDRODYNAMICS

flow area, CROSS SECTIONAL AREA (perpendicular to velocity, v) A(m^2)

Slope - S



α	sin(α)	tan(α)
0°	0	0
5°	0.087	0.087
10°	0.174	0.176
20°	0.342	0.346
30°	0.500	0.577
40°	0.643	0.839
50°	0.766	1.192

$$S = \frac{dh}{L} \Rightarrow \frac{dh}{l}$$

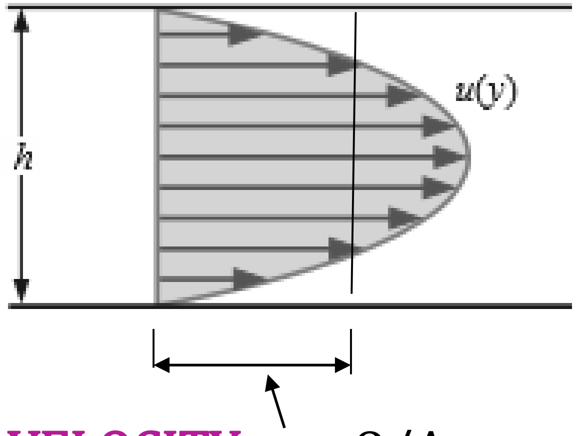
For small α (cca 8-10°)

$$sin\alpha \approx tg\alpha$$

CHARACTERISTIC OF HYDRODYNAMICS

POINT VELOCITY $\boldsymbol{u = \dfrac{ds}{dt}}$

THE AVERAGE (MEAN) VELOCITY - v - is defined as the average speed through a cross section.



elementary volume **discharge** $\boldsymbol{dQ = u\,dA}$

$$v = \frac{1}{A}\int_S u.\,dA = \frac{Q}{A}$$

MEAN VELOCITY v=Q/A

DISCHARGE (mass) = ρ . v . A

MASS RATE PAST A CROSS-SECTION: Q_m (kg/s)

DISCHARGE (volume) = v . A = Q

VOLUME FLOW RATE PAST A CROSS- SECTION: Q (m^3/s)

KINDS AND FORMS OF FLOW

A. - UNSTEADY FLOW ….. Q = Q(x,y,z,t), v= v(x,y,z,t) $\frac{\partial Q}{\partial t} \neq 0$ $\frac{\partial Q}{\partial x_i} \neq 0$ $\frac{\partial v}{\partial t} \neq 0$ $\frac{\partial v}{\partial x_i} \neq 0$

- STEADY FLOW ….. Q = const. $\frac{\partial Q}{\partial t} = 0$ $\frac{\partial Q}{\partial x_i} = 0$

a) **UNIFORM** flow … $\frac{\partial v}{\partial t} = 0$ $A = const. \quad v = const.$

b) **NON – UNIFORM** flow $\frac{\partial v}{\partial x_i} \neq 0$ $A \neq const. \quad v \neq const.$

B. - WITH FREE LEVEL – flow limited by solid walls, free level on surface, motion caused by own weight of liquid

- PRESSURE – flow limited by solid walls from all sides, motion caused by difference of pressures

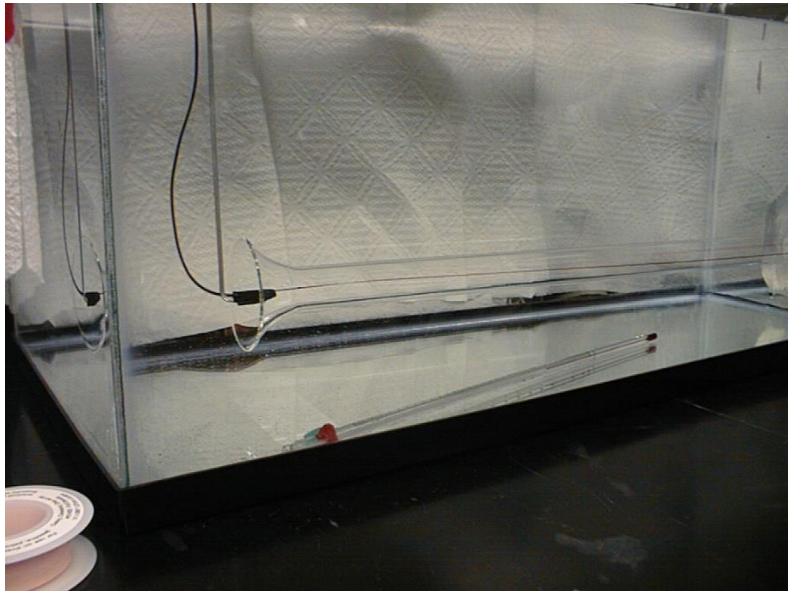
C. - LAMINAR flow

- TURBULENT flow

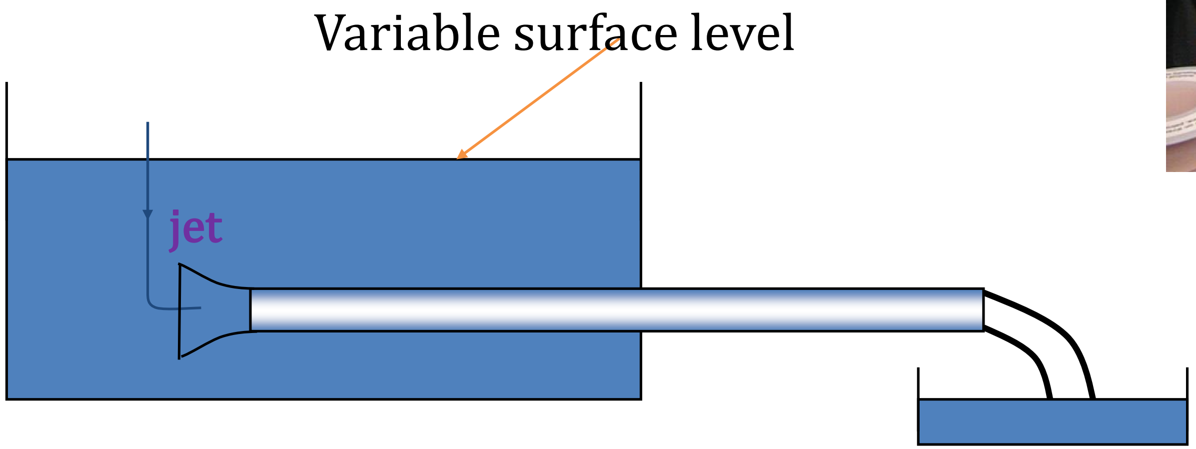
REAL FLUID

"A fluid in which there is *friction* *i.e* viscosity."

LAMINAR AND TURBULENT FLOW



Reynolds experiment **1883:**



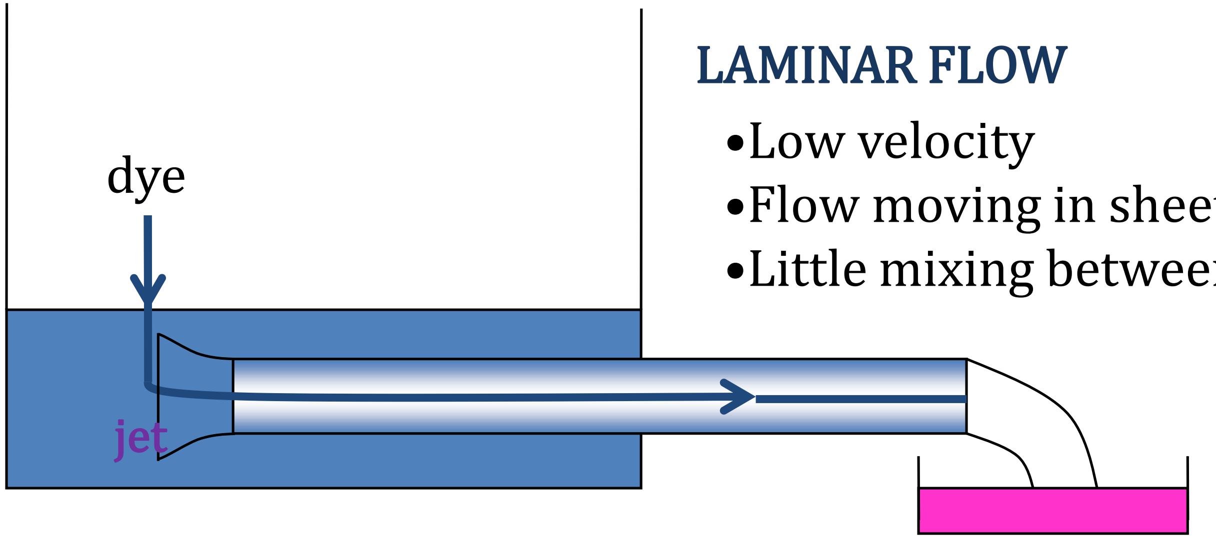
Two different, distinct **flow regimes**:

A) **LAMINAR FLOW**

B) **TURBULENT FLOW**

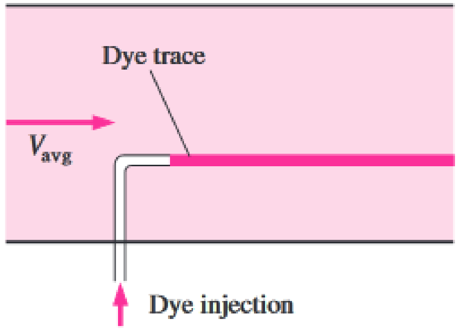
Osborne Reynolds (1842-1912)

REYNOLDS EXPERIMENT 1883:

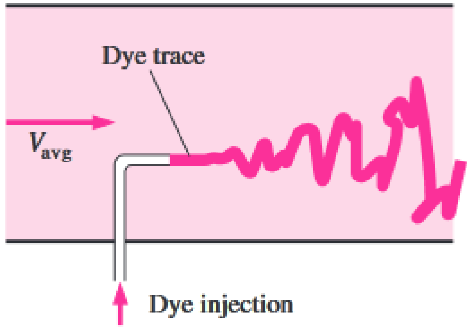


LAMINAR FLOW

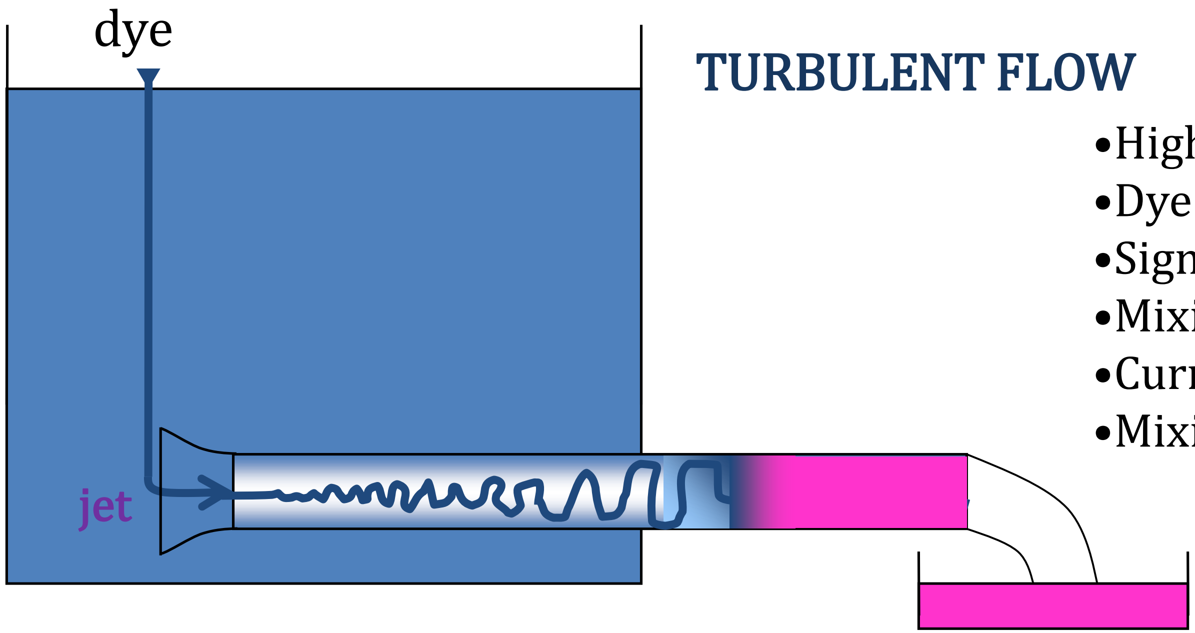
- Low velocity
- Flow moving in sheets
- Little mixing between sheets



(a) Laminar flow



(b) Turbulent flow



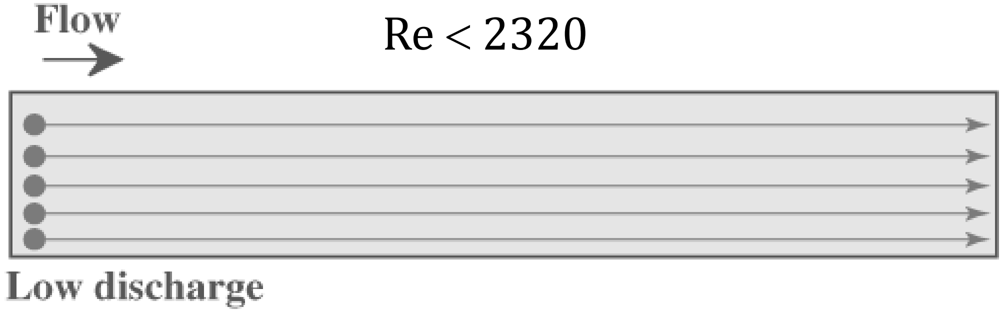
TURBULENT FLOW

- High velocity
- Dye breaks up in a diffuse cloud
- Significant mixing
- Mixing increases with velocity
- Currents perpendicular to pipe
- Mixing is initiated by roughness on the flow boundaries

REYNOLDS CLASSIFIED THE FLOW TYPE ACCORDING TO THE MOTION OF THE FLUID.

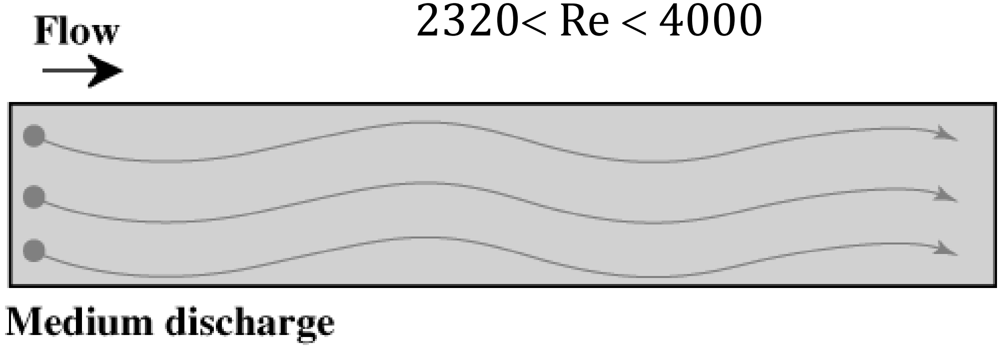
$$Reynolds\ number\ for\ pipe\ \ \boldsymbol{Re} = \frac{\boldsymbol{v\,D}}{\upsilon}$$

Re < 2320



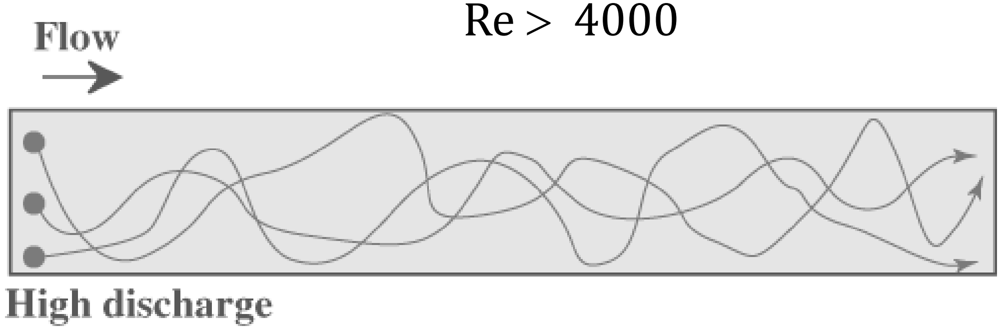
LAMINAR FLOW: every fluid molecule followed a straight path that was parallel to the boundaries of the tube.

2320< Re < 4000



TRANSITIONAL FLOW: every fluid molecule followed wavy but parallel path that was not parallel to the boundaries of the tube.

Re > 4000

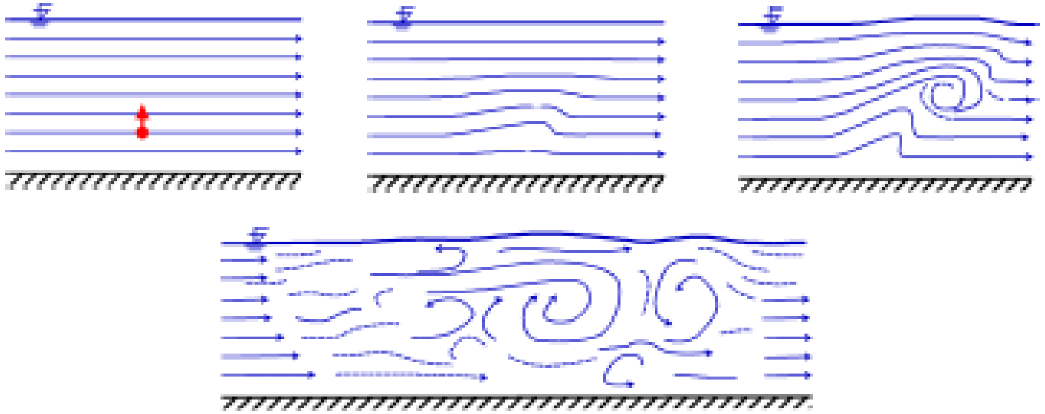


TURBULENT FLOW: every fluid molecule followed very complex path that led to a mixing of the dye.

LAMINAR AND TURBULENT FLOW

• **laminar – particles of liquid move at parallel paths**
• **turbulent – motion of particles of liquid: irregular and**
inordinate, fluctuations of velocity vector in time and space, mixing inside flow

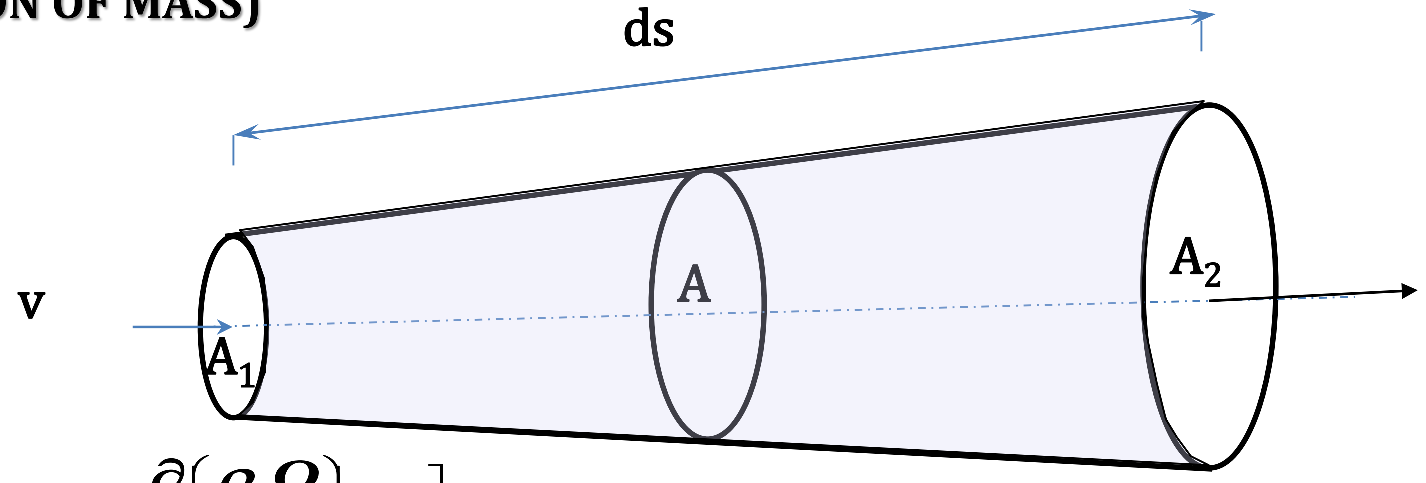
• Criterion – **Reynolds number**
L – characteristic length:
diameter D for pipelines, hydraulic radius R
Critical Reynolds Number - for pipe $Re_{cr} = 2320$
for open channel $Re_{cr} = 580$
for groundwater flow $Re_{cr} = 1$



CONTINUITY EQUATION

mass leaving - mass entering = - rate of increase of mass in cv

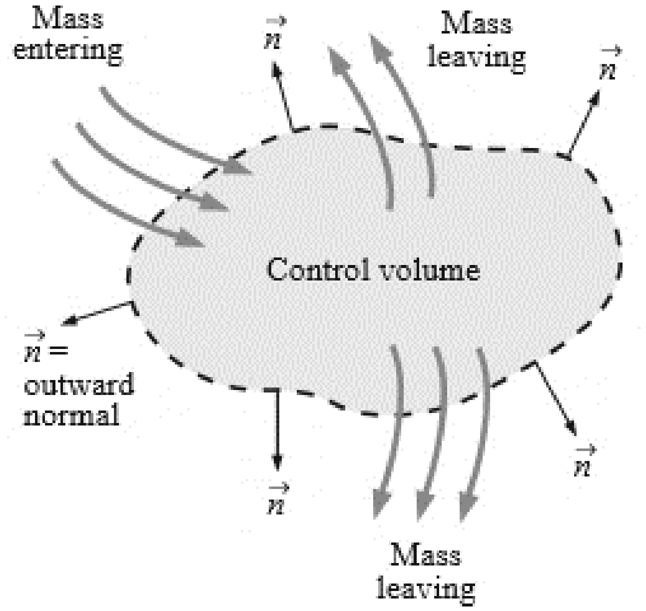
(LAW OF CONSERVATION OF MASS)



Input mass A_1 : $\rho.Q.dt$

Output mass A_2 : $\left[(\rho.Q) + \frac{\partial(\rho.Q)}{\partial s} ds \right] dt$

Change of mass inside V in time dt $\frac{\partial(\rho.A.ds)}{\partial t} dt$



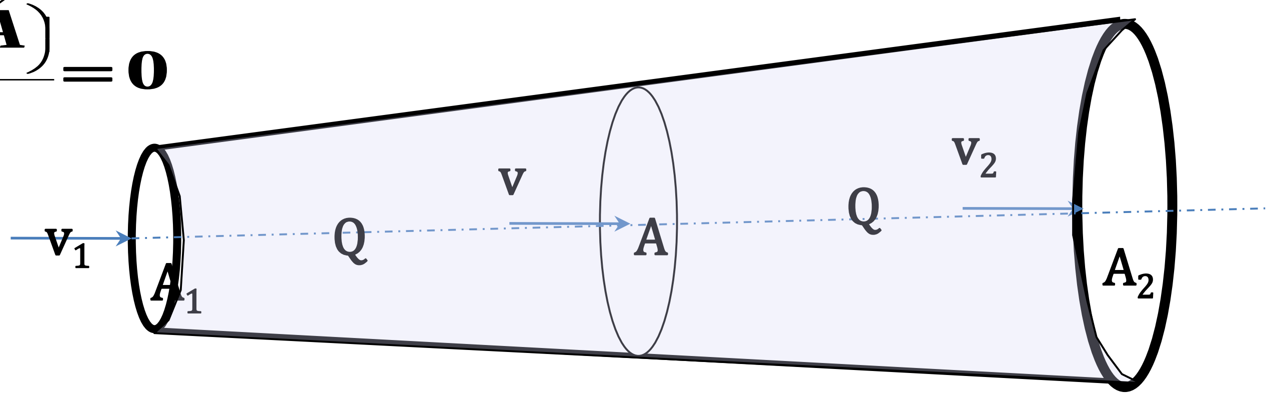
CONTINUITY EQUATION FOR UNSTEADY FLOW

$$\frac{\partial(\rho.Q)}{\partial s} + \frac{\partial(\rho.A)}{\partial t} = 0$$

CONTINUITY EQUATION - STEADY FLOW

$$\frac{\partial(\rho.Q)}{\partial s}+\cancel{\frac{\partial(\rho.A)}{\partial t}}=0$$

$$\frac{\partial(\rho.Q)}{\partial s}=0$$



steady flow compressible liquid – no dependency on time

$$\rho.Q=const.$$

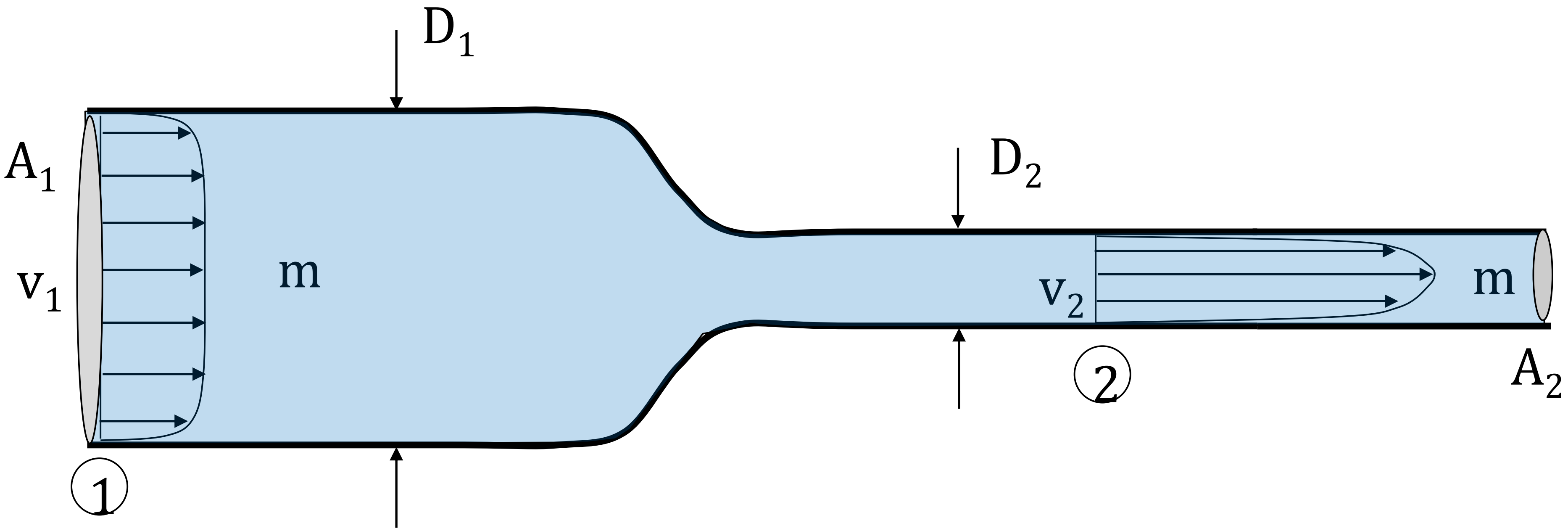
$$Q=\rho_1.A_1.v_1=\rho_2.A_2.v_2=\rho_i.A_i.v_i=konst$$

STEADY FLOW of incompressible liquid

$Q = const.$ $\rho = \text{const.}$

$$\boxed{A_1.v_1=A_2.v_2=\text{konst.}=Q_v}$$

- For pipes with variable diameter, $\dot{m}$ is still the same due to conservation of mass, but $v_1 \neq v_2$



$$Q = A_1.v_1 = A_2.v_2 = \text{konst.}$$

BERNOULLI EQ. FOR IDEAL FLUID

(LAW OF CONSERVATION OF ENERGY)

THE DERIVATION OF BERNOULLI EQUATION (ENERGY CONSERVATION)

($G = mg$)

KINETIC ENERGY ... $\frac{1}{2} mv^2$ kinetic energy per unit weight $v^2/2g$ x ($1/mg$)

POTENTIAL ENERGY ... mgh

Pressure energy ($h = p/\rho g$).... Pressure energy per unit weight ... $p/\rho g$ x ($1/mg$)

Elevation energy ... mgh Elevation energy per unit weight .... h x ($1/mg$)

BERNOULLI´S EQUATION (ideal fluid)

$$\boxed{h + \frac{p}{\rho g} + \frac{v^2}{2g} = const.}$$

Total (mechanical) energy per unit weight

BERNOULLI EQUATION FOR IDEAL FLUID

expresses the **principle of conservation of energy**

The Bernoulli equation is a statement of the **conservation** of **mechanical energy**

$$\underbrace{h+\frac{p}{\rho g}}_{\text{pot. e.}}+\underbrace{\frac{v^2}{2g}}_{\text{kinet.e.}}$$

$$h+\frac{p}{\rho g}+\frac{v^2}{2g}=Const.=ME$$

$\frac{p}{\rho g}$ = PRESSURE HEAD

$$z+\frac{p}{\rho g}=$$ Piezometric head

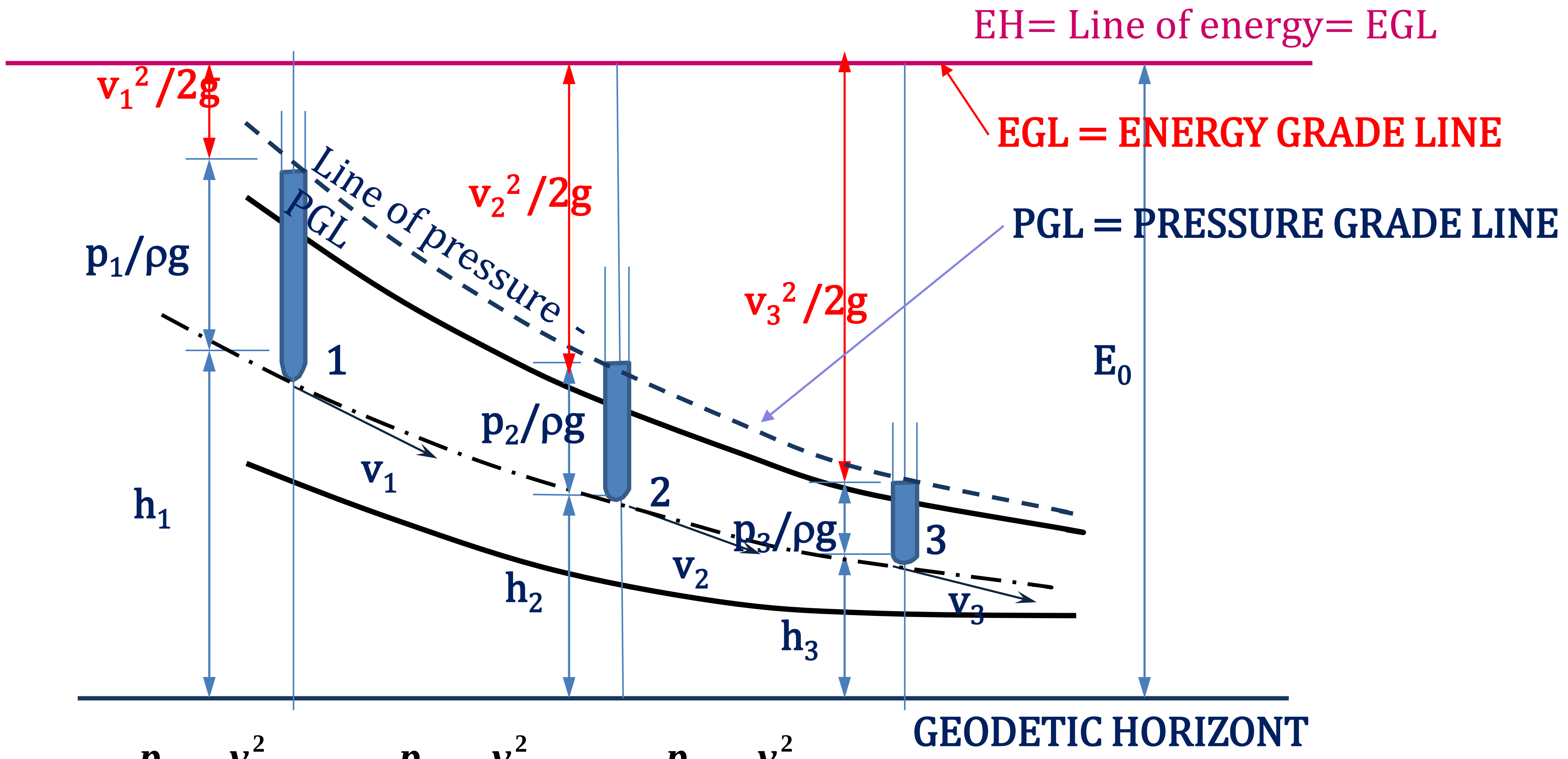
h = ELEVATION (GEODETIC)HEAD

$\frac{v^2}{2g}$ = VELOCITY HEAD

$\boxed{h+\frac{p}{\rho g}=}$ HYDRAULIC GRADE LINE – HGL or PRESSURE GRADE LINE – PGL“

$\boxed{h+\frac{p}{\rho g}+\frac{v^2}{2g}}$ = Total head - ENERGY GRADE LINE - EGL

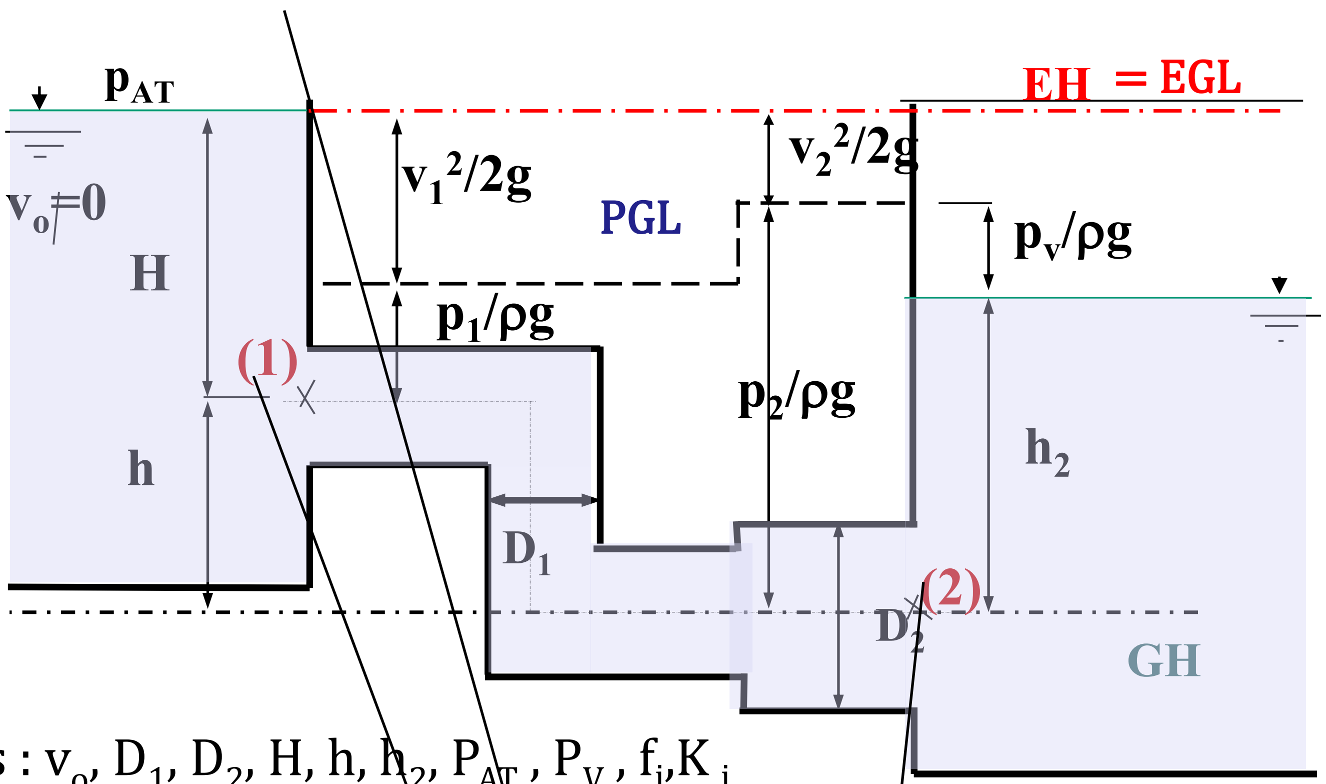
BERNOULLI EQ. FOR IDEAL FLUID



$$h_1 + \frac{p_1}{\rho g} + \frac{v_1^2}{2g} = h_2 + \frac{p_2}{\rho g} + \frac{v_2^2}{2g} = h_3 + \frac{p_3}{\rho g} + \frac{v_3^2}{2g} = E_0$$

h – Elevation (geodetic) head p/ρg - Pressure head $v^2/2g$ - Velocity head

BERNOULLI EQ. FOR IDEAL FLUID



Given values : v_o, D_1, D_2, H, h, h_2, P_{AT} , P_V , f_i,K_i

?: Q, v_1, v_2

EGL, PGL

$$h+H+\frac{p_{AT}}{\rho g}+\frac{v_o^2}{2g}=h_2+\frac{p_V}{\rho g}+\frac{v_2^2}{2g}$$

$$S_2=\frac{\pi D_2^2}{4}$$

$$v_2=\sqrt{2g.\left[\left(h+H+\frac{p_{AT}}{\rho g}+\frac{v_o^2}{2g}\right)-\left(h_2+\frac{p_V}{\rho g}\right)\right]}$$

Discharge: $Q = v_2.S_2$

BERNOULLI EQ. FOR REAL FLUID

CALCULATION OF HEAD (ENERGY) LOSSES:

In General:

When a fluid is flowing through a pipe, the fluid experiences some resistance due to which some of energy (head) of fluid is lost.

Energy Losses (Head losses)

- Major Losses
- Minor losses

Major Losses: loss of head **due to pipe friction** and to viscous dissipation in flowing water

$$Darcy - \text{ } Weisbach \text{ } equation \Rightarrow h_l = f \frac{L}{D} \frac{V^2}{2g}$$

Minor losses: Loss due to the **change of the velocity** of the flowing fluid in the **magnitude** or in **direction** as it moves through fitting like Valves, Tees, Bends and Reducers.

$$h_{lm} = K_L \frac{V^2}{2g}$$

FRICTION LOSS

- Most useful head loss equation for closed-conduit flow – Darcy-Weisbach equation

Friction head loss

Pipe length

Pipe velocity

$$h_f = f\,\frac{L}{D}\,\frac{V^2}{2g}$$

Dimensionless Friction coefficient

Pipe diameter

Gravitational acceleration

BERNOULLI EQ. FOR REAL FLUID

$$h+\frac{p_1}{\rho g}+\frac{\alpha v_1^2}{2g}=h+\frac{p_2}{\rho g}+\frac{\alpha v_2^2}{2g}+\sum_{i=1}^{2}\left(h_{zmi}+h_{zti} \right)$$

Head loss

$$h_L=h_{L,major}+h_{L,minor}$$

If the piping system has constant diameter

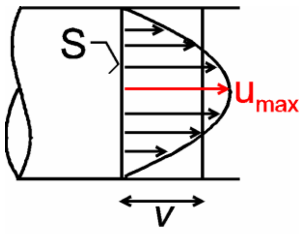
FRICTION FACTOR f

$$h_L=\left(f\,\frac{L}{D}+\sum K_L\right)\frac{v^2}{2g}$$

CORIOLIS NUMBER - α

point velocity u

average velocity v



in technical calculations – kinetic energy head is expressed from **mean velocity** v

$$\frac{\alpha v^2}{2g}$$

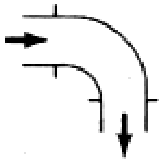
α - coefficient of kinetic energy - **Coriolis number** depends on the shape of cross section and on form of velocity profile

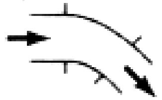
circular pipelines and regular channels α = 1,05 , 1,2,
LAMINAR FLOW α = 2,

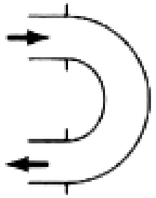
current technical calculations of pipelines (TURBULENT FLOW) α 1,0

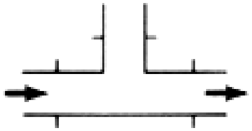
MINOR LOSSES

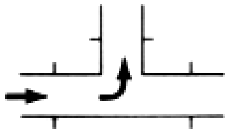
Component	K_L
Elbows	
Regular 90°, flanged	0.3
Regular 90°, threaded	1.5
Long radius 90°, flanged	0.2
Long radius 90°, threaded	0.7
Long radius 45°, flanged	0.2
Regular 45°, threaded	0.4
180° return bends	
180° return bend, threaded	0.2
180° return bend, flanged	1.5
Tees	
Line flow, flanged	0.2
Line flow, threaded	0.9
Branch flow, flanged	1.0
Branch flow, threaded	2.0



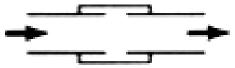








Component	K_L
Union, threaded	0.8
Valves	
Globe, fully open	10
Angle, fully open	2
Gate, fully open	0.15
Gate, ¼ closed	0.26
Gate, ½ closed	2.1
Gate, ¾ closed	17
Ball valve, fully open	0.05
Ball valve, 1/3 closed	5.5
Ball valve, 2/3 closed	210

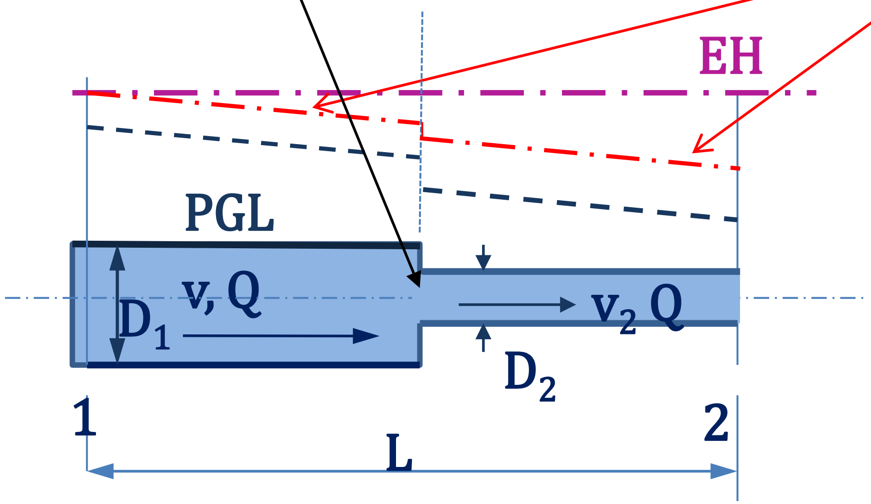


Source: Munson et al. (1998)

LOCAL (MINOR) LOSSES IN PIPELINES

Minor losses

i_E Slope of EGL (friction losses)



$$i_E = \frac{h_{lt}}{L} \Rightarrow h_{lt} = i_E . L$$

MINOR LOSSES

$$h_{lM} = \boxed{K_{lM}} \frac{v^2}{2g}$$

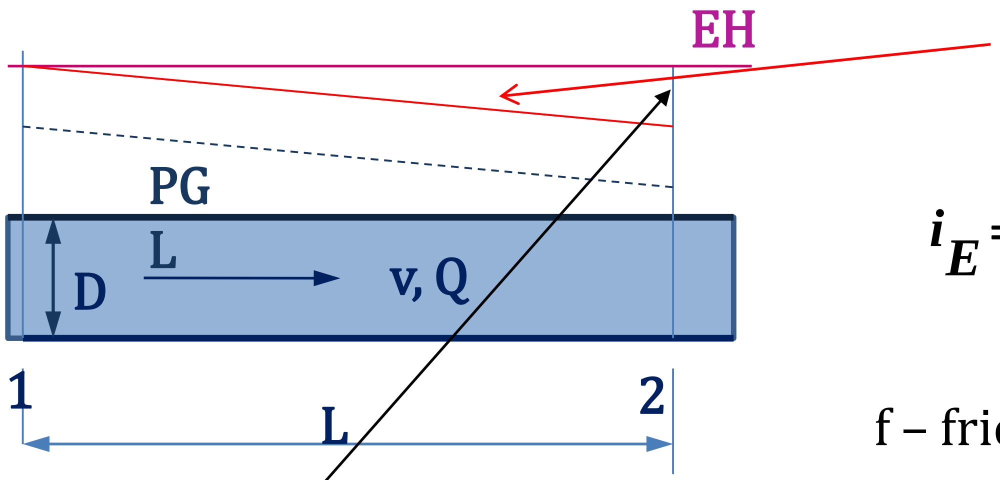
Coef. for minor loss

Reynolds number

$$\mathbf{Re} = \frac{v.D}{\upsilon}$$

FRICTION (MAJOR) LOSSES IN PIPELINES

MAJOR LOSSES



i_E Slope of EGL

$$i_E = \frac{h_{lt}}{L} \Rightarrow h_{lt} = i_E . L$$

f – friction coefficient

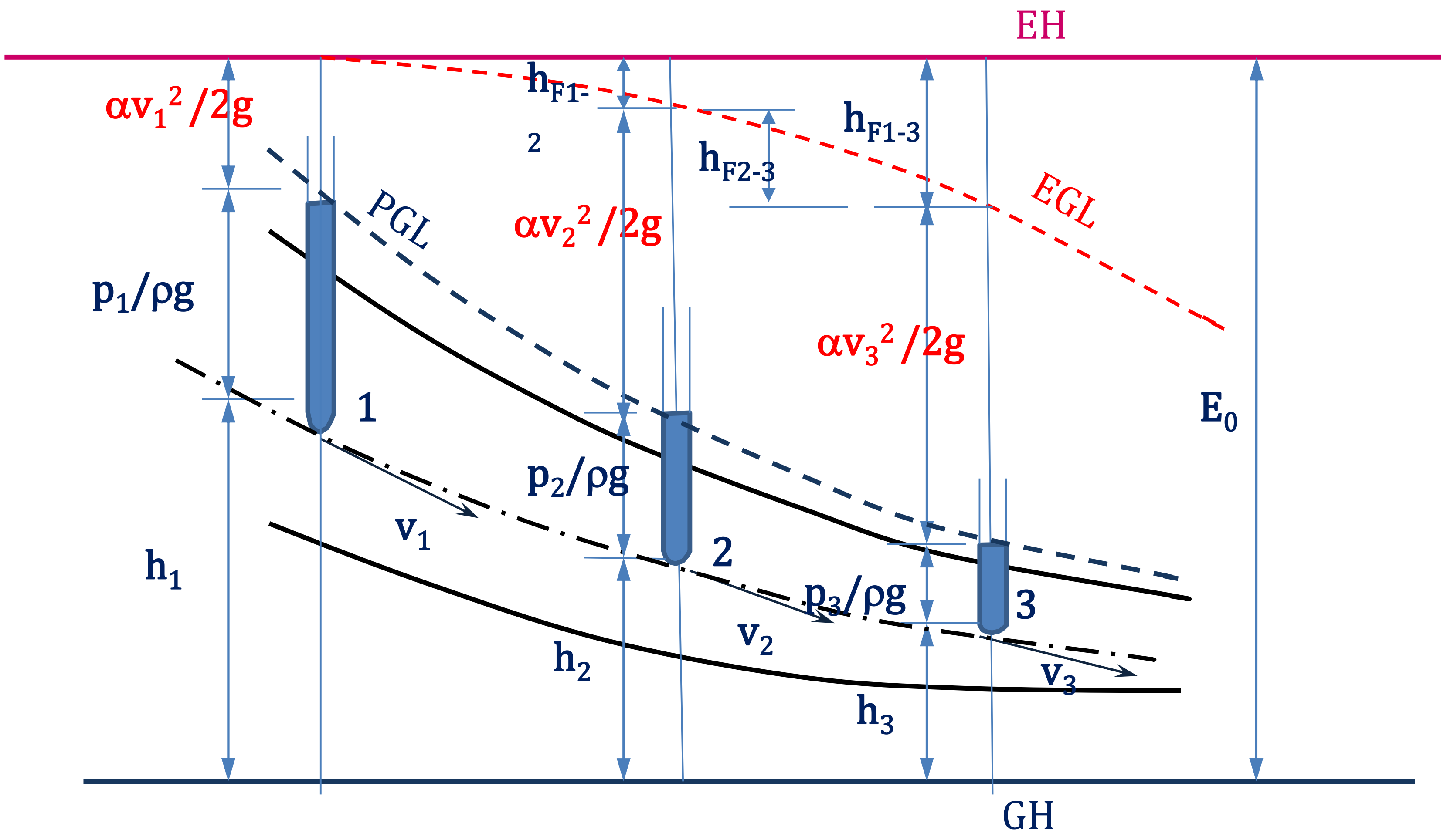
Darcy – Weisbach equation

$$h_{lf} = f\,\frac{L}{D}\,\frac{v^2}{2g} \quad (m)$$

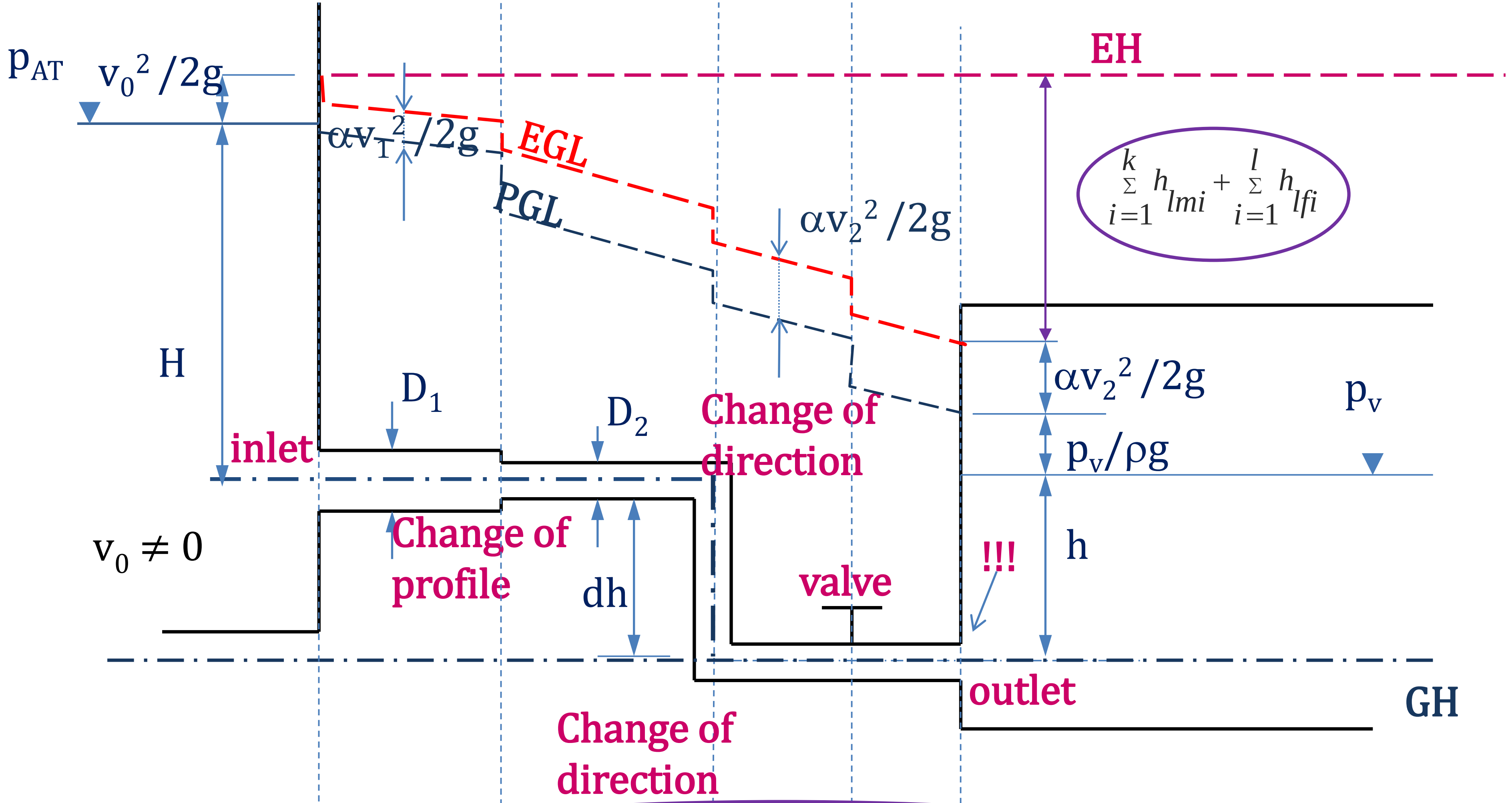
Reynolds number

$$\text{Re} = \frac{v.D}{\upsilon}$$

BERNOULLI EQ. FOR REAL FLUID



BERNOULLI EQ. FOR REAL FLUID – PGL, EGL



$$dh + H + \frac{p_{at}}{\rho g} + \frac{v_o^2}{2g} = h + \frac{p_v}{\rho g} + \frac{v_2^2}{2g} + \sum_{i=1}^{k} h_{lmi} + \sum_{i=1}^{l} h_{lfi}$$

BERNOULLI EQ. for (A) a (B)

$$dh+H+\frac{p_{at}}{\rho g}+\frac{v_0^2}{2g}=dh+H+\frac{p_{at}}{\rho g}+\frac{v_2^2}{2g}+\sum_{i=1}^{k} h_{lmi}+\sum_{i=1}^{l} h_{lfi}$$

Sections of pipe

1.sec $\left(K_{inlet}+K_{change}+f_1\frac{l_1}{D_1}\right)\frac{v_1^2}{2g}$

2. sec $\left(2.K_{ch.\_of\_dir}+K_{ch\_of\_D}+f_2\frac{\left(l_2+dh+l_3\right)}{D_2}\right)\frac{v_2^2}{2g}$

Continuity eq.

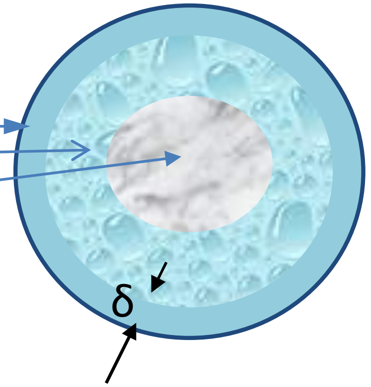
$$Q=v_1.S_1=v_2.S_2 \quad \Rightarrow \quad v_2=v_1\frac{S_1}{S_2}$$

$$S_1=\frac{\pi D_1^2}{4} \qquad S_2=\frac{\pi D_2^2}{4}$$

$$v_2=v_1\frac{D_1^2}{D_2^2}$$

TURBULENT FLOW

a) **Viscous sublayer** -laminar flow($\tau = \tau_L ; \tau_T = 0$)
b) **Overlap layer**
c) **Turbulent sublayer** -turbulent flow ($\tau = \tau_T ; \tau_L = 0$)



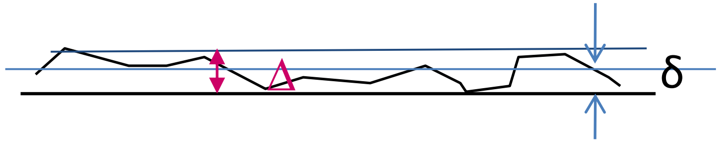
Thickness of the viscous sublayer

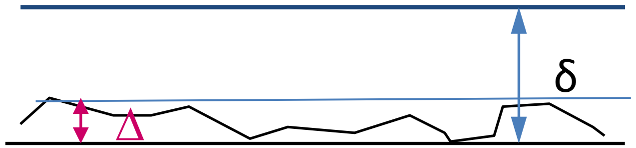
$$\boxed{\delta = 33{,}4 \frac{D}{\mathrm{Re}\ f^{1/2}}}$$

$\delta = f(D, \mathrm{Re}, f)$

Thickness of the viscous sublayer depends on D, Re and fl:

Roughness of pipe wall
1) **Absolute roughness** (Δ)

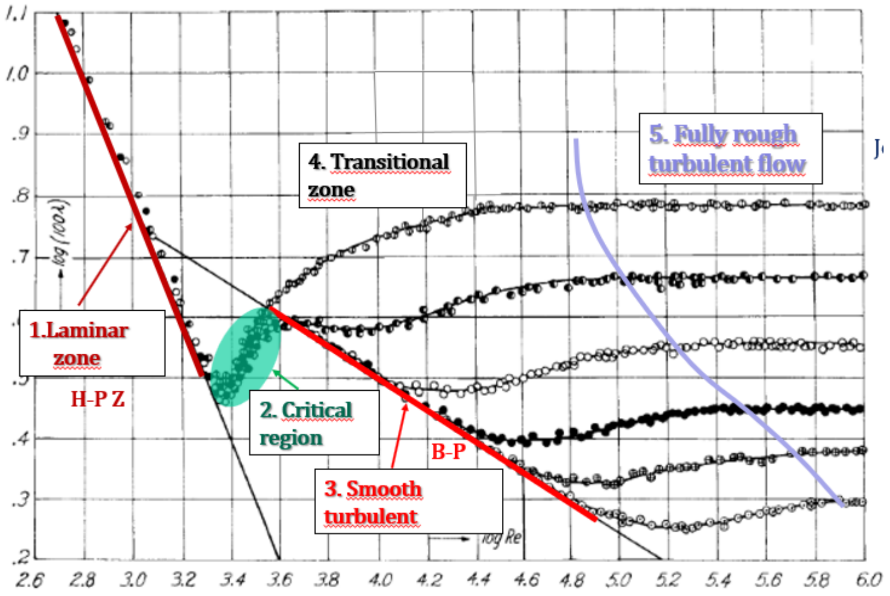




2) **Relative roughness**
Δ/D , Δ/r, Δ/R, D/Δ ....

A. (1) LINEAR ZONE – – Hagen-Poiseuille´s law

$f = 64/Re$ - line 1 $\boxed{f = f(Re)}$



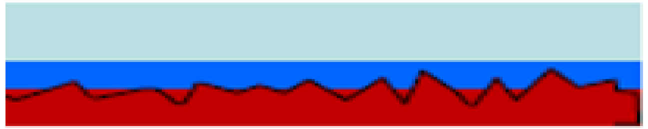
B. (2) CRITICAL ZONE (Re =2320 - 4000) $\boxed{f = f(Re)}$

instability zone - lamin. ???? turb. Flow .. jump - Frenkel

$$f = 2{,}7 / Re^{0{,}53}$$

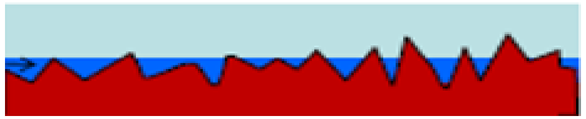
C. (3) SMOOTH PIPES ZONE – - $f = f(Re)$ $\boxed{\delta > 5.\Delta}$

Blasius $f = 0{,}3164 / Re^{0{,}25}$ Re …. 4000…….(10^5)



D. (4) TRANSITIONAL ZONE from Blasius - up to $\Delta/\delta = 5$

$\boxed{f = f(Re, r/\Delta)}$

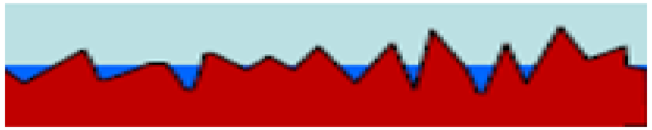


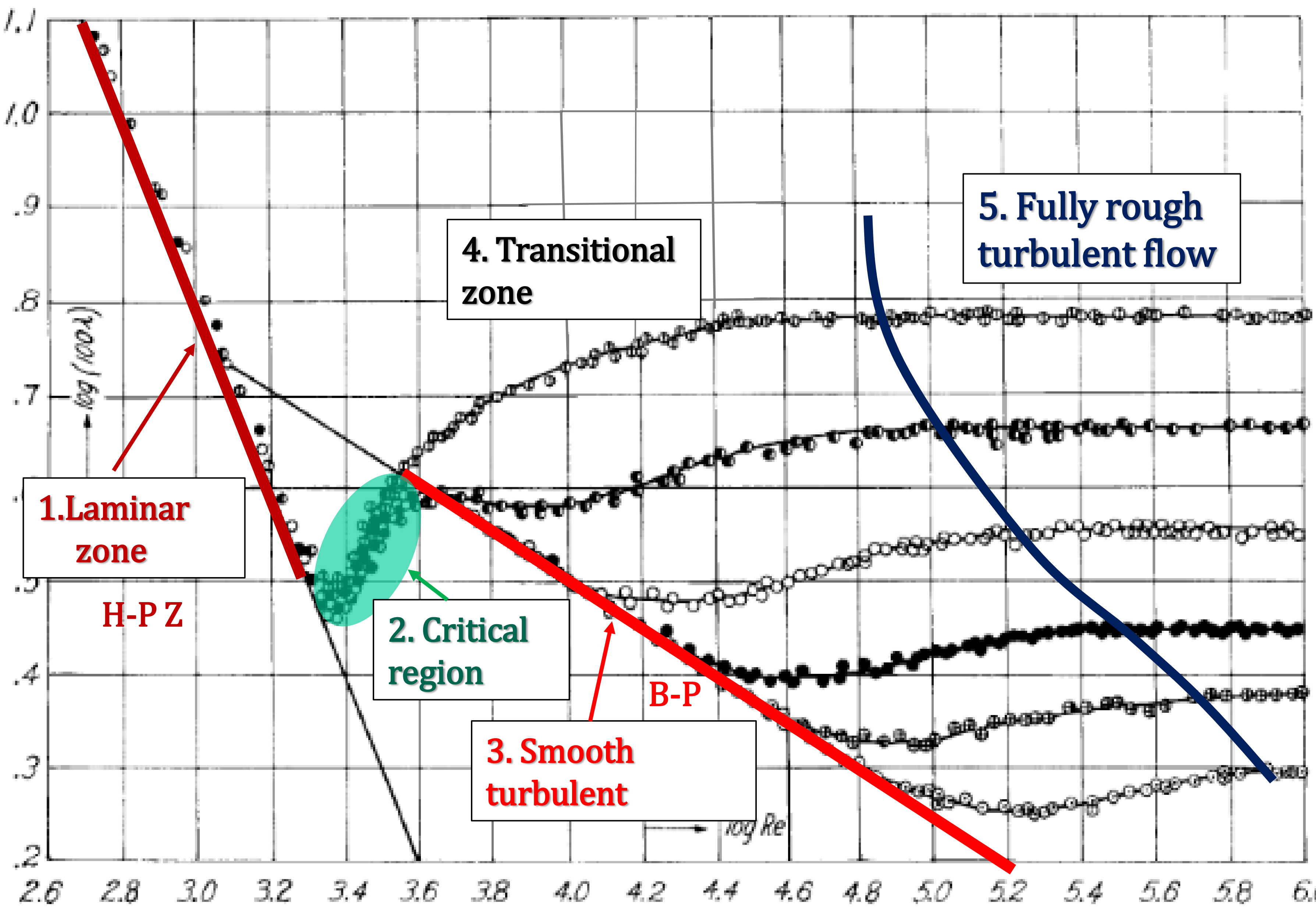
Pipe wall

Viscous sublayer

E. (5) FULLY ROUGH TURBULENT ZONE – …. $\delta < \Delta/5$

$\boxed{f = f(r/\Delta)}$





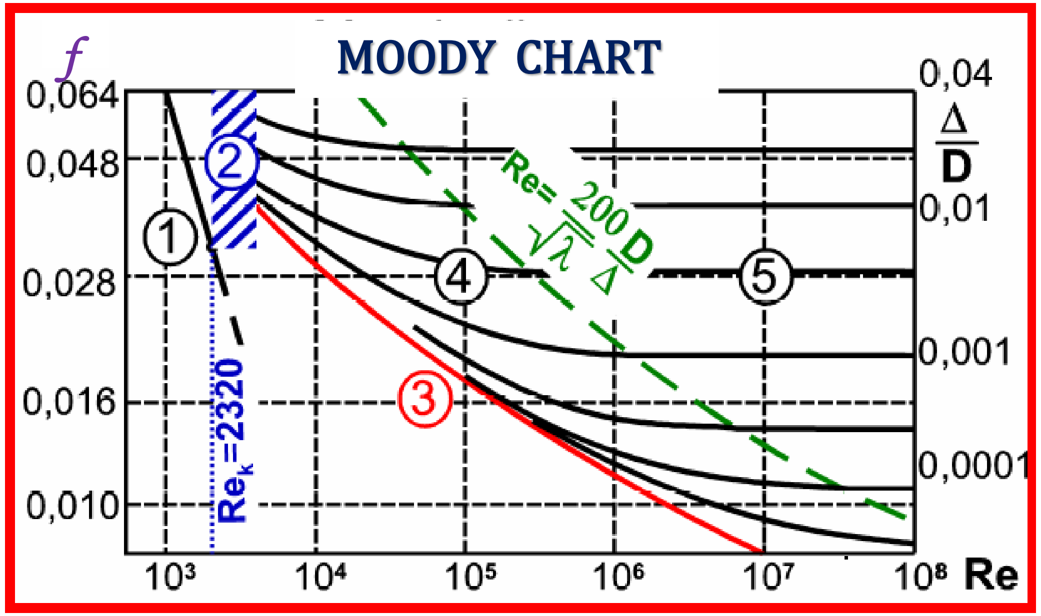
Johann Nikuradse, 1933

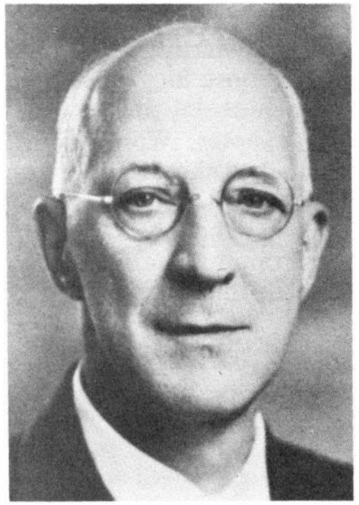
$\lambda = f$

DIAGRAM - NIKURADSE

r – poloměr; Δ – absolutní drsnost

COMMERCIALLY AVAILABLE PIPES





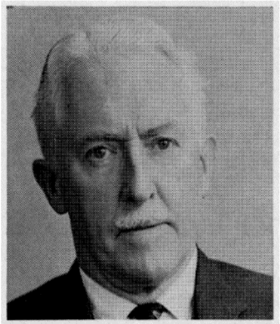
Lewis Moody, 1944

Moody chart presents the friction factor **f** for pipe flow as a function of the Re and relative roughness (Δ/D)

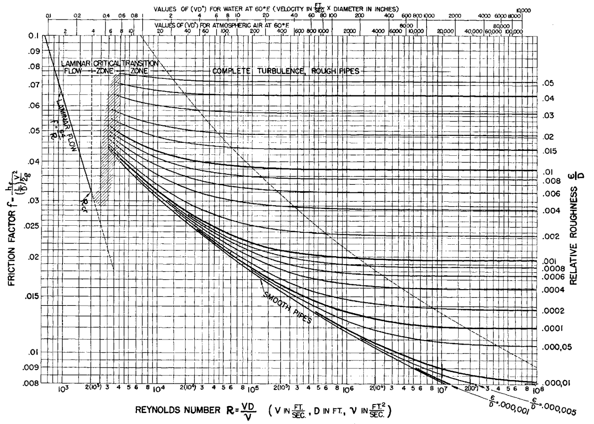
for commercial pipe in transition zone (4):

COLEBROOK-WHITE EQUATION

$$\frac{1}{\sqrt{f}} = -2\log\left[\frac{2,51}{\mathrm{Re}\sqrt{f}} + \frac{\Delta}{3,7\,D}\right]$$



Cyril F. Colebrook, 1939



END (PART 1)