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Derivation and application of mathematical model for well test analysis with variable skin factor in hydrocarbon reservoirs

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Skin factor is often regarded as a constant in most of the mathematical model for well test analysis in oilfields, but this is only a kind of simplified treatment with the actual skin factor changeable. This paper defined the average permeability of a damaged area as a function of time by using the definition of skin factor. Therefore a relationship between a variable skin factor and time was established. The variable skin factor derived was introduced into existing traditional models rather than using a constant skin factor, then, this newly derived mathematical model for well test analysis considering variable skin factor was solved by Laplace transform. The dimensionless wellbore pressure and its derivative changed with dimensionless time were plotted with double logarithm and these plots can be used for type curve fitting. The effects of all the parameters in the expression of variable skin factor were analyzed based on the dimensionless wellbore pressure and its derivative. Finally, actual well testing data were used to fit the type curves developed which validates the applicability of the mathematical model from Sheng-2 Block, Shengli Oilfield, China. © 2016 Author(s). All article content, except where otherwise noted, is licensed under a Creative Commons Attribution (CC BY) license (http://creativecommons.org/licenses/by/4.0/).
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I. INTRODUCTION

Skin factor is a parameter which is used to evaluate the degree of influence of formation permeability around the wellbore by external factors.[1,2] The conception of "thin-skin mathematical model" was initially proposed to adjust the error of bottom hole pressure between theoretical calculation and measured data by van Everdingen.[3] However, this conception does not have reasonable explanation in percolation mechanics theory when it was used to explain an increase in permeability around wellbore. Hawkins introduced an improved model called "thick-skin mathematical model" which was accepted by researchers and engineers and developed further into what is now called "skin factor".[2–4] There are many factors that causes skin effect which includes invasion of mud during drilling process, incomplete perforation, fines migration, hydraulic fracturing, acidification and so on.[5–8]

Skin factor is an important parameter that is obtained in well test analysis. In most of the well test mathematical models, the skin factor is regarded as a constant.[9–12] In fact, due to turbulence effect of fluids and fines migration caused by the interaction between fluids and sands; hence the value of skin factor is variable. Some researchers believed that skin factor is related to turbulence effect of fluids due to the fact that high speed non-Darcy flow will produce additional pressure drop at the bottom of the well.[13–17] Moreover, the authors also proposed their own mathematical model of skin factor that changes with production rate. However, variable skin factor caused by

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sand migration lacked systematic study. Larsen and Kviljo studied cleanup effects which could change skin factor.[18] The variable skin factor was represented by a simple hyperbolic function of time which was absent in theoretical evidence. In addition, some researchers built filtration models which describe the particles capture and detachment.[19–21] As a result, these mathematical models are difficult to apply in well test analysis in oilfields.

The paper proposed mathematical model of the variable skin factor which was introduced into existing traditional well test models instead of using a constant skin factor to establish a new well test mathematical model. Also, the type curves generated from the new mathematical model came about as a result of using Laplace transform to solve this model. It can be used universally to conduct well test in single oil phase flow, especially in super-low and -high permeability oil reservoirs, their permeabilities around well area change significantly in short time for these two kinds of reservoirs. In effect, these results supplement the content of modern well test analysis.

II. MATHEMATICAL MODEL

A. Derivation of variable skin factor

In light of the definition of the skin factor in steady radial fluid flow of vertical wells, the mathematical equation is given as[4,22]

$$S = \left(\frac{k}{k_s} - 1 \right) \ln \frac{r_s}{r_w} \tag{1}$$

Where S is skin factor, dimensionless; k is average permeability, μm^2; k_s is average permeability of the damaged area, μm^2; r_s is the radius of the damaged area, m; r_w is the radius of wellbore, m.

Assuming that the radius and average permeability of a damaged area are constant, that is to say, r_s and k_s are changeless, so the skin factor is a constant which is the usual treatment of skin factor in most literatures and production. In fact, k_s is a variable which changes with production time, and its change regulation is as follows.

In the early flow period, the rate of k_s changing is fast, because the fines and uncemented particles are easily washed into the wellbore (the radius of particles are smaller than the radius of pores and throats. Generally, the average permeability of reservoir in this case is high) or accumulated around the wellbore (the radius of particles are larger than the radius of pores and throats. Generally, average permeability of reservoir in this case is low). Furthermore, with the elapse of time, these fines and uncemented particles continue to lessen, and the rate of k_s changing become more and more slow until k_s tends to a constant. This variation can be approximated as

$$\frac{k_{si}}{k_s} = -\beta \left(1 - e^{-v_i t} \right) + 1 \tag{2}$$

Where, k_{si} is the average permeability of damaged area at initial time, μm^2; v_i is initial rate of permeability changing, per day; t is time, day; β is a constant that is smaller than a unit, dimensionless.

The effects of β and v_i on the average permeability of damaged area are shown in Figs. 1 and 2.

According to Fig. 1, the limit of k_{si}/k_s is controlled by β, that is

$$\lim_{t \to +\infty} \frac{k_{si}}{k_s} = 1 - \beta \tag{3}$$

and if β is positive, it represents k_s increasing gradually; conversely, if β is negative, it represents k_s decreasing gradually. According to Fig. 2, the speed of tending to the limit of k_{si}/k_s is controlled by v_i, the larger v_i is, the shorter the time of k_{si}/k_s tending to 1-β becomes, the steeper is the curve, and the faster is the change speed of k_s.
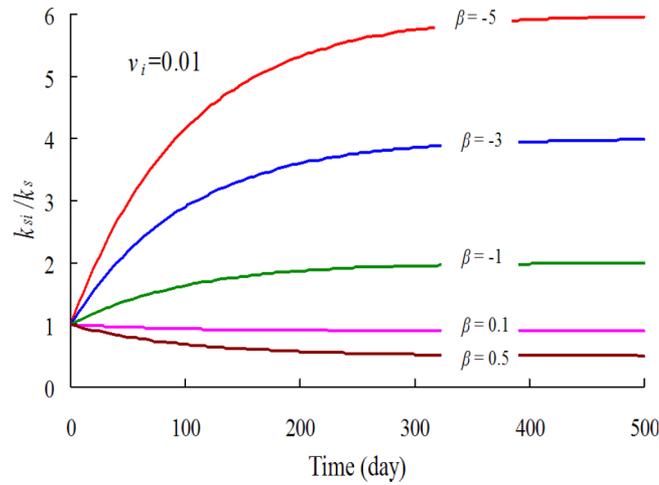
FIG. 1. Effect of β on the average permeability of damaged area ($v_i = 0.01$).

If we define the initial skin factor as S_i, Eq (1) can be rewritten as

$$S_\text{i} = \left(\frac{k}{k_\text{si}} - 1 \right) \ln \frac{r_s}{r_w} \tag{4}$$

S_i is a constant, S, however, is a variable which changes with time (because k_s changes with time). Therefore, the relationship of S and S_i can be established by combining Eqs (1), (2), and (4), it yields

$$\frac{S + \ln \frac{r_s}{r_w}}{S_\text{i} + \ln \frac{r_s}{r_w}} = \frac{k_\text{si}}{k_s} \tag{5}$$

Therefore, the expression of variable skin factor was derived through the above analysis.

B. Assumption of the mathematical model

The assumptions of the mathematical model are as follows.

There is an infinite-acting reservoir in plane; its pay thickness is h (m), the reservoir is homogeneous, and there is a production well with complete perforation in the reservoir, whose production
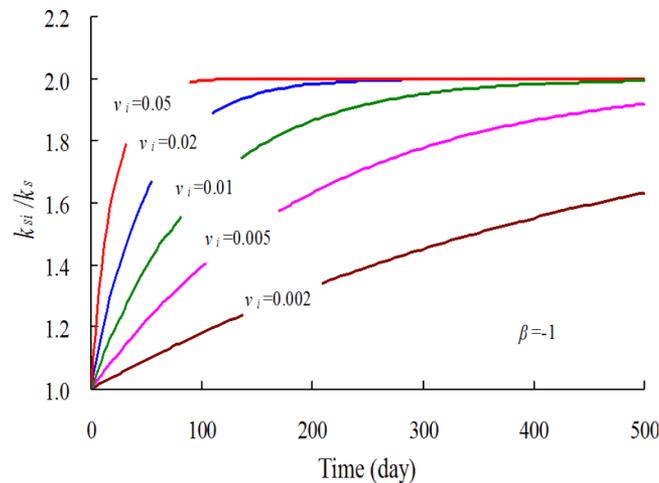

FIG. 2. Effect of v_i on the average permeability of damaged area ($\beta = -1$).

rate is q (m^3/d). The initial pressure of the reservoir is p_i (MPa); the bottom hole pressure is p_{wf} (MPa); its porosity and permeability are ϕ (fraction) and k(μm^2) respectively; the radius of wellbore is r_w(m); the viscosity of the fluid is μ(mPa·s); the volume factor of the fluid is B (m^3/m^3); the total compression coefficient is c_t (1/MPa); the skin factor is S (dimensionless); the wellbore storage coefficient is C (m^3/MPa). Assuming that radial flow occurs in the formation and the flow follows Darcy's Law.

C. Establishment of the mathematical model

In the early flow period, the effect of wellbore storage is obvious, all or most of the fluid produced is supplied by the wellbore. The flow is not completely spread to the reservoir at this time, and the skin factor can be viewed as a constant in this period. However, when the effect of wellbore storage weakened gradually, the flow spreads completely to the reservoir, and the skin factor should be taken as variable. In light of Rammy *et al.*,[23] the end time of the wellbore storage effect is

$$t_{\alpha D} = C_D(60 + 3.5S_i) \tag{6}$$

Where, $t_{\alpha D}$ is the time of wellbore storage effect terminates. The well testing model was established based upon above assumption and analysis. The dimensionless parameters are defined as follows.

$$p_D = \frac{kh(p_i - p)}{1.842 \times 10^{-3}q\mu B}; \quad p_{wD} = \frac{kh(p_i - p_{wf})}{1.842 \times 10^{-3}q\mu B}; \quad t_D = \frac{3.6kt}{\phi\mu c_t r_w^2}; \quad r_D = \frac{r}{r_w};$$

$$C_D = \frac{0.1592C}{\phi c_t h r_w^2}; v_{iD} = \frac{\phi\mu c_t r_w^2}{3.6kv_i}; \quad r_{sD} = \frac{r_s}{r_w} \tag{7}$$

The governing equation is:

$$\frac{\partial^2 p_D}{\partial r_D^2} + \frac{1}{r_D}\frac{\partial p_D}{\partial r_D} = \frac{\partial p_D}{\partial t_D} \tag{8}$$

The initial condition is:

$$p_D(r_D, t_D = 0) = 0 \tag{9}$$

The outer boundary condition is:

$$p_D(r_D \to \infty, t_D) = 0 \tag{10}$$

The inner boundary condition is divided into two kinds of situation depends on time.

1) When $t_D < t_{\alpha D}$, the effect of wellbore storage plays a major role and the skin factor is viewed as constant.

$$\begin{cases} C_D\dfrac{dp_{wD}}{dt_D} - r_D\dfrac{\partial p_D}{\partial r_D}\bigg|_{r_D=1} = 1 \\ p_{wD} = p_D(r_D = 1, t_D) - S_i\left[r_D\dfrac{\partial p_D}{\partial r_D}\right]_{r_D=1} \end{cases} \tag{11}$$

2) When $t_D \geq t_{\alpha D}$, the effect of wellbore storage can be neglected and the skin factor is viewed as variable.

$$\begin{cases} r_D\dfrac{\partial p_D}{\partial r_D}\bigg|_{r_D=1} = -1 \\ p_{wD} = p_D(r_D = 1, t_D) + S \end{cases} \tag{12}$$

A solvable partial differential equation was set up by combining from Eq (8) to Eq (12). The equation can be solved by Laplace transform, and its solution in Laplace domain yields:[1,24]

$$\begin{cases} \tilde{p}_{wD}(s) = \dfrac{1}{s}\left\{\dfrac{K_0(\sqrt{s}) + S_i\sqrt{s}K_1(\sqrt{s})}{\sqrt{s}K_1(\sqrt{s}) + C_D s\left[K_0(\sqrt{s}) + S_i\sqrt{s}K_1(\sqrt{s})\right]}\right\}; & t_D < t_{\alpha D} \\[4mm] \tilde{p}_{wD}(s) = \dfrac{K_0(\sqrt{s})}{s\sqrt{s}K_1(\sqrt{s})} + \dfrac{\beta v_{iD}}{s(s + v_{iD})}(S_i + \ln r_{sD}) - \dfrac{S_i}{s}; & t_D \geq t_{\alpha D} \end{cases} \tag{13}$$

Where, K_0 and K_1 are the zero and unit order modified Bessel functions respectively; s is the Laplace operator; $\tilde{p}_{wD}(s)$ is the dimensionless wellbore pressure in Laplace domain.

D. Solution of the mathematical model

$p_{wD}(t_D)$ can be obtained by Stehfest numerical inversion.[25]

$$p_{wD}(t_D) = \frac{\ln(2)}{t}\sum_{i=1}^{N} V(i)\tilde{p}_{wD}(s) \tag{14}$$

$$s = i\frac{\ln(2)}{t}; \tag{15}$$

$$V(i) = (-1)^{N/2+i}\sum_{k=[(i+1)/2]}^{\min(i,N/2)} \frac{k^{n/2}(2k+1)!}{k!(k+1)!(n/2-k+1)!(i-k+1)!(2k-i+1)!} \tag{16}$$

Where, $p_{wD}(t_D)$ *is the dimensionless wellbore pressure in real domain.*

III. RESULTS AND DISCUSSION

The double logarithmic diagrams of $p_{wD}(t_D)$ and its derivatives ($\mathrm{d}p_{wD}(t_D)/\mathrm{d}\ln(t_D)$) versus dimensionless time (t_D) were obtained from the solution of the model. These diagrams were shown from Figs. 3-5. Four different initial skin factor (S_i) values were selected in each of the three Figures, which were 1, 5, 10, and 20, respectively. As shown in Figs. 3-5, the greater the value of S_i is, the greater the value of the dimensionless wellbore pressure curve is, and the greater the peak value of derivative of dimensionless pressure is. The effect of each variable on $p_{wD}(t_D)$ and $\mathrm{d}p_{wD}(t_D)/\mathrm{d}\ln(t_D)$ was analyzed in next sections.

A. Effect of β

Fig. 3 shows the effect of β on $p_{wD}(t_D)$ and $\mathrm{d}p_{wD}(t_D)/\mathrm{d}\ln(t_D)$, in which the value of r_{sD}=40, v_{iD}=0.00001, and C_D=10, respectively. Three different values of β (-0.5, 0, and 0.5) were chosen to analyze the effect of β. The value of β reflects the ultimate changing degree of the average permeability of damaged area. The greater the β, the smaller the k_{si}/k_s, and the smaller the final skin factor becomes. This regular pattern was also reflected in Fig. 3.

As shown in Fig. 3(a), when β equals 0.5, $p_{wD}(t_D)$ decreases dramatically after wellbore storage effect finishes, then $p_{wD}(t_D)$ increases slowly with production. There will be negative among the values of $\mathrm{d}p_{wD}(t_D)/\mathrm{d}\ln(t_D)$, therefore, it is not continuous in the double logarithmic graph, and the larger the S_i, the more the negative values among $\mathrm{d}p_{wD}(t_D)/\mathrm{d}\ln(t_D)$, so the wider the "gap" of $\mathrm{d}p_{wD}(t_D)/\mathrm{d}\ln(t_D)$ curve in Fig. 3(b). When β= -0.5, the trends of $p_{wD}(t_D)$ and $\mathrm{d}p_{wD}(t_D)/\mathrm{d}\ln(t_D)$ were opposite as compared with β = 0.5. $p_{wD}(t_D)$ will increase significantly after wellbore storage effect is complete, then the increasing speed becomes slow. The second peak value will appear in the curve of $\mathrm{d}p_{wD}(t_D)/\mathrm{d}\ln(t_D)$ in Fig. 3(b), and the larger the S_i is, the greater the peak value is. When β=0, it represents that the skin factor is a constant. The $p_{wD}(t_D)$ will increase slowly after wellbore storage effect is complete, and the values of $\mathrm{d}p_{wD}(t_D)/\mathrm{d}\ln(t_D)$ equal 0.5 gradually after achieving the first peak.
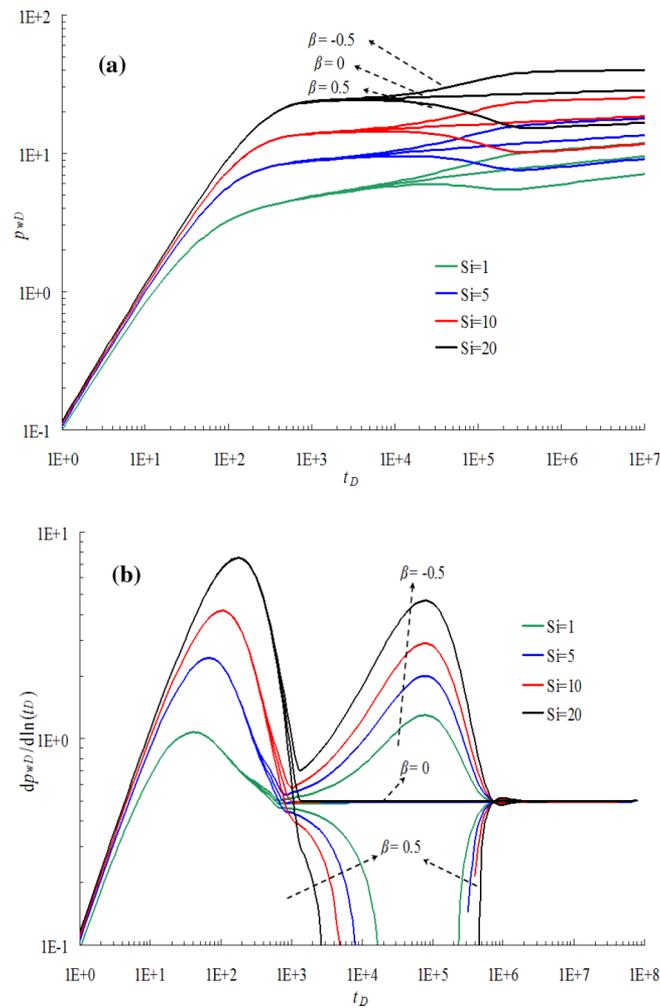
FIG. 3. Effects of β on p_{wD} and $dp_{wD}/d\ln(t_D)$: (a) Effect of β on p_{wD}; (b) Effect of β on $dp_{wD}/d\ln(t_D)$.

B. Effect of v_{iD}

Fig. 4 shows the effect of v_{iD} on $p_{wD}(t_D)$ and $dp_{wD}(t_D)/d\ln(t_D)$, in which the value of r_{sD}, β, and C_D were 40, -0.3, and 10, respectively. Three different values of v_{iD} (0.0001, 0.00001, and 0.000001) were chosen to analyze the effect of v_{iD}. v_{iD} reflects the change rate of k_s, the larger v_{iD} is, the faster the change rates of k_s and S.

As shown in Figs. 4(a) and 4(b), the larger the v_{iD}, the faster the change rates of $p_{wD}(t_D)$, the earlier the second peak value (β is negative) or "gap" (β is positive) appears. However, when S_i and β are invariant, all the curves of $p_{wD}(t_D)$ will eventually converge into one curve and all the curves of $dp_{wD}(t_D)/d\ln(t_D)$ will converge to 0.5 finally.

C. Effect of r_{sD}

Fig. 5 shows the effect of r_{sD} on $p_{wD}(t_D)$ and $dp_{wD}(t_D)/d\ln(t_D)$, in which the value of v_{iD} = 0.00005, β = -0.3, and C_D = 10, respectively. Three different values of r_{sD} (10, 100, and 1000) were chosen to analyze the effect of r_{sD}.

The value of r_{sD} reflects the size of damaged area. The larger the size of damaged area, the more particles that can be washed into the wellbore or piled up around the wellbore and finally the change in skin factor become more obvious. These characteristics can be seen in Figs. 5(a) and 5(b) as well. In these two figures, the larger the value of r_{sD}, the greater the magnitude of p_{wD}
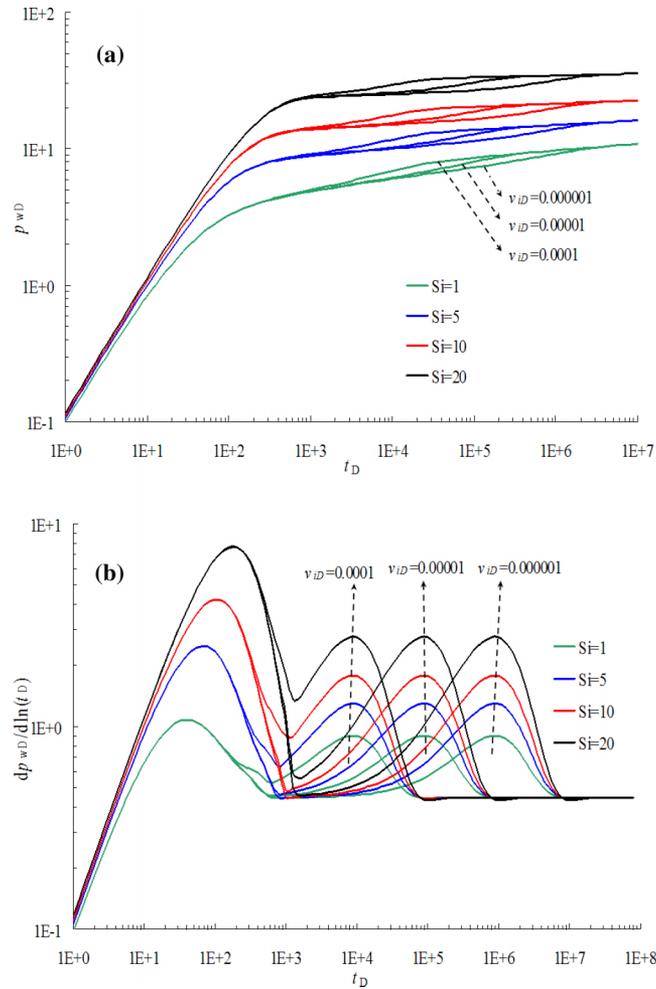
FIG. 4. Effects of v_{iD} on p_{wD} and $dp_{wD}/d\ln(t_D)$: (a) Effect of v_{iD} on p_{wD}; (b) Effect of v_{iD} on $dp_{wD}/d\ln(t_D)$.

(t_D) increases, the greater the ultimate value of $p_{wD}(t_D)$, and the greater the second peak value (β is negative) or "gap" (β is positive). However, as shown in Fig. 5(a), the effect of r_{sD} on $p_{wD}(t_D)$ is small, especially on the case of the larger skin factor. According to the study of Mohamed et al. (2014), the radius of damaged area is smaller than 6 meters generally, and its range of variation is small as well. Therefore, the effect of r_{sD} can be neglected.

IV. APPLICATIONS IN THE OILFIELDS

There is a well in Sheng-2 Block, Shengli Oilfield, China which was used to validate the variable skin factor well test model derived. A layer of this well has been tested in 1994. The thickness of the layer is 5.6 meters; the radius of the wellbore is 0.1 meters. Daily oil production of this well is 450 m³/d during the test; the oil viscosity is 4.2 mPa•s, the volume coefficient of the oil is 1.10 m³/m³, and its compression coefficient is 8.9×10^{-4} MPa⁻¹. The bottom-hole pressure is always higher than the bubble point pressure of the oil during the test, so it is single phase of oil flow during production period.

The following procedures depict how type curve matching is employed to calculate the essential parameters of reservoirs in oilfields. Firstly, prepare two pieces of log-log graph paper which are exactly the same, one of which is plotted by pressure difference (Δp) and its derivative value ($d\Delta p$) versus test time of well test data, the other one is plotted by a family of type curve. Secondly, put the
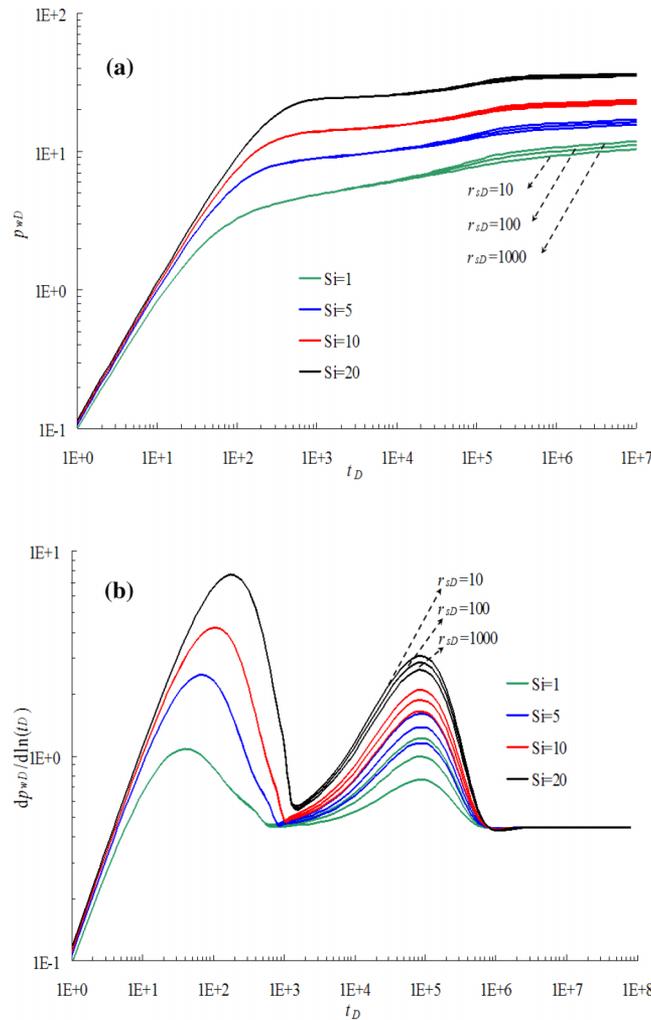
FIG. 5. Effects of r_{sD} on p_{wD} and $dp_{wD}/dln\,(t_D)$: (a) Effect of r_sD on p_{wD}; (b) Effect of r_{sD} on $dp_{wD}/dln\,(t_D)$.

graph contained test data to the type curve graph paper, and then move the paper to obtain the best match of data with one of the type curves. Finally, read from a random match point. Fig. 6 shows the result of fitting the type curves of the proposed model in this paper with the oilfield well test data.

The parameters of the type curve fitted well to the well test data were as follows: $v_{iD} = 0.001$, $\beta = 0.7$, $C_D = 100$, $S_i = 9.0$ and $r_{sD} = 40$. Choosing one point from the plot of the type curve overlapped the well test data arbitrarily. The coordinate of this point in the type curve is $(10^6, 5.12)$ and that of the well test data is $(18, 4.66)$. Therefore, unknown parameters can be calculated as follows.

$$k_{si} = 1.842 \times 10^{-3} \frac{q\mu B}{h} \left(\frac{p_D}{\Delta p} \right)_{match} = 0.75 \mu m^2 \tag{17}$$

$$\phi c_t h = \frac{3.6kh}{\mu r_w^2} \left(\frac{t}{t_D} \right)_{match} = 7.26 \times 10^{-3} m/MPa \tag{18}$$

$$C = 2\pi \phi c_t h r_w^2 (C_D)_{match} = 4.56 \times 10^{-2} m^3/MPa \tag{19}$$

$$S_i = (S)_{match} = 9.0 \tag{20}$$

The calculated results show that the initial skin factor is 9.0, and the initial permeability of damaged area is 0.75 μm^2. However, according to the theory of variable skin factor in this paper,
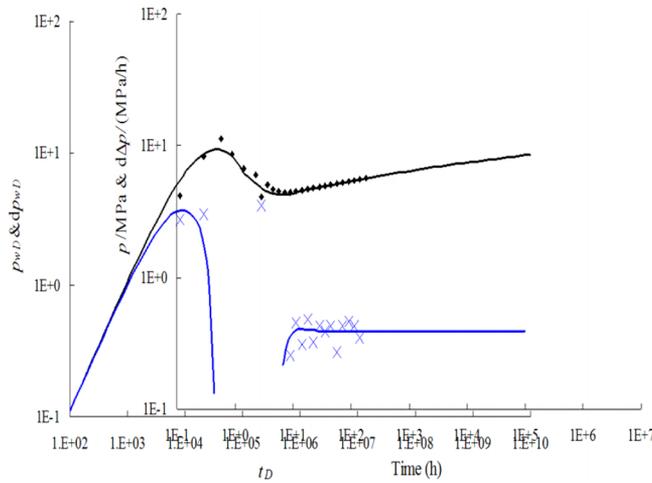
FIG. 6.   Result of fitting the type curves with well test data.

the ultimate permeability around the wellbore should be $0.75/(1-\beta) = 2.5$ μm². This result agrees closely with the 2.8 μm² which was verified by production performance in the later stage, and it indicated that the model used in this case is suitable.

As it was shown in Fig. 6 and the parameters of the reservoir, fluid flow rate was fast during well test, and the permeability of the layer was high. The cleanup effects made the particles around the wellbore washed into it resulting in skin factor decreasing gradually.[18,21] The variation of skin factor with time is shown in Fig. 7.

As shown in Fig. 7 and above analysis, the permeability around wellbore increased from 0.75 μm² to about 2.5 μm² and the skin factor decreased from 9 to a small value with an increase in production time. If there is no account for the variation of the skin around the well, it may cause too small oil production allocation or unnecessary operations (plugging removal, fracturing, and so on) resulting in economic losses.

Because different pressure differences occur, it is important to note that the type curve of $dp_{wD}(t_D)/d\ln(t_D)$ is not continuously smooth and the values sometimes are negative. The different type curve results from the piecewise function. In the former part of the piecewise function, the skin factor is considered as a constant whereas that is variable in the latter part of the piecewise function.

Strictly speaking, skin factor should be viewed as variable parameter in the former part as well, but if we do so, the equation is too difficult to solve. In addition, the time that represented by the
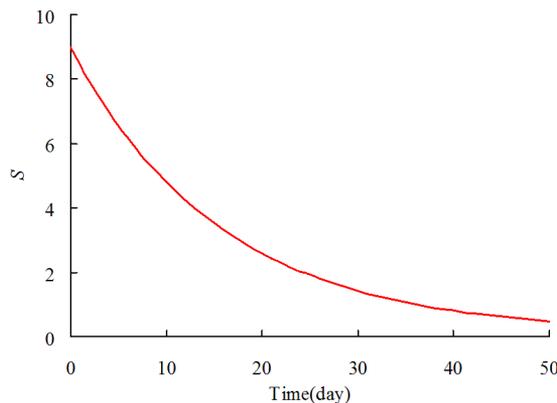

FIG. 7.   Temporal variation of skin factor.

former part of the function is too short, only occurs in the beginning of the production process. Finally, the data used in type curve matching procedure are usually in the middle and late periods. So the early period data is not as important as the data of middle and late periods. Therefore, this model has some drawbacks, but they do not affect the applicability of the type curves.

V. CONCLUSIONS

1) This paper established a well test analysis model of considering variable skin factor with the model expressed as a relationship between skin factor and time by defining two parameters that are β and v_{iD}. It was proven that the definitions of these two parameters were reasonable by case analysis.

2) Laplace transform was used to solve the model and the type curves were plotted based upon the results. These plots were variation of $p_{wD}(t_D)$ and $dp_{wD}(t_D)/d\ln(t_D)$ with t_D. The influence of the parameters in the expression of the variable skin factor (β, v_{iD}, and r_{sD}) on $p_{wD}(t_D)$ and $dp_{wD}(t_D)/d\ln(t_D)$ were discussed.

3) The flow of the reservoir is not fully extended to the formation in early flow period in the proposed mode. The effect of wellbore storage played a major role and the skin factor in this period was considered as a constant. When the effect of wellbore storage is neglected, the skin factor can be viewed as a variable.

4) A case analysis of well test data of a well was conducted to illustrate and validate the applicability of the model of Sheng-2 Block in Shengli Oilfield, China.

5) The solution in Laplace domain is a piecewise function, and there exist the cases of dimensionless pressure drop. Therefore, the curve of $dp_{wD}(t_D)/d\ln(t_D)$ will not be smooth and sometimes the values are negative, but these shortcomings do not affect the applicability of the type curves.

ACKNOWLEDGMENT

[1] A. F. Van Everdingen and W. Hurst, "The skin effect and its influence on productive capacity of a well," J. Pet. Technol. **1**(12), 305–324 (1949).

[2] A. R. Groeneman and F. F. Wright, "Analysis of fluid input wells by shut-in pressures," J. Pet. Technol. **8**(07), 21–24 (1956).

[3] A. F. van Everdingen, "The skin effect and its influence on productive capacity of a well," J. Pet. Technol. **5**(06), 171–176 (1953).

[4] M. F. Hawkins, Jr., "A note on the skin effect," J. Pet. Technol. **8**(12), 65–66 (1956).

[5] H. M. Elshahawi and K. H. Gad, "Estimation of skin for high deliverability gas well tests," in *SPE Middle East Oil Show*. (2001), Paper No. SPE 68144-MS.

[6] J. Lu, T. Zhu, D. Tiab, and J. Owayed, "Productivity formulas for a partially penetrating vertical well in a circular cylinder drainage volume," Math. Probl. Eng. Article ID 626154:1–34 (2009).

[7] M. Lastre, A. Milne, and C. Pinto, "Productivity improvement in highly unconsolidated formation: skin characterization and mitigation, case history," in *SPE European Formation Damage Conference & Exhibition* (2013), Paper No. SPE 165123-MS.

[8] R. G. Agarwal, R. Al-Hussainy, and H. J. Ramey, Jr., "An investigation of wellbore storage and skin effect in unsteady liquid flow: I. Analytical treatment," SPE J. **10**(03), 279–290 (1970).

[9] D. Bourdet, T. M. Whittle, A. A. Douglas, and Y.M. Pirard, "A new set of type curves simplifies well test analysis," World Oil **196**(6), 95–106 (1983).

[10] S. Vongvuthipornchai and R. Raghavan, "Well test analysis of data dominated by storage and skin: non-Newtonian power-law fluids," SPE Form. Eval. **2**(04), 618–628 (1987).

[11] H. J. Ramey, Jr., "Short-time well test data interpretation in the presence of skin effect and wellbore storage," J. Pet. Technol. **22**(01), 97–104 (1970).

[12] A. F. Moench and P. A. Hsieh, "Analysis of slug test data in a well with finite thickness skin," in *Memoirs of the 17th international congress on the hydrogeology of rocks of low permeability* (1985), Vol. 17(01), pp. 1729–1735.

[13] L. K. Blacker, "An analysis of rate-sensitive skin in oil wells," in *SPE Annual Technical Conference and Exhibition* (1982), Paper No. SPE11187-MS.

[14] M. Y. Soliman, S. Hedayati, and T. Daneshi, "Analysis of buildup and drawdown tests with variable wellbore storage and skin," in *SPE Production Operations Symposium* (1991), Paper No. SPE 21708-MS.

[15] L. Ma, R. T. Johns, D. Zhu, and A.D. Hill, "Fast method for real-time interpretation of variable-rate wells with changing skin: application to matrix acidizing," in *International Oil and Gas Conference and Exhibition in China* (2000), Paper No. SPE 64651-MS.

[16] R. D. Gdanski, D. D. Fulton, and C. Shen, "Fracture face skin evolution during cleanup," in *SPE Annual Technical Conference and Exhibition* (2006), Paper No. SPE101083-MS.

[17] I. M. Mohamed, G. I. Block, O. A. Abou-Sayed, S.M. Elkatatny, and A.S. Abou-Sayed, "Flow rate dependent skin in water disposal injection well: case study," in *48th US Rock Mechanics/Geomechanics Symposium* (2014), ARMA-2014-7758.

[18] L. Larsen and K. Kviljo, "Variable-skin and cleanup effects in well-test data," SPE Form. Eval. **5**(03), 272–276 (1990).

[19] F. F. Zinati, R. Farajzadeh, P. K. Currie, and P. L. J. Zitha, "Modeling of external filter cake build-up in radial geometry," Pet. Sci. Technol. **27**(07), 746–763 (2009).

[20] F. Civan, "Non-isothermal permeability impairment by fines migration and deposition in porous media including dispersive transport," J. Transport Porous Med. **85**(01), 233–258 (2010).

[21] P. Bedrikovetsky, A. Vaz, F. Machado, A. Zeinijahromi, and S. Borazjani, "Skin due to fines mobilization, migration, and straining during steady-state oil production," Pet. Sci. Technol. **30**(15), 1539–1547 (2012).

[22] G.B. Thomas, "Analysis of pressure build-up data," J. Pet. Technol. **5**(04), 125–128 (1953).

[23] H. J. Ramey, A. Kumar, and M. S. Gulati, "Gas well test analysis under water-drive conditions," American Gas Association, Washington D. C., USA (1973).

[24] M. F. Gardner and J. L. Barnes, "Transients in linear systems, studied by the Laplace transform," Bull. Amer. Math. Soc. **49**, 837–840 (1943).

[25] H. Stehfest, "Algorithm 368: Numerical inversion of Laplace transforms [D5]," Communications of the ACM. **13**(01), 47–49 (1970).