ZVX146E Environmental Hydraulics

Lecture_4

2025_2026

Environmental Hydraulics 2025_2026

X=	Name	E1	E2	E3	E4	E5	Ext.	Test	Σ	Results
1	Aznar-Etchepare Gilen	4	4						8	
2	Eslami Talemi Zeynab	4							4	
3	Hanif Muhammad Tayyab								0	
4	Chaure Anmol	4							4	
5	Kabeleka Chisenga								0	
6	Khan Ijaz Azeem	4	3						7	
7	Mensching Una Milena	4	4						8	
8	Naghizadeh Avilagh Fariba	4							4	
9	Oluwatudimu Olayinka Anita								0	
10	Thorand Jean-Noé	4							4	
11	Thu Kaung Si	4							4	
12										

ARCHIMEDES’ PRINCIPLE

The buoyant force acting on a submerged object is equal to the weight of the fluid the object displaces.

ARCHIMEDES’ PRINCIPLE states that the **WEIGHT** of the amount of water displaced is **equal** to the **BUOYANT FORCE**.

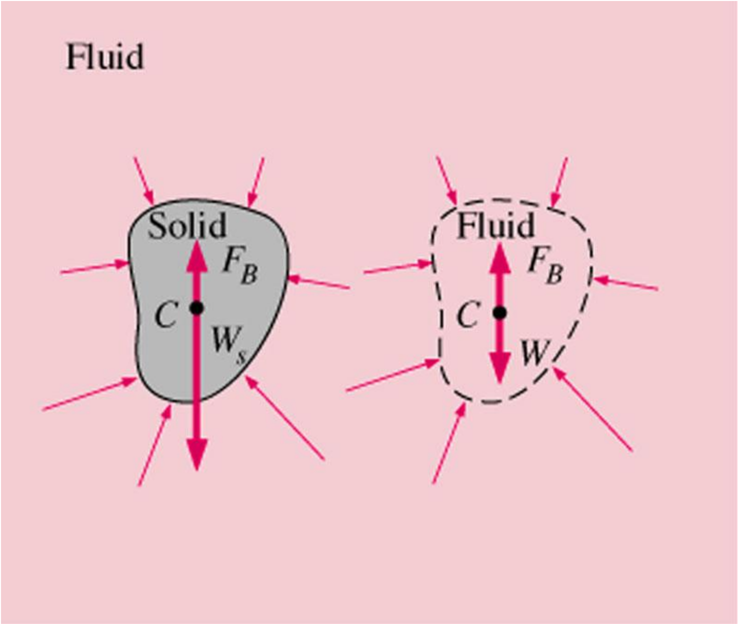
BUOYANCY

The tendency of fluid to exert a supporting force on a body placed in the fluid.

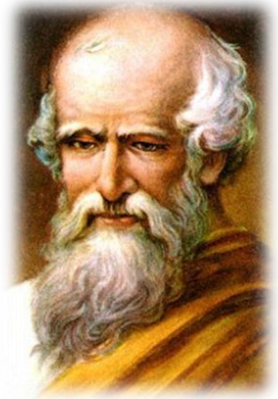
The force = ***weight of the fluid displaced by the body***. Its act upward through centroid of the displaced volume.

$$F_B = \rho_f . g . V_f$$

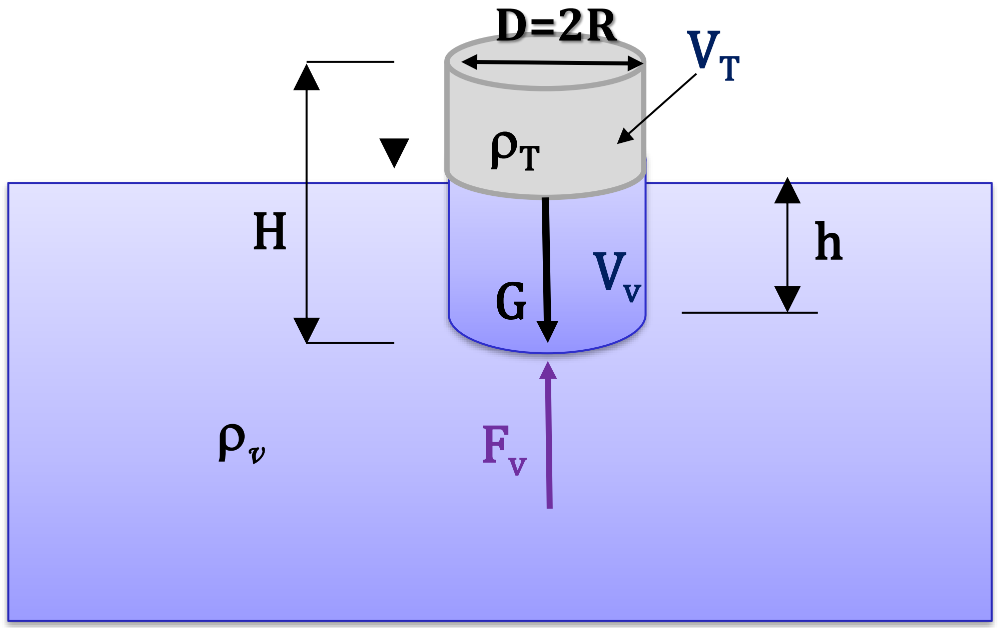
Density of fluid



Buoyancy



Archimédés (287–212)



F_b – buoyancy force

G – gravity force (weight of body)

V_v - volume of the submerged part of the body

Laws of buoyancy discovered by Archimedes:

A body immersed in a fluid experiences a vertical buoyant force equal to the weight of the fluid it displaces

A floating body displaces its own weight in the fluid in which it floats

ρ_{WATER} - density of liquid

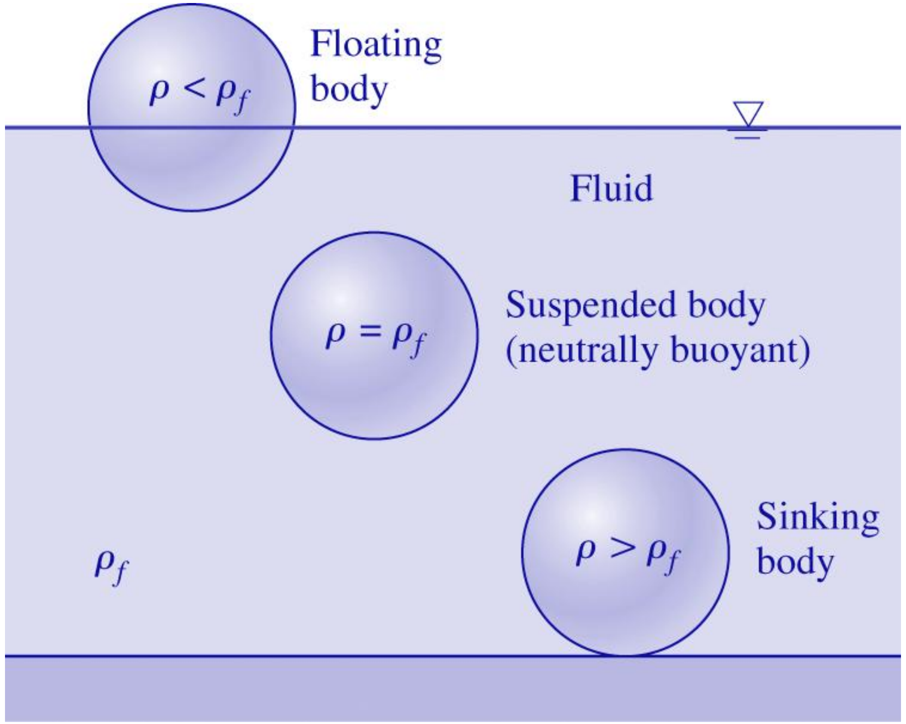
ρ_{SOLID} - density of body

$$F_b = G = \rho_{WATER}\, g\, V_W = \rho_{SOLID}\, g\, V_T$$

$$V_W = \pi R^2 h$$

$$V_T = \pi R^2 H$$

BUOYANCY AND STABILITY



- Buoyancy force F_B is equal only to the displaced volume $\rho_\phi g V_{displaced}$.
- Three scenarios possible
  1. $\rho_{\beta ody} < \rho_{\phi luid}$: **Floating** body
  2. $\rho_{body} = \rho_{fluid}$: **Neutrally** buoyant
  3. $\rho_{\beta ody} > \rho_{\phi luid}$: **Sinking** body

BUOYANCY

- **BUOYANCY** = the ability to float in a fluid.
- Examples of fluids = water, air
- **BUOYANT FORCE** = the upward force that acts on a submerged object.
  - It acts opposite of gravity

Hydrodynamics - introduction

VISCOSITY

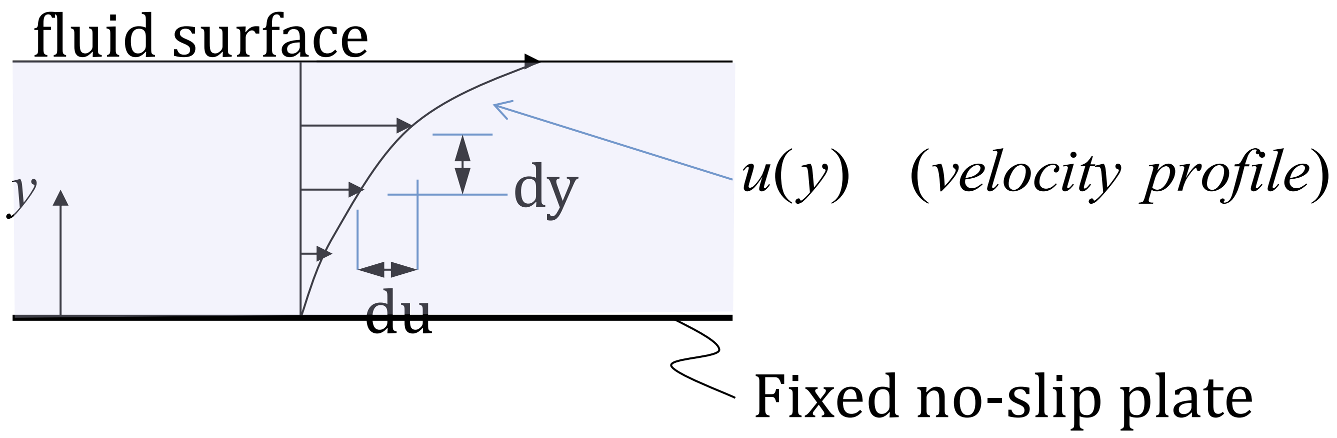
NEWTON'S EQUATION OF VISCOSITY

Viscosity is a measure of the resistance of a fluid to deform under shear stress.

Shear stress due to viscosity between layers: $\tau = \mu \dfrac{du}{dy}$

μ - **dynamic viscosity** (coeff. of viscosity)

$\nu = \dfrac{\mu}{\rho}$ - **kinematic viscosity**



Use definition of shear force: $\boldsymbol{F} = \tau A = \boldsymbol{\mu A \dfrac{du}{dy}}$

VISCOSITY

	Dynamic Viscosity (Pa·s)		Dynamic Viscosity (Pa·s)
Gases		**Heat Transfer Liquids**	
Air	18.3×10^{-6}	FC-87	0.4×10^{-3}
Nitrogen	17.8×10^{-6}	Galden ZT 85	0.8×10^{-3}
Oxygen	20.2×10^{-6}	R134a	0.2×10^{-3}
Hydrogen	8.8×10^{-6}	Ammonia	0.12×10^{-3}
Xenon	21.2×10^{-6}	**Other Liquids**	
Ammonia	9.8×10^{-6}	Glycerol	934×10^{-3}
Liquids		Olive oil	80
Water	0.28×10^{-3}	Ketchup	50
Ethanol	1.1×10^{-3}	Peanut butter	150
Acetone	0.31×10^{-3}	**'Solids'**	10^{5}

Viscosity (Temp)

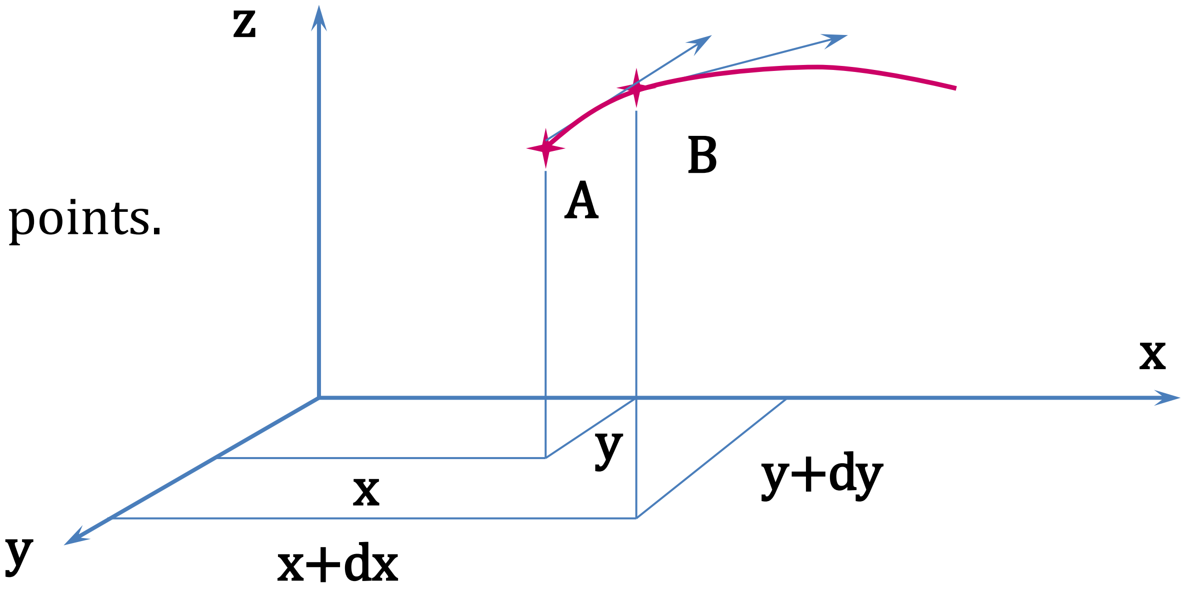
Temp. (°C)	Water: Viscosity, μ (Pa s $\times 10^5$)	Water: Kinematic viscosity, ν ($m^2/s \times 10^6$)	Air: Viscosity, μ (Pa s $\times 10^5$)	Air: Kinematic viscosity, ν ($m^2/s \times 10^6$)
0	179.2	1.792	1.724	13.33
10	130.7	1.307	1.773	14.21
20	100.2	1.004	1.822	15.12
30	79.7	0.801	1.869	16.04
40	65.3	0.658	1.915	16.98

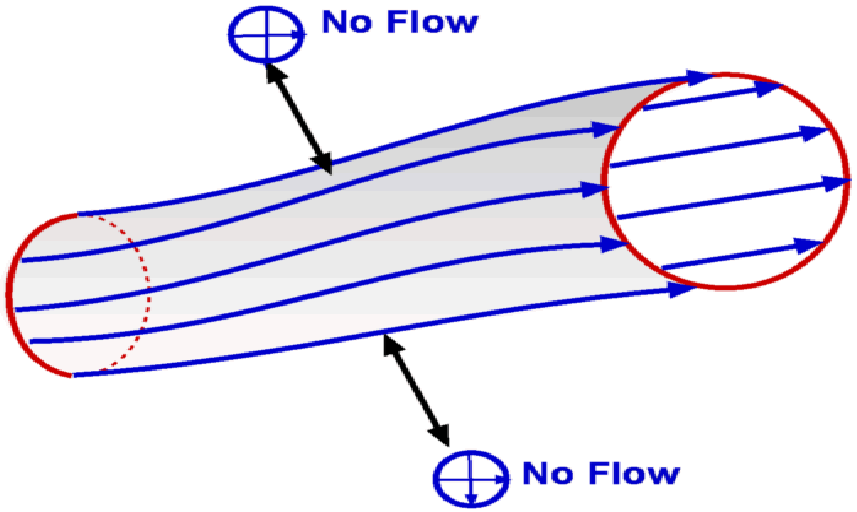
Fluid	Dynamic Viscosity (Pa.s)**
Water	$\mu_W = \left(\dfrac{997.2}{2.443299 \times 10^{-2} \times T - 6.153676} \right)$
Oil	$\mu_O = 0.6402 + 18.9612 \times e^{(-0.074 \times T)}$
Air	$\mu_g = 2.8 \times 10^{-7} \times T^{0.735476}$

STREAMLINE; STREAMTUBE

STREAMLINE is a curve that is everywhere tangent to the *instantaneous local velocity vector.*

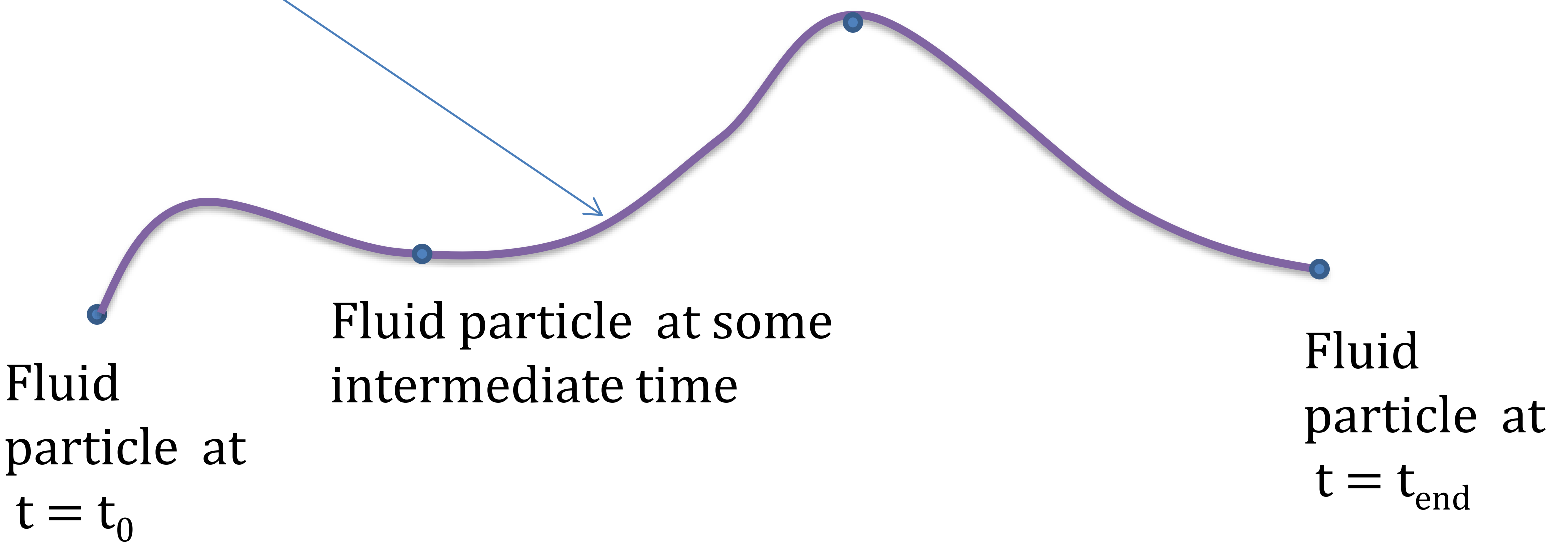
velocity vectors
are tangent to it at all points.





STREAMTUBE

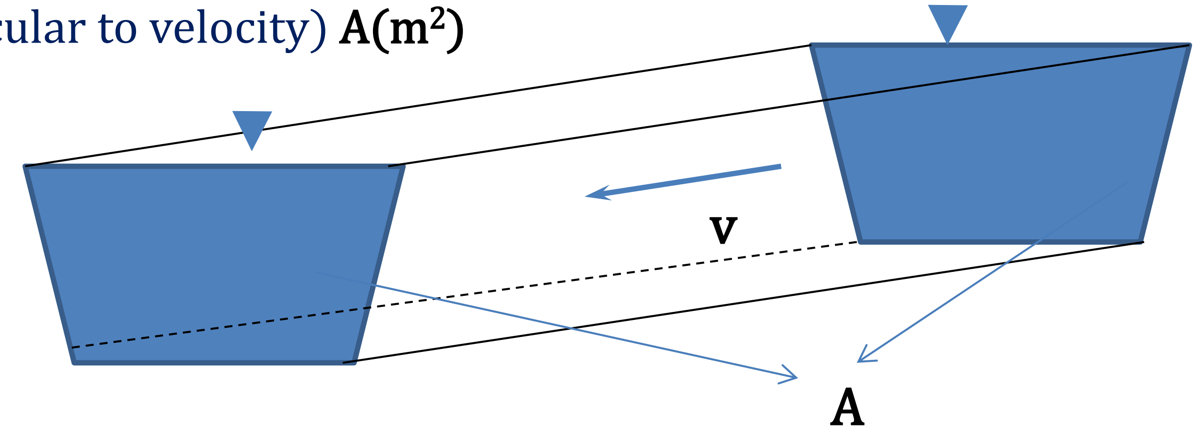
A **PATHLINE (TRAJECTORY) is the actual path** traveled by an individual fluid particle over some time period.



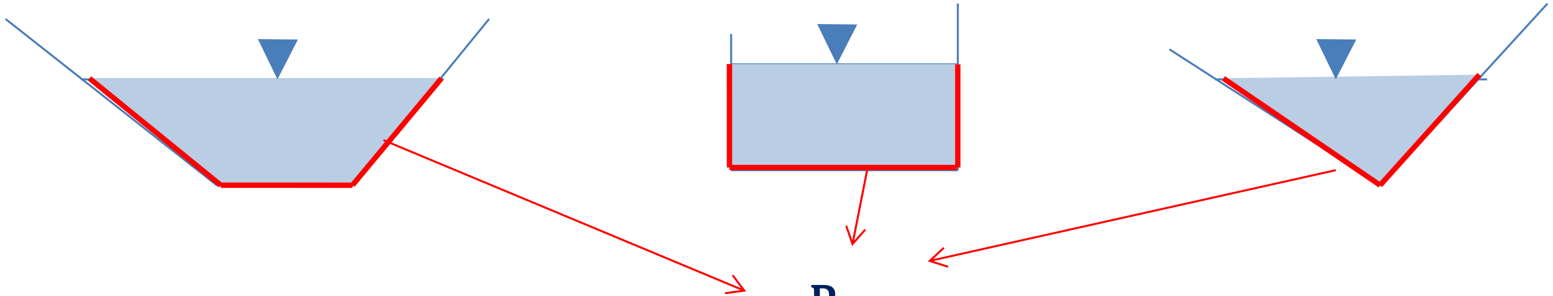
For **steady flow**, **streamlines** and **pathlines** are **identical.**
For **unsteady flow**, they can be **very different**.

CHARACTERISTICS OF HYDRODYNAMICS

Flow area, **CROSS SECTIONAL AREA**
(perpendicular to velocity) $A(m^2)$



WETTED PERIMETER P (m)



HYDRAULIC RADIUS $R = A/P$ (m)

circular pipeline with diameter D:

$$R = \frac{A}{P} = \frac{\pi D^2}{4 \pi D} = \frac{D}{4}$$

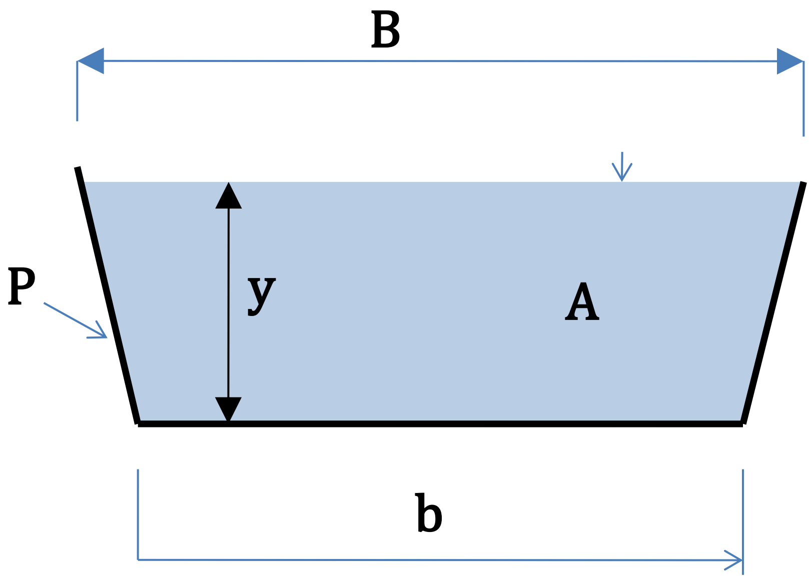
CHARACTERISTICS OF HYDRODYNAMICS

Depth y, h [m]

Channel width at bottom b [m],

Channel width at water level B [m]

Mean depth $y_m = A/B$ [m]



Discharge (mass , volume discharge) Q_M ($kg.s^{-1}$)

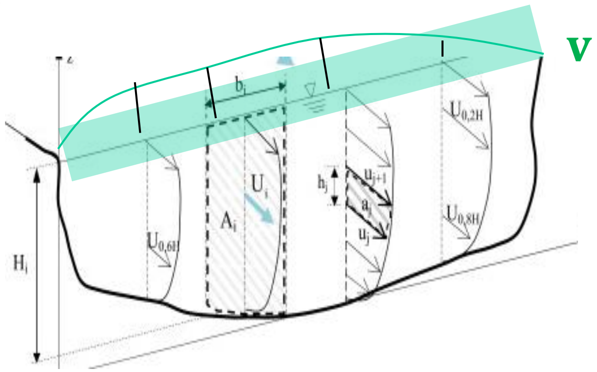
Q_V ($m^3.s^{-1}$) = **Q**

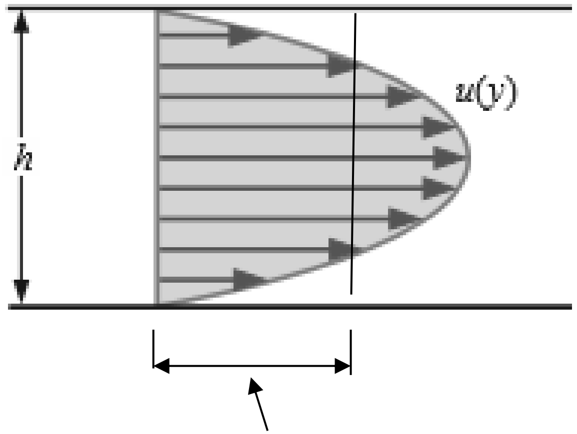
CHARACTERISTICS OF HYDRODYNAMICS

POINT VELOCITY

$$u = \frac{ds}{dt}$$

THE AVERAGE (MEAN) VELOCITY - v - is defined as the average speed through a cross section.





elementary volume **discharge**

$$dQ = u\,dA$$

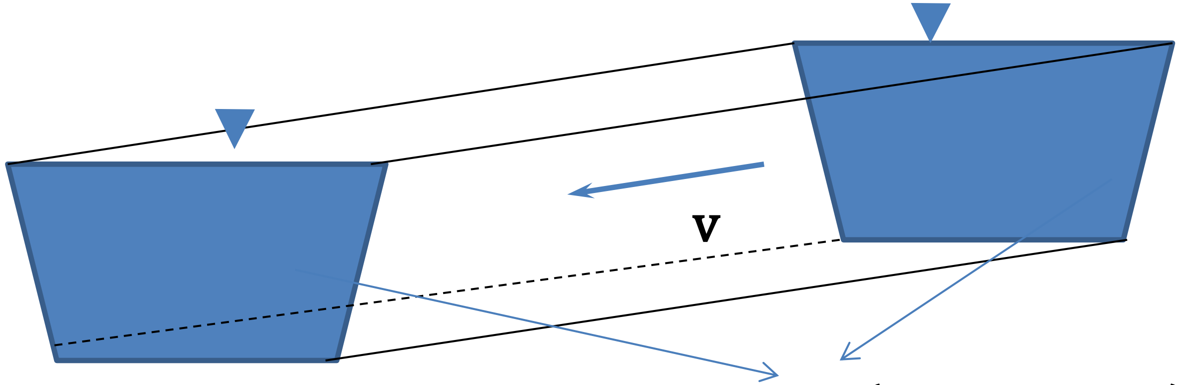
MEAN VELOCITY $v = Q/A$

DISCHARGE (mass) $= \rho \, . \, v \, . \, A$

MASS RATE PAST A CROSS-SECTION: Q_m (kg/s)

DISCHARGE (volume) $= v \, . \, A = Q$

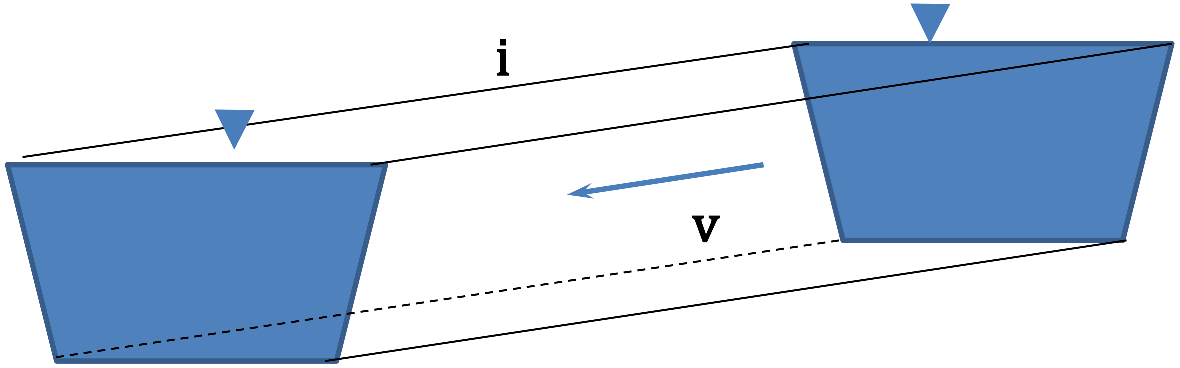
VOLUME FLOW RATE PAST A CROSS- SECTION: Q (m^3/s)



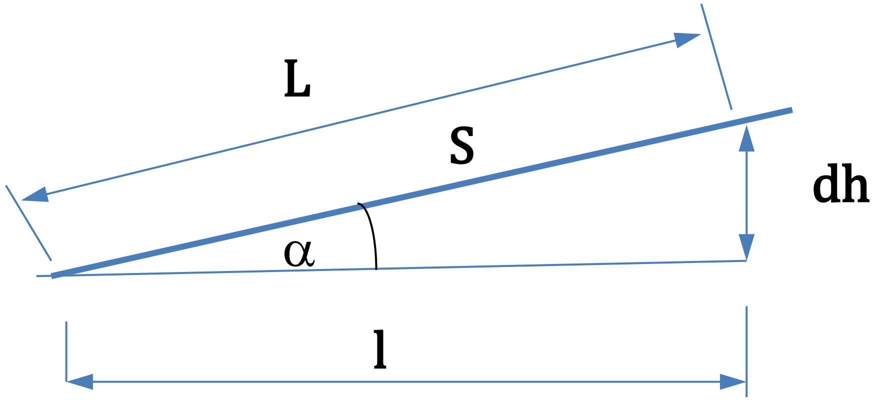
A (cross setions)

CHARACTERISTICS OF HYDRODYNAMICS

SLOPE of bottom - S



α	sin(α)	tan(α)
0°	0	0
5°	0.087	0.087
10°	0.174	0.176
20°	0.342	0.346
30°	0.500	0.577
40°	0.643	0.839
50°	0.766	1.192



$$S = \frac{dh}{L} \quad \Rightarrow \quad \frac{dh}{l}$$

For small α (cca 8-10°)

$$\sin\alpha \approx tg\,\alpha$$

KINDS AND FORMS OF FLOW

UNSTEADY FLOW ….. Q = Q(t), v= v(t)

STEADY FLOW ….. Q = const.

a) **UNIFORM** flow … $A = const. \quad v = const.$

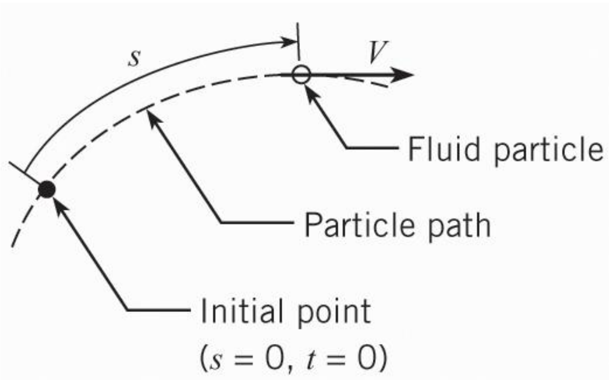
b) **NON – UNIFORM** flow $A \neq const. \quad v \neq const.$

- **WITH FREE LEVEL** – flow limited by solid walls, free level on surface, motion caused by own weight of liquid
- **PRESSURE** – flow limited by solid walls from all sides, motion caused by difference of pressures
- **JETS** – limited by liquid or gas surroundings, motion by own weight or by delayed action (inertia)

SOLUTION OF FLOW:

- **SPACE FLOW** (3D numeric models)
- **PLANAR FLOW** (2D – simplified solution)
- **ONE DIMENSIONAL** (1D – current routine calculations of pipelines, channels, structures)

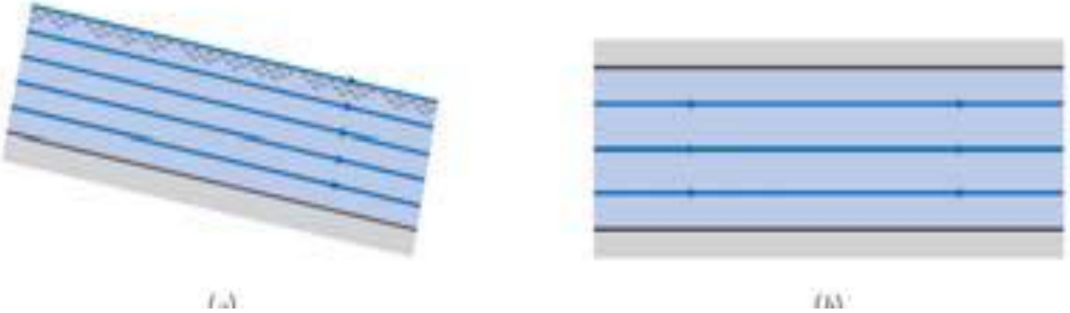
TYPES OF STEADY FLOW



Express velocity v = v(s,t)

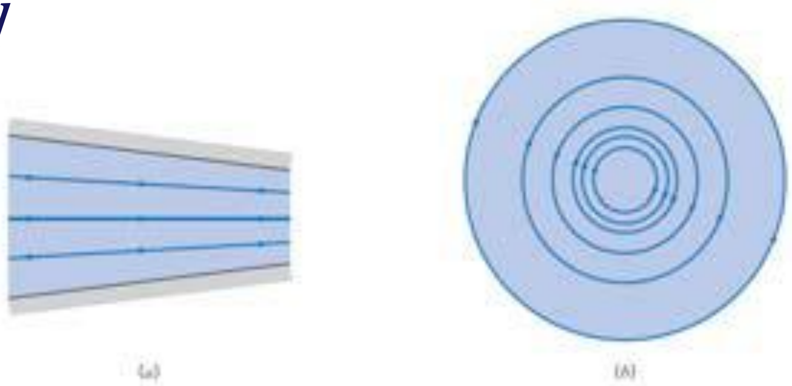
Uniform: Velocity is constant along a streamline (Streamlines are *straight and parallel*)

$$\frac{\partial V}{\partial s} = 0$$



Non-uniform: Velocity changes along a streamline (Streamlines *are curved and/or not parallel*)

$$\frac{\partial V}{\partial s} \neq 0$$



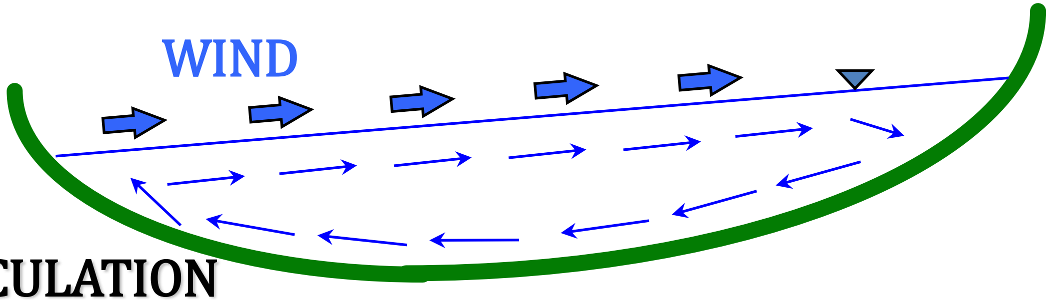
Vortex flow

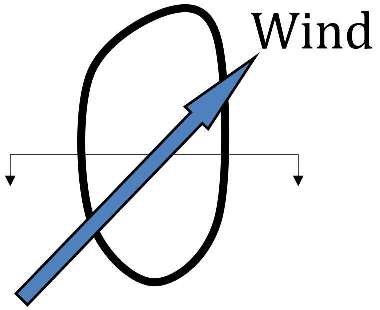
FLOW DIMENSIONALITY

- Most of the real flows are
  3-dimensional and unsteady : u (x,y,z,t)
- For many situations simplifications can be made :
  2-dimensional unsteady and steady flow
  u (x,y,t)
  1-dimensional unsteady and steady flow
  u (x,t)

1-D, 2-D, 3-D FLOW

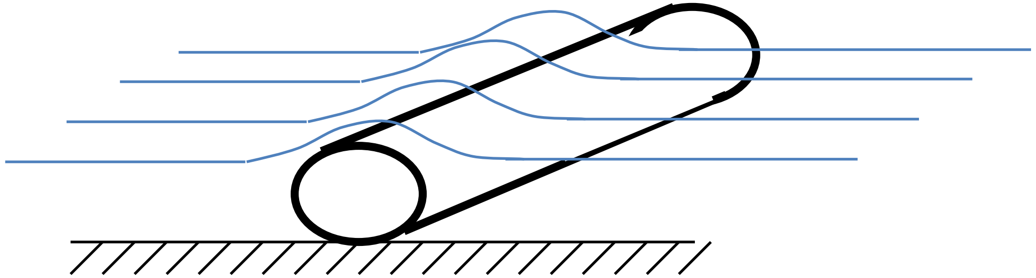
3-D FLOW

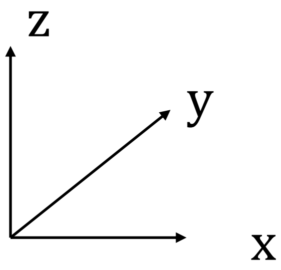




Ex: LAKE CIRCULATION

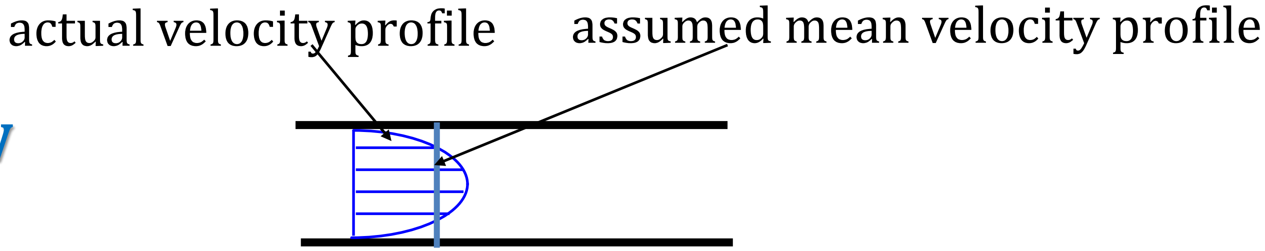
2-D FLOW





Ex: LONG OBSTACLE

1-D FLOW

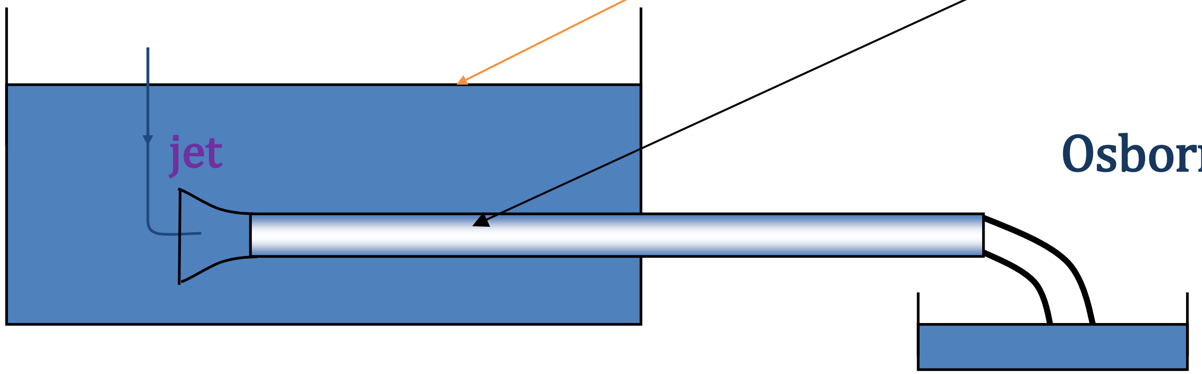


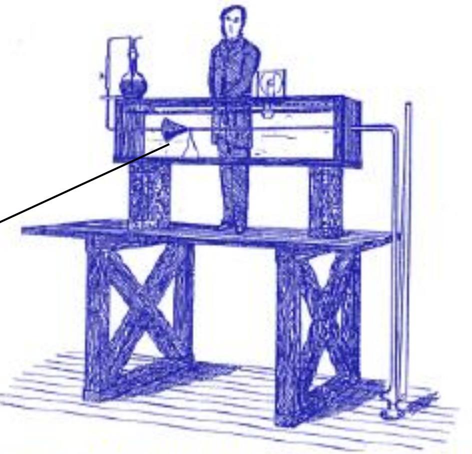
Ex: PIPE FLOW

REAL FLUID

LAMINAR AND TURBULENT FLOW

Reynolds experiment 1883:





Reynolds' Turbulent flow experimental apparatus

Osborne Reynolds (1842-1912)

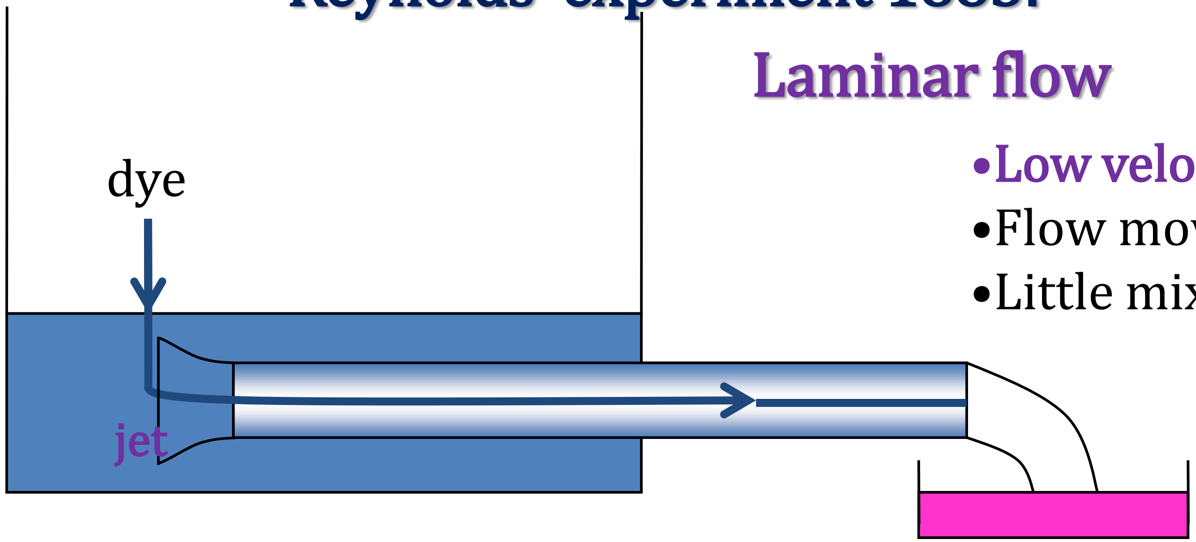
Two different, distinct **flow regimes**:

A) **LAMINAR FLOW**

B) **TURBULENT FLOW**

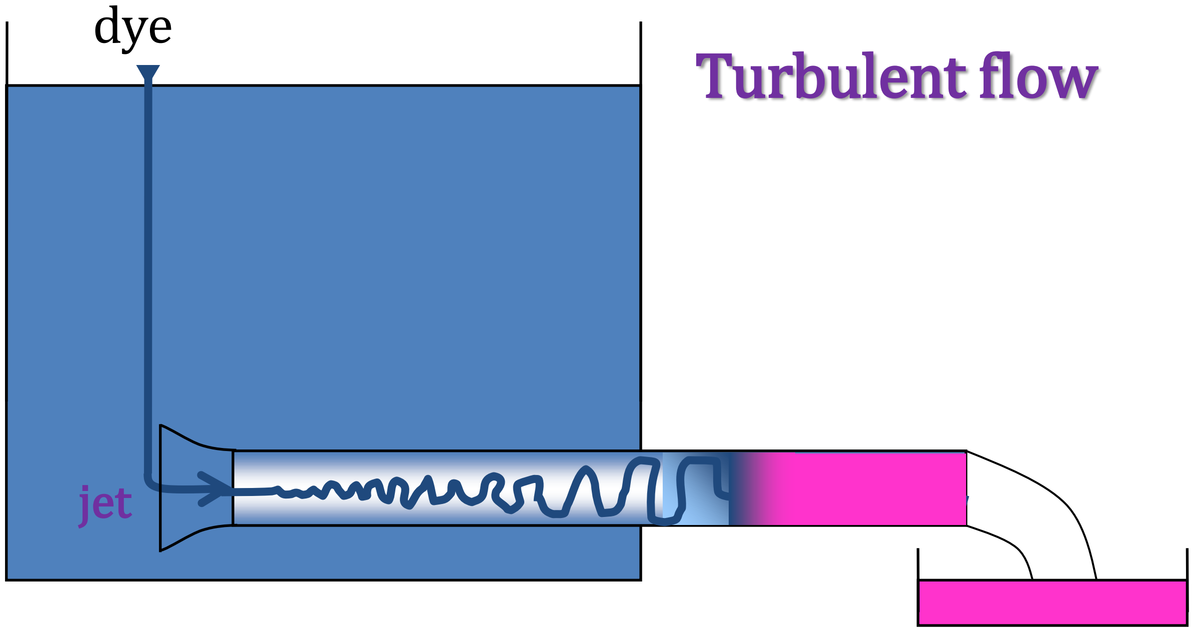
Reynolds experiment 1883:

Laminar flow



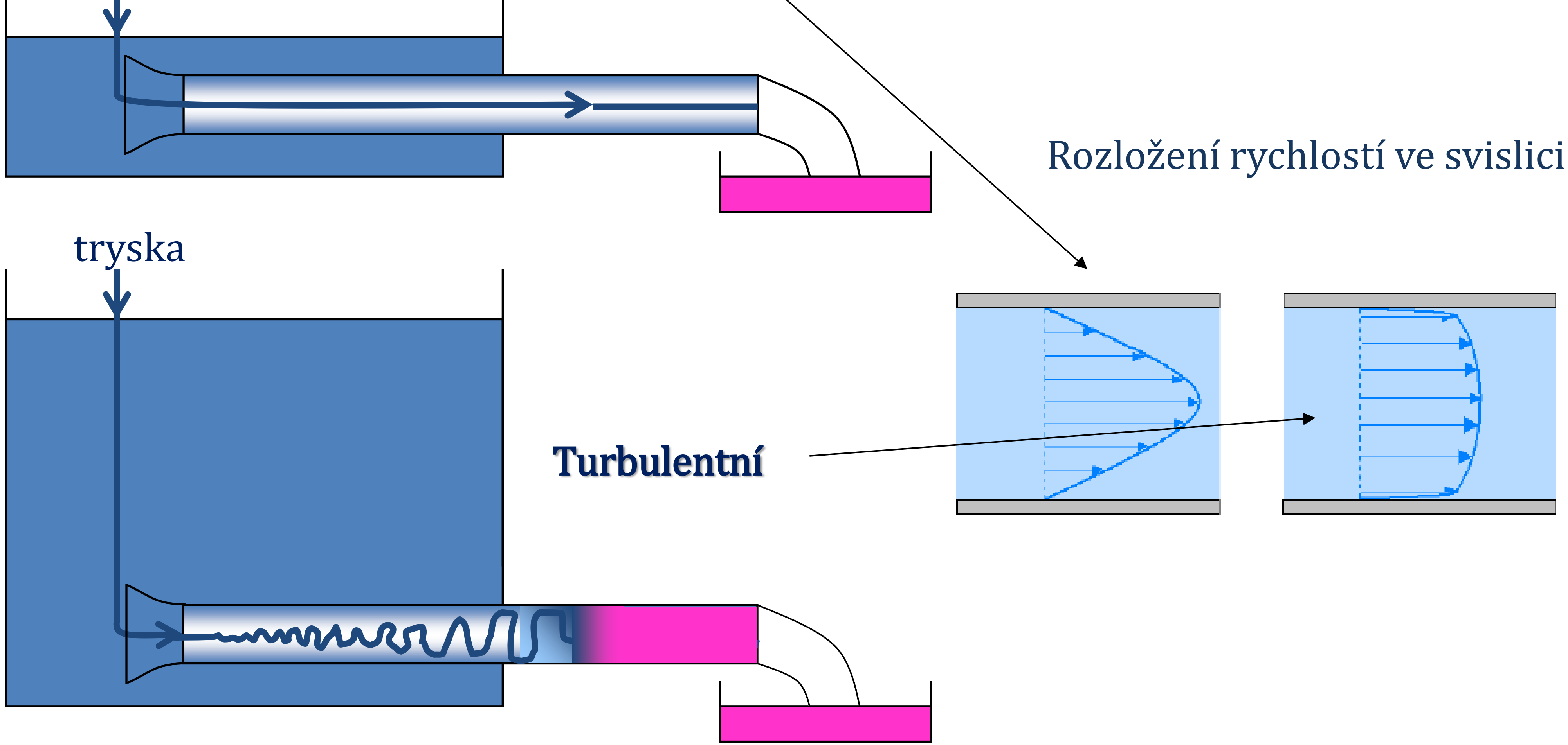
- Low velocity
- Flow moving in sheets
- Little mixing between sheets

Turbulent flow



- High velocity
- Dye breaks up in a diffuse cloud
- Significant mixing
- Mixing increases with velocity
- Currents perpendicular to pipe

Reynoldsův experiment - 1883



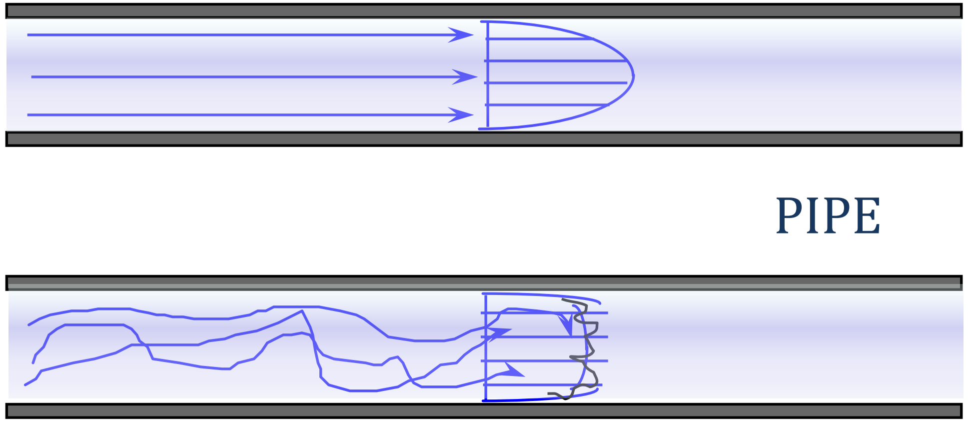
DEFINITIONS

- **LAMINAR:** FLOW IS LAMINAR IF PARTICLES MOVE IN DEFINED LAYERS IN DEFINED PATH,(NO CROSSING OF LAYERS, flow on a skin)

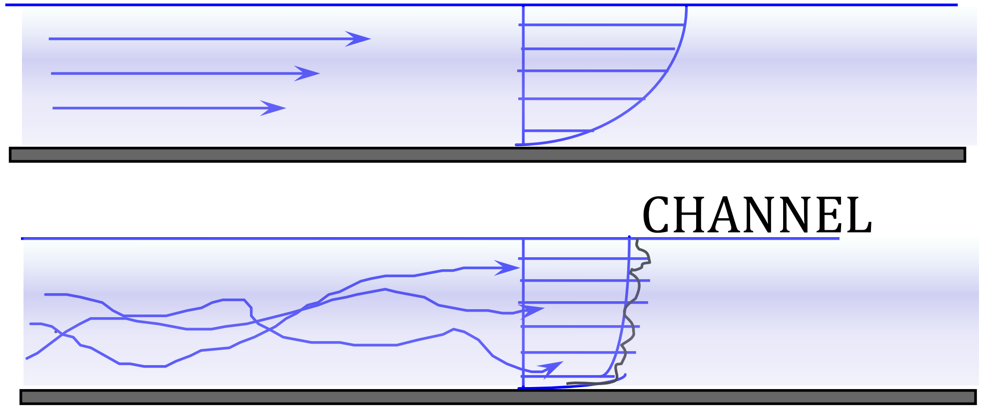
- **TURBULENT:** PARTICLES MOVE IN A ZIG- ZAG WAY (PARTICLES CROSS EACH OTHER/ LAYERS, high speed flow in pipe)

LAMINAR AND **TURBULENT** FLOW

- **LAMINAR** FLOW
- **TURBULENT** FLOW



- **LAMINAR** FLOW
- **TURBULENT** FLOW



LAMINAR AND TURBULENT FLOW

- **LAMINAR** – particles of liquid move at parallel paths
- **TURBULENT** – motion of particles of liquid: irregular and

inordinate, fluctuations of velocity vector in time and space, mixing inside flow

- Criterion – **Reynolds number**

L – characteristic length:

diameter D for pipelines, hydraulic radius R

Critical Reynolds Number - **for pipe** $\mathbf{Re_{cr} = 2320}$

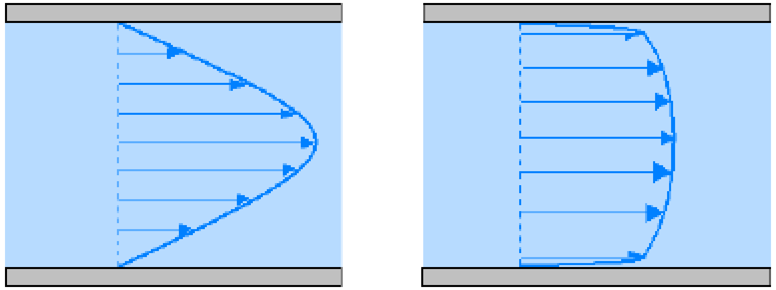
for open channel $\mathbf{Re_{cr} = 580}$

for groundwater flow $\mathbf{Re_{cr} = 1}$

$$Re = \frac{vD}{\upsilon}$$ **pipe** $$Re = \frac{\rho v D}{\mu}$$

$$Re = \frac{vR}{\upsilon}$$ **Flow wit free water level**

Hydraulic radius



OPEN CHANNEL FLOW

$$Re = \frac{vR}{\nu}$$

$$R = \frac{A}{P} = \frac{\frac{\pi D^2}{4}}{\pi D} = \frac{D}{4}$$

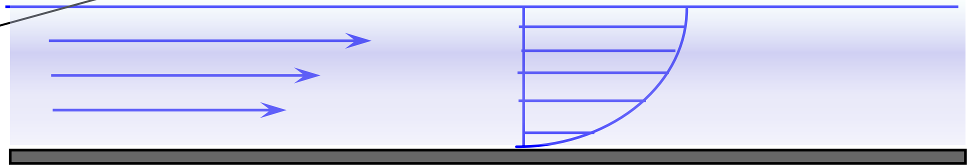
For flow in open channel

Re < 580 laminar

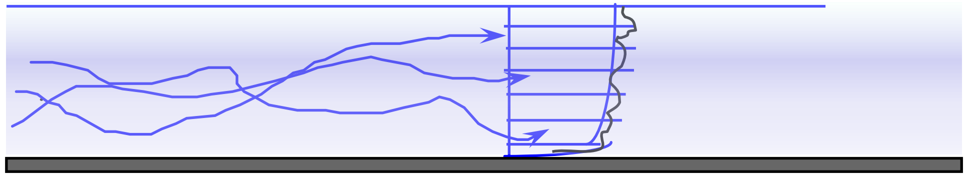
580 < Re < 1000 transitional

Re>1000 ... turbulent

laminar



turbulent



REYNOLDS NUMBER

The *Reynolds number* can be used as a criterion to distinguish between laminar and turbulent flow:

$$\mathbf{Re} = \frac{\rho\, v\, D}{\mu} = \frac{v\, D}{\upsilon}$$

v=mean velocity in pipe
D=diameter of pipe
ν=kinematic viscosity
μ = dynamic viscosity
ρ = density

For flow in a pipe

- **Laminar** flow if Re < 2320
- **Turbulent** flow if Re > 4000
- **Transitional** flow if 2320< Re < 4000

➢ For very high Reynolds numbers, viscous forces are negligible: ***inviscid flow***

➢ For very low Reynolds numbers (Re<<1) viscous forces are dominant: ***creeping flow***

OPEN CHANNEL FLOW

$$Re = \frac{vR}{\upsilon}$$

$$R = \frac{A}{P} = \frac{\frac{\pi D^2}{4}}{\pi D} = \frac{D}{4}$$

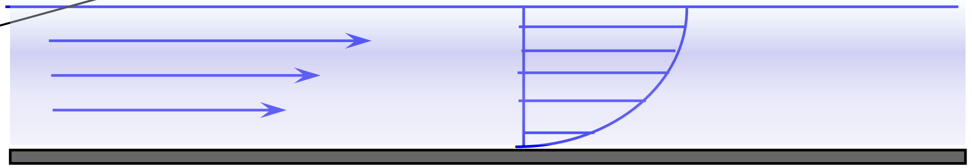
For flow in open channel

Re < 580 **laminar**

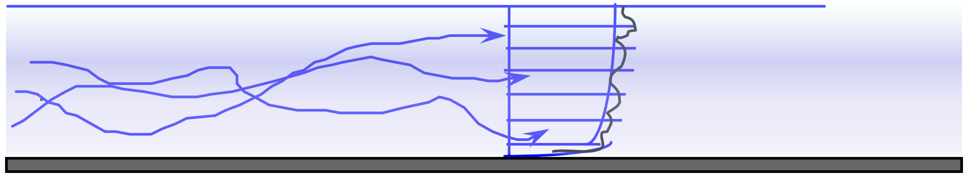
580 < Re < 1000 **transitional**

Re>1000 **... turbulent**

laminar



turbulent

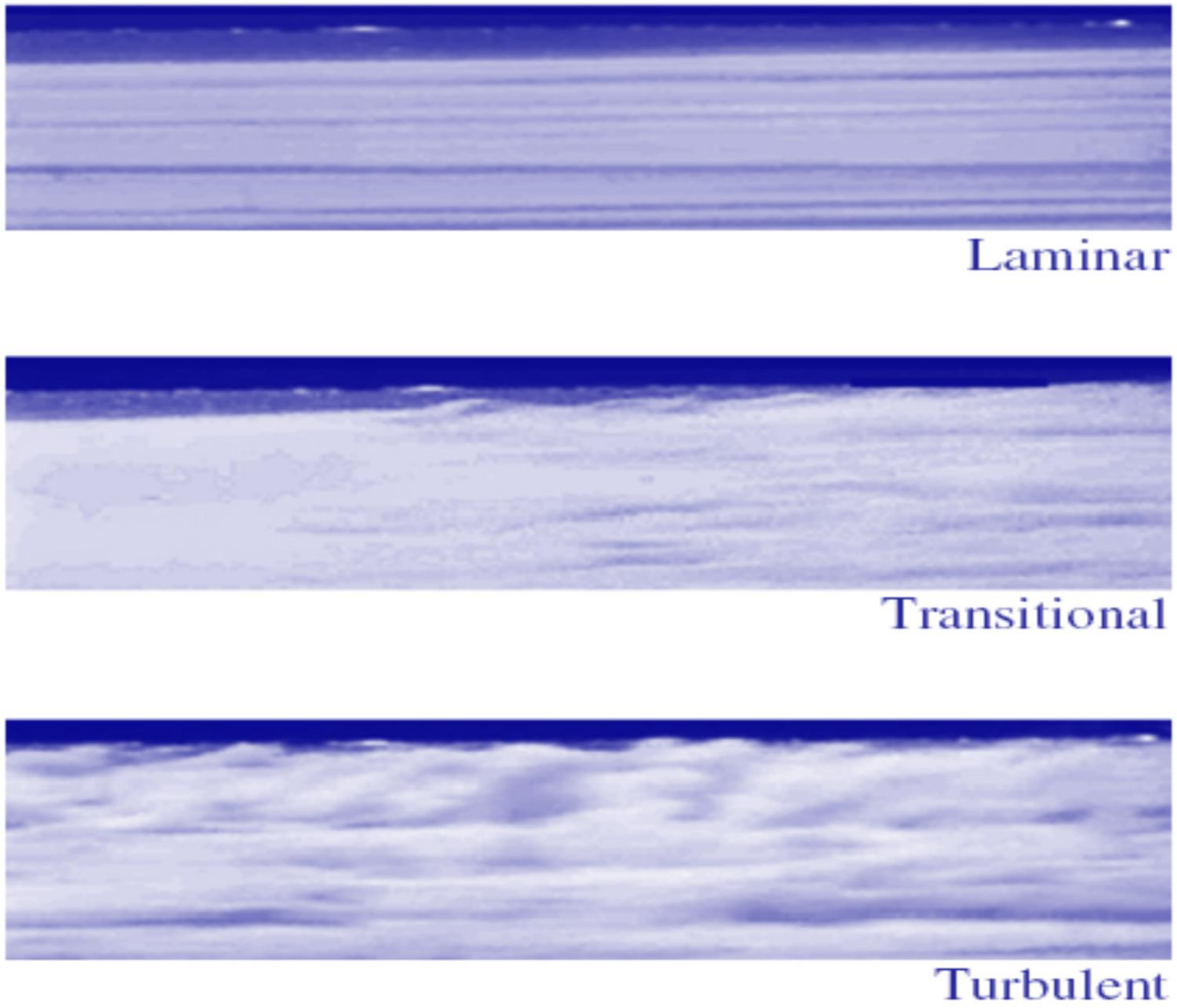


LAMINAR VERSUS TURBULENT FLOW

Laminar flow: The highly ordered fluid motion characterized by smooth layers of fluid. The flow of high-viscosity fluids such as oils at low velocities is typically laminar.

Turbulent flow: The highly disordered fluid motion that typically occurs at high velocities and is characterized by velocity fluctuations. The flow of low-viscosity fluids such as air at high velocities is typically turbulent.

Transitional flow: A flow that alternates between being laminar and turbulent.

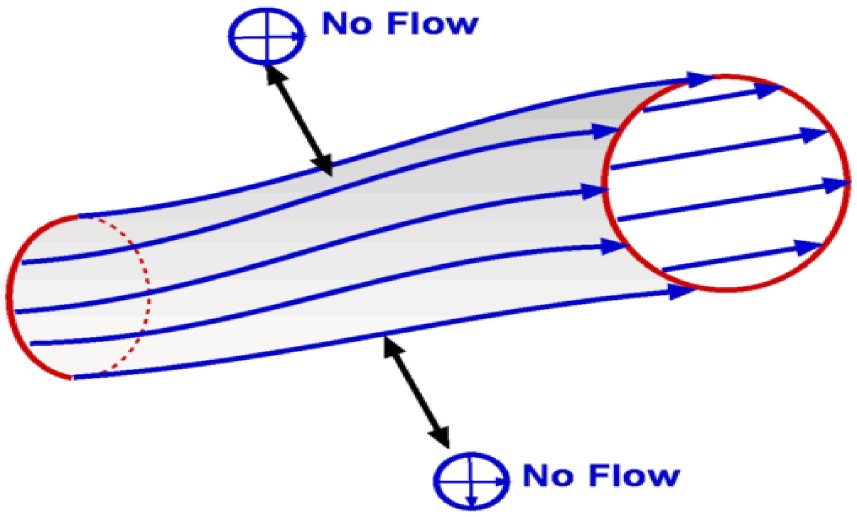


Laminar, transitional, and turbulent flows over a flat plate.

CONTINUITY EQUATION

STREAM-TUBE AND CONTINUITY EQUATION

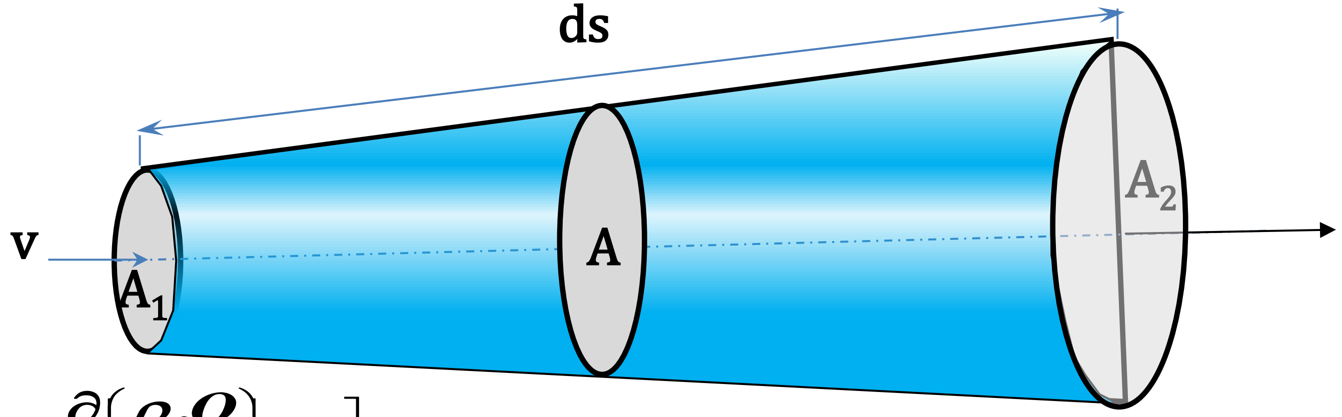
- STREAM-TUBE
- CONTINUITY EQUATION



Is the surface formed instantaneously by all the streamlines that pass through a given closed curve in the fluid.

CONTINUITY EQUATION

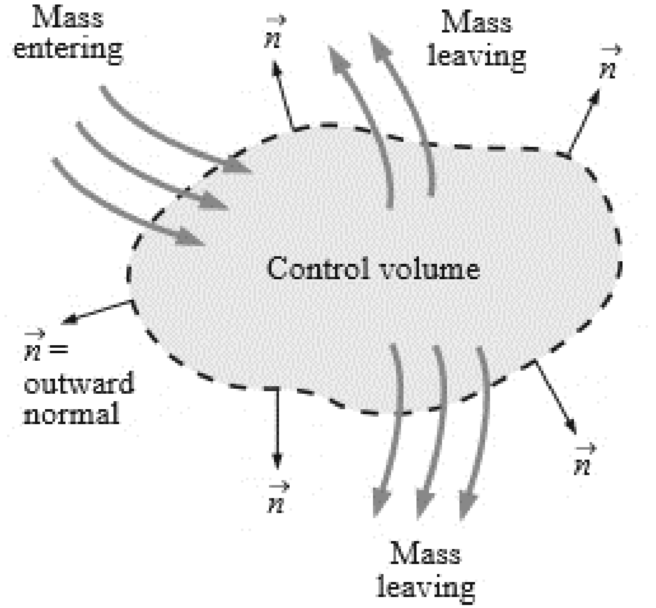
mass leaving - mass entering = - rate of increase of mass in cv



Input mass A_1 : $\rho.Q.dt$

Output mass A_2 : $\left[(\rho.Q) + \dfrac{\partial(\rho.Q)}{\partial s} ds \right] dt$

Change of mass inside V in time dt $\dfrac{\partial(\rho.A.ds)}{\partial t} dt$



CONTINUITY EQUATION FOR AN UNSTEADY FLOW

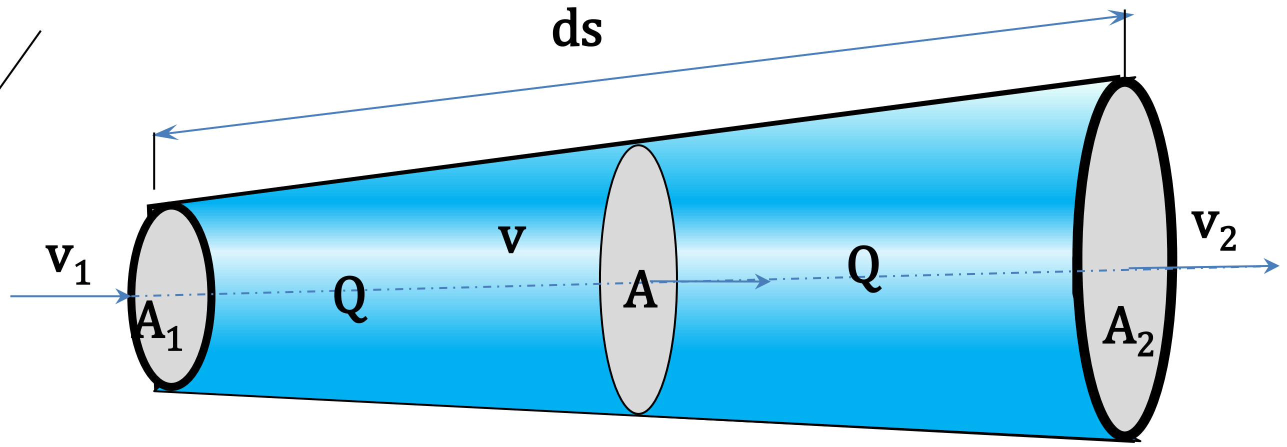
$$\frac{\partial(\rho.Q)}{\partial s} + \frac{\partial(\rho.A)}{\partial t} = 0$$

CONTINUITY EQUATION - STEADY FLOW

$$\frac{\partial(\rho.Q)}{\partial s} + \frac{\partial(\rho.A)}{\partial t} = 0$$

$$\frac{\partial(\rho.Q)}{\partial s} = 0$$

$$\rho.Q = const.$$



steady flow compressible liquid – no dependency on time

$$Q = \rho_1.A_1.v_1 = \rho_2.A_2.v_2 = \rho_i.A_i.v_i = const$$

STEADY FLOW of incompressible liquid

$Q = const.$ $\rho = const.$

$$A_1.v_1 = A_2.v_2 = \;......\; A_i.v_i = const. = Q_v$$

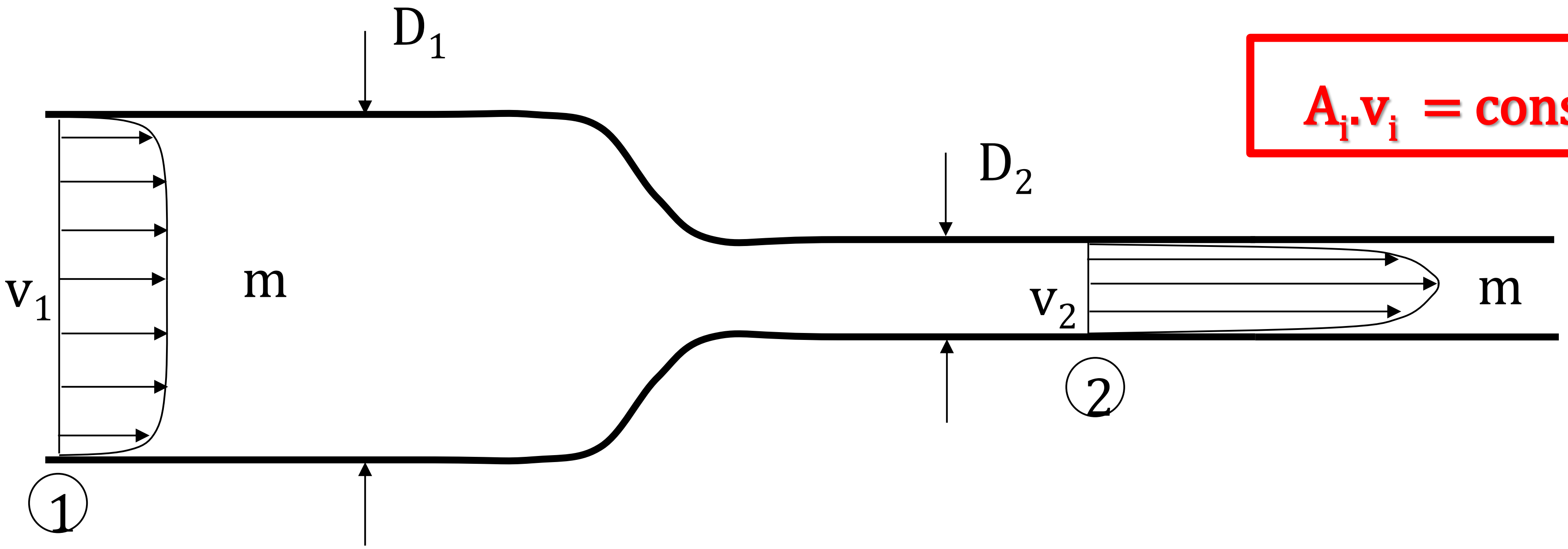
CONTINUITY EQUATION - STEADY FLOW

$$\boxed{A_i.v_i = const. = Q}$$

- For pipes with **variable diameter**, $\dot{m}$ is still the same due to conservation of mass, but $v_1 \neq v_2$

CONTINUITY EQUATION
- STEADY FLOW

$$\boxed{A_i . v_i = const. = Q}$$

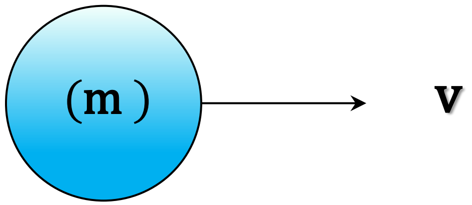


$D_1 > D_2$

$A_1 > A_2$ ...... $v_1 < v_2$

BERNOULLI´S EQUATION

Mechanical energy



(G =mg)

Kinetic energy ... ½ mv^2 kinetic energy per unit weight $\mathbf{v^2/2g}$ x (1/mg)

Potential energy ... mgh

Pressure energy (h = p/ρg).... Pressure energy per unit weight **... p/ρg** x (1/mg)

Elevation energy ... mgh Elevation energy per unit weight .... **h** x (1/mg)

Bernoulli´s equation (Ideal fluid)

$$\boxed{h + \frac{p}{\rho g} + \frac{v^2}{2g} = const.}$$

Total (mechanical) energy per unit weight

BERNOULLI EQUATION FOR IDEAL FLUID

expresses the **principle of conservation of energy**

The Bernoulli Equation is a statement of the **conservation** of **mechanical energy**

$$\underbrace{h+\frac{p}{\rho g}}_{\text{pot. e.}}+\underbrace{\frac{v^2}{2g}}_{\text{kinet.e.}}$$

$$h+\frac{p}{\rho g}+\frac{v^2}{2g} = Const. = ME$$

$\boxed{\frac{p}{\rho g}}$ = **PRESSURE HEAD**

$z+\frac{p}{\rho g}$ = **Piezometric head**

$\boxed{h}$ = **ELEVATION (GEODETIC)HEAD**

$\boxed{h+\frac{p}{\rho g}}$ = **HYDRAULIC GRADE LINE – HGL** or **PRESSURE GRADE LINE – PGL"**

$\boxed{\frac{v^2}{2g}}$ = **VELOCITY HEAD**

$\boxed{h+\frac{p}{\rho g}+\frac{v^2}{2g}}$ = Total head - **ENERGY GRADE LINE - EGL**

Each term in the BE is called **„head"**

BERNOULLI EQUATION FOR IDEAL FLUID

$$\underbrace{\underbrace{h + \frac{p}{\rho g}}_{\text{PGL}} + \frac{v^2}{2g}}_{\text{EGL}} = const. = E$$

PGL – pressure grade line

EGL – energy grade line

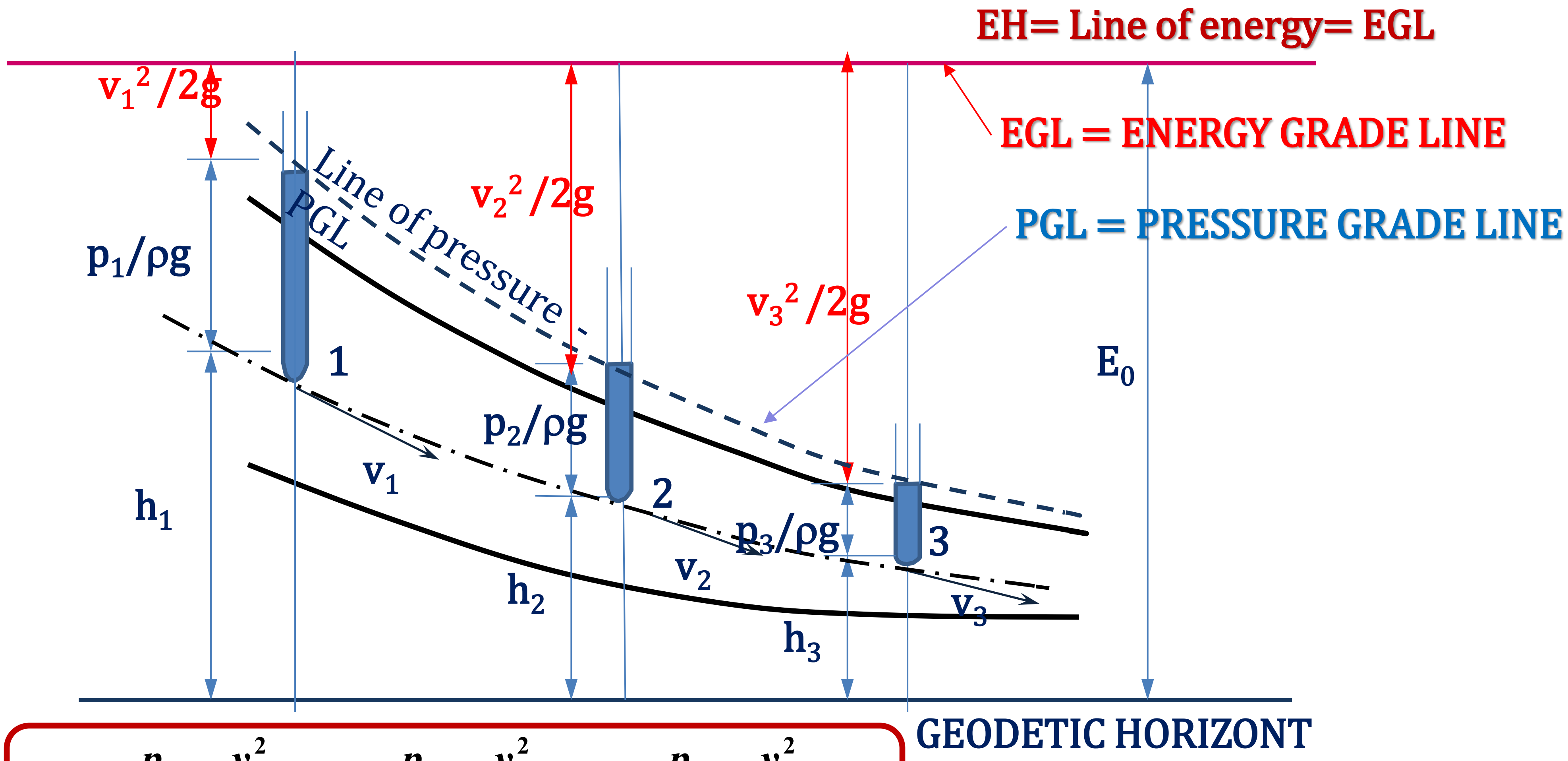
EH – energy horizont

pressure head (p/ρg) !!!! $p = p_{out} + \rho g z$!!!

PGL = (pressure head) + (elevation head)

EGL = (elevation head) + (pressure head)+ (velocity head)

BERNOULLI EQ. FOR IDEAL FLUID



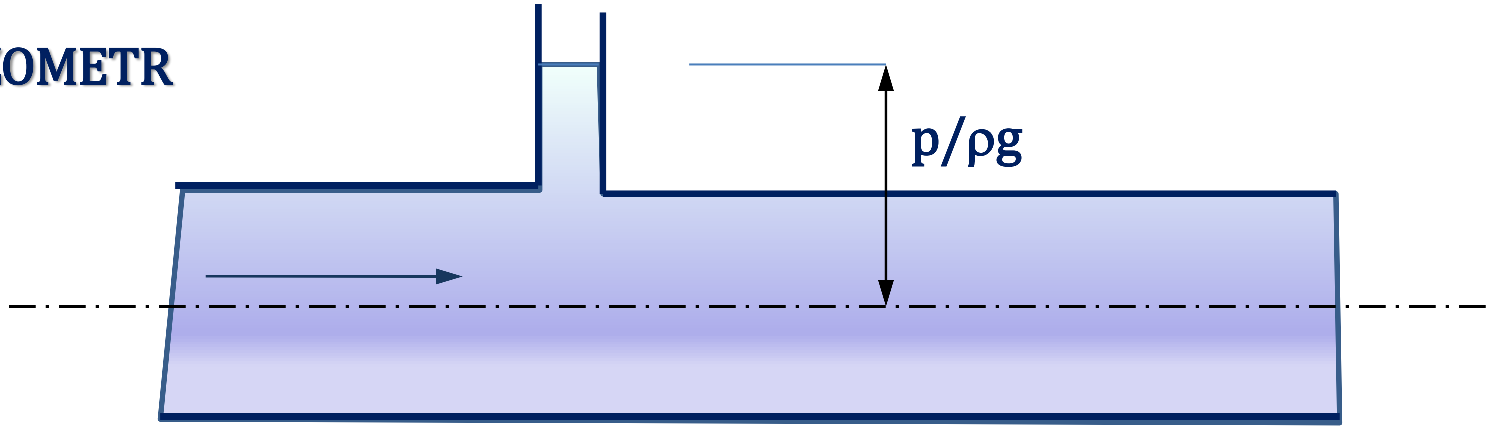
$$\boxed{h_1 + \frac{p_1}{\rho g} + \frac{v_1^2}{2g} = h_2 + \frac{p_2}{\rho g} + \frac{v_2^2}{2g} = h_3 + \frac{p_3}{\rho g} + \frac{v_3^2}{2g} = E_0}$$

h – elevation (geodetic) head **p/ρg** - pressure head $v^2/2g$ - velocity head

APPLICATION OF THE B.E.

PIEZOMETER TUBE - (measure piezometric head)

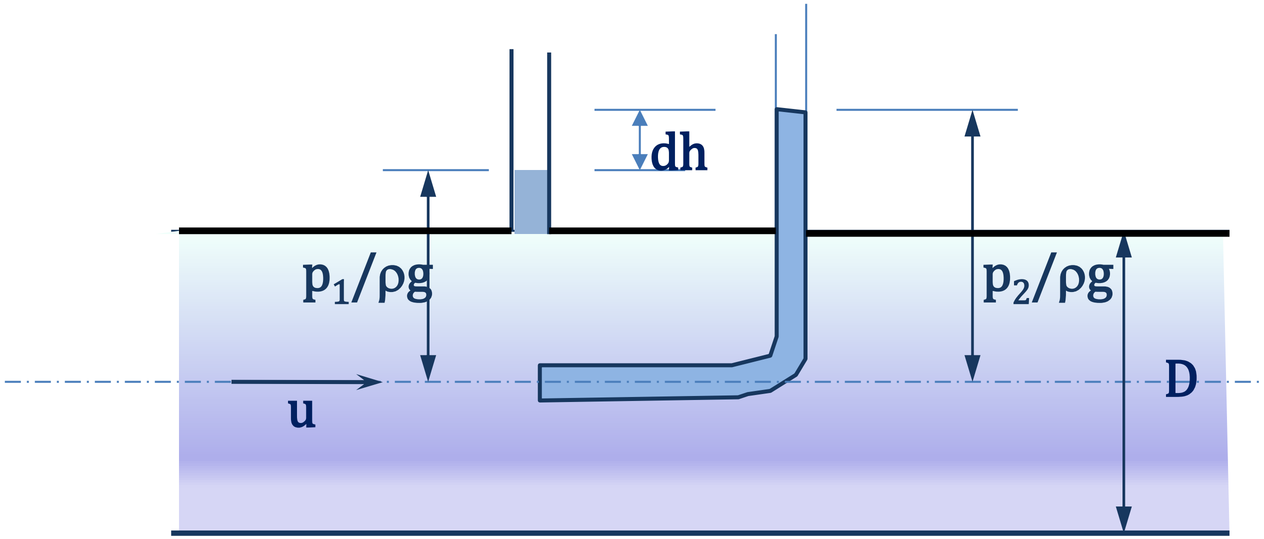
PIEZOMETR



APPLICATION OF THE B.E.

PITOT PROBE (TUBE)
(measure velocity head)

$$dh = \frac{u^2}{2g}$$



$$\frac{p_1}{\rho g} + \frac{u^2}{2g} = \frac{p_2}{\rho g}$$

$$\frac{p_2 - p_1}{\rho g} = dh = \frac{u^2}{2g}$$

$$u = \sqrt{2g\ dh}$$ **Pitot formula**

IDEAL FLUID

HYDRAULIC CALCULATIONS OF PIPELINES

2 kinds of equations:

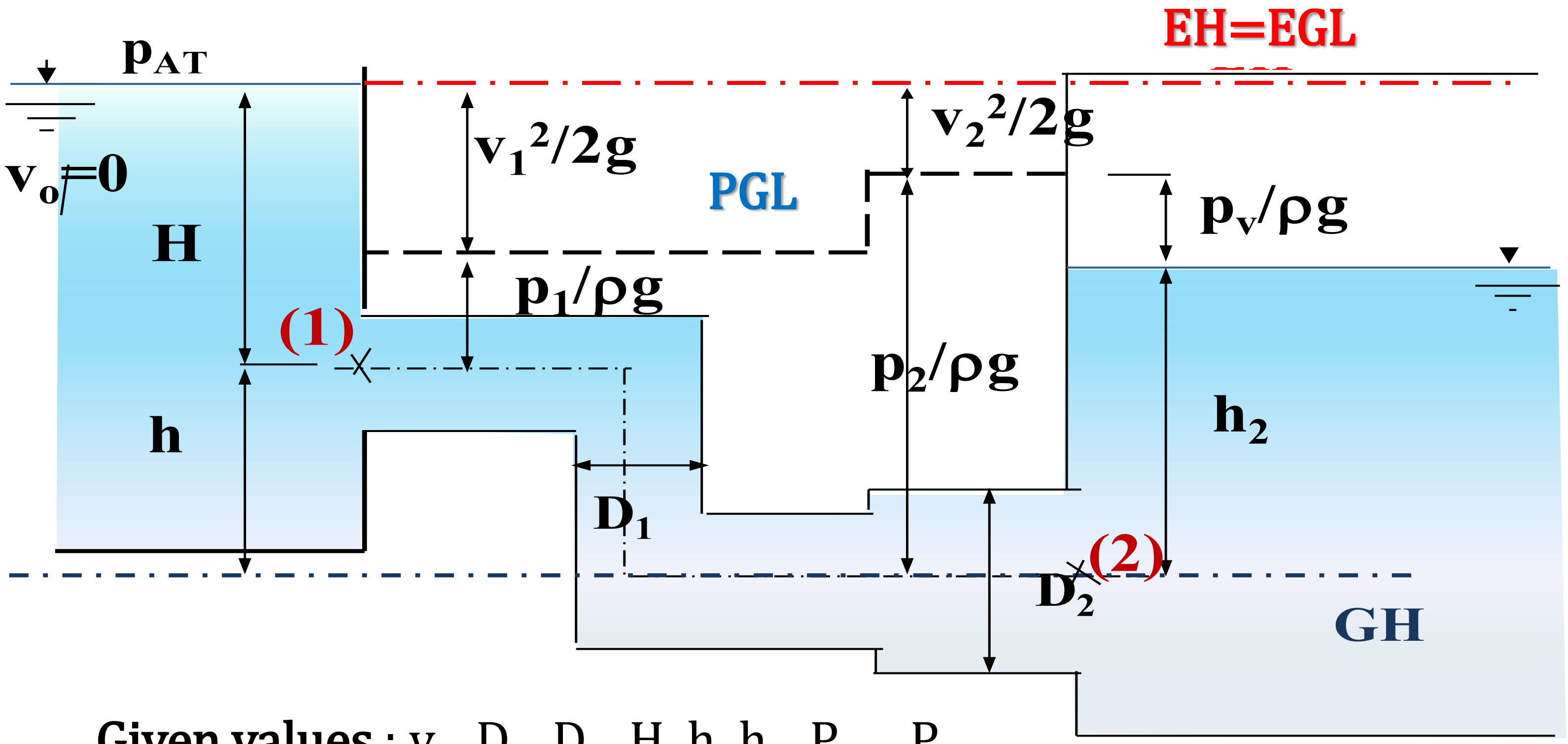
Bernoulli equation ← elevations and pressure relations,

Continuity equation - boundary conditions

calculation: Q, v, D, L, H, p

PGL, EGL

BERNOULLI EQ. FOR IDEAL FLUID



Procedure:

1. GH.
2. Choose (A) and (B).
3. BE for (A) and (B).
4. Sections.
5. Calculation v and Q
6. EGL and PGL

Given values : v_o, D_1, D_2, H, h, h_2, P_{AT} , P_V

?: Q, v_1, v_2

EGL, PGL

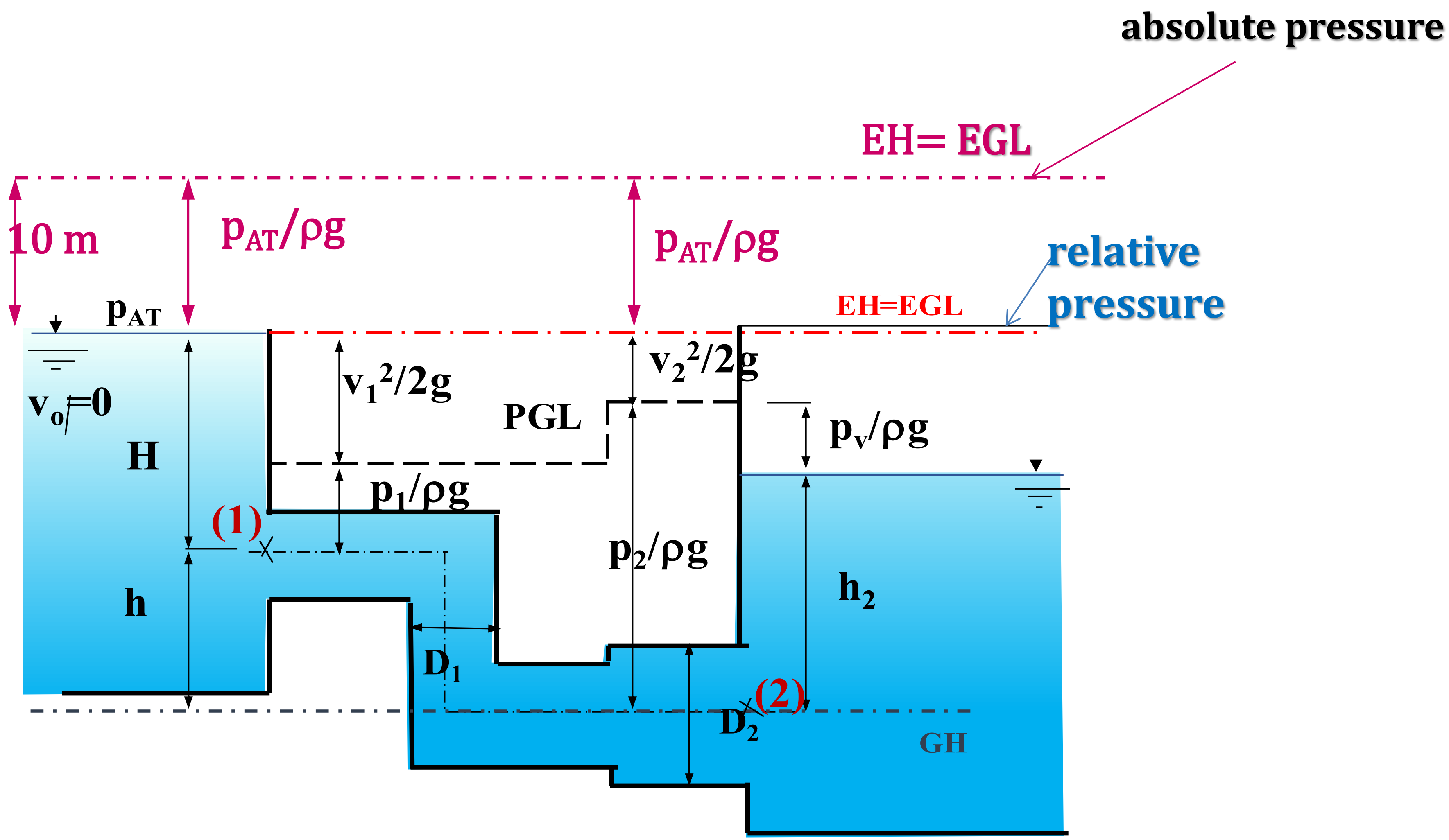
$$h+H+\frac{p_{AT}}{\rho g}+\frac{v_o^2}{2g}=h_2+\frac{p_V}{\rho g}+\frac{v_2^2}{2g}$$

$$v_2=\sqrt{2g.\left[\left(h+H+\frac{p_{AT}}{\rho g}+\frac{v_o^2}{2g}\right)-\left(h_2+\frac{p_V}{\rho g}\right)\right]}$$

Discharge: $Q = v_2.S_2$

$$S_2=\frac{\pi D_2^2}{4}$$

EGL and **PGL** for ideal fluid

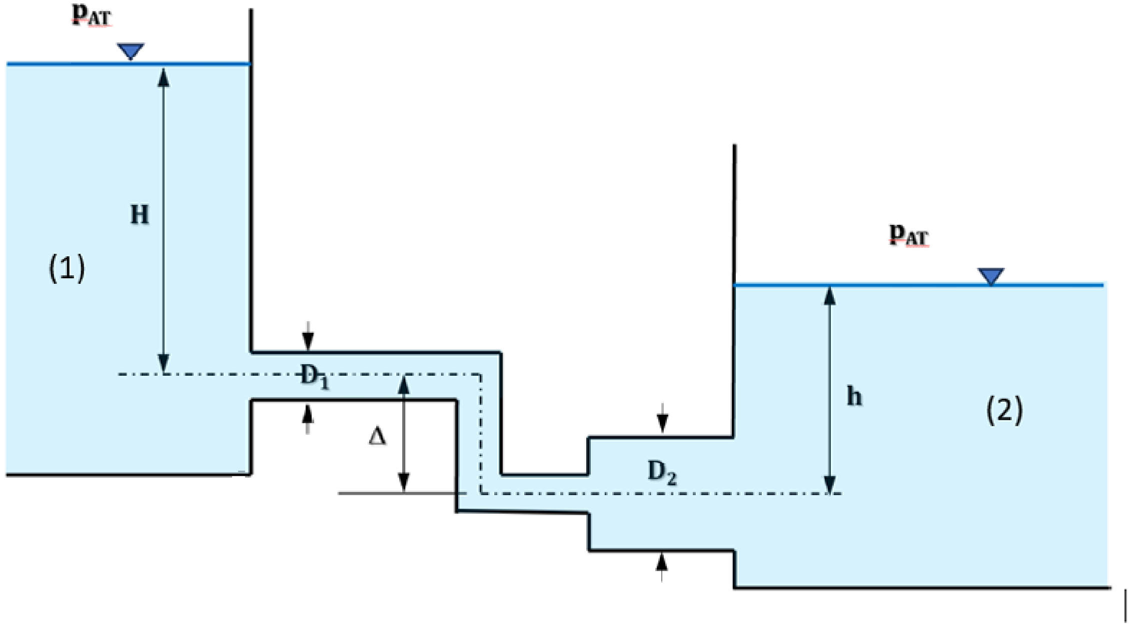


Environmental Hydraulics 2025/2026

Example 3a

Water flows steadily from reservoir (1) to reservoir (2) (**Ideal fluid**). Determine the discharge and the mean velocities. Draw energy grade line (**EGL**) and pressure grade line (**PGL**)

Known parameters:
$\Delta = 2$ m; $H = (2 + 0.1 \, . \, X)$ m; $h = 3$ m; $v_0 = 0.1$ m.s^{-1}; $D_1 = 0.2$ m; $D_2 = 0{,}25$ m;
$p_{AT} = 1.013 \, . \, 10^5$ Pa; $\rho = 1000$ kg.m^{-3};




End