ground water

Technical Note/

An Improved Straight-Line Fitting Method for Analyzing Pumping Test Recovery Data

by Li Zheng[1], Jian-Qing Guo[2], and Yuping Lei[1]

Abstract

Theis (1935) derived an exact solution for the residual drawdown in a well after the cessation of a pumping test by summing two drawdowns: one (s_1), caused by imaginary continuation of the original pumping and the other (s_2), due to an imaginary injection at the same constant rate. We approximated the Theis solution to obtain a simple linear relation for determining the transmissivity and storage coefficient from recovery data. Unlike other existing straight-line fitting methods, in our method, we applied different approximations to the well functions in the solutions of s_1 and s_2. We used the well-known Cooper-Jacob approximation for s_1, truncating the expansion of the well function in s_2 to its first three terms. For the same level of truncation errors, while the Cooper-Jacob approximation requires the argument $u_1 \leq 0.01$, the second approximation requires the argument $u_2 \leq 0.2$, thus dramatically improving the utility of short-term recovery data. Application of our method requires only recovery data from a single observation well and no knowledge of the drawdown at the moment of pumping cessation.

Introduction

Ground water hydrologists frequently use pumping tests to determine transmissivity, T, and storage coefficient, S. They often consider recovery data desirable for interpreting test results and estimating aquifer parameters (de Marsily 1986). Due to possible irregularities in running the pump, to skin effects, and the quadratic head loss across the well screen, recovery data are less prone to error than water level measurements taken during the pumping phase.

In a classic paper, Theis (1935) applied the principle of superposition to derive an exact solution for the recovering water levels in a well by summing two drawdowns: one (s_1), caused by imaginary continuation of the original pumping and the other (s_2), due to an imaginary injection at the same constant rate. Since then, many have developed simplified methods for estimating T and S from recovery data by approximating the exact Theis solution. Detailed surveys of these methods can be found in Chapuis (1992), Banton and Bangoy (1996), Chenaf and Chapuis (2002), Agarwal (1980), Samani and Pasandi (2003, also see *Ground Water* 42, no. 1, for Notice of Plagiarism with regard to this paper), and Singh (2003). Either a type curve matching or a straight-line fitting approach is most frequently adopted in these methods. In this article, we limit our discussions to the straight-line fitting approach.

Both Chenaf and Chapuis (2002) and Samani and Pasandi (2003) used the Cooper-Jacob approximation of the Theis solution and gave straight-line plots in semilog graphs for calculating T and S from recovery data. While their work represented progress in using recovery data for pumping test interpretation, their methods were limited by the use of the Cooper-Jacob approximation, which requires that both pumping and injection phases have proceeded for long enough that both arguments u_1 and u_2 (see Methodology for definitions) are no more than 0.01, to ensure sufficiently small truncation errors. While it is often reasonable to expect $u_1 \leq 0.01$ to hold during the recovery phase, the requirement of $u_2 \leq 0.01$ would exclude the use of many early-time recovery data. Banton

[1]Center for Agriculture Resources Research, IGDB, Chinese Academy of Sciences, Shijiazhuang, Hebei 050021, People's Republic of China.

[2]Corresponding author: Department of Water Resources Engineering, Chang An University, Xi'an, Shanxi Province 710054, People's Republic of China; 36-29-32339836; guojianging@yahoo.com.cn

Received July 2004, accepted February 2005.

doi: 10.1111/j.1745-6584.2005.00094.x

and Bangoy (1996) also reported a straight-line fitting method that did not require long pumping and injection periods. Their method, however, requires recovery data from at least two observation wells and straight-line fitting of data for three times to calculate the four unknown linear coefficients in order to estimate T and S.

In this study, we propose a simple straight-line fitting approach for estimating T and S from pumping test recovery data. Our method applies the Cooper-Jacob approximation to drawdown in the Theis solution caused by pumping and approximates the injection used to simulate recovery by truncating the Theis well function expansion to the first three terms. The approximate solution involves a linear regression on a set of recovery data from a single observation well and requires no knowledge of the drawdown at the moment of pumping cessation. Further, our solution requires $u_1 \leq 0.01$ and $u_2 \leq 0.2$ to achieve the same level of accuracy as the previously mentioned methods. Thus, it is more suitable for analyzing data from short-term recovery.

Methodology

Theis Solution of Residual Drawdown During Recovery

In a confined aquifer, the horizontal flow toward a fully penetrating pumping well is governed by the following equation:

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r} = \frac{S}{T}\frac{\partial s}{\partial t} \tag{1}$$

where r is the distance to the pumping well [L]; s, the water level drawdown [L]; S, the storage coefficient, dimensionless; T, the transmissivity [L^2T^{-1}]; and t, the time [T]. By the principle of superposition, the recovery of water levels or the residual drawdown (s') after the pump engine is turned off can be represented by the summation of two drawdowns, s_1 and s_2: s_1 caused by the imaginary continuation of the original pumping at a constant rate Q and s_2 due to an imaginary injection at the same rate Q starting at time t_p when the pump is turned off. For a homogeneous isotropic aquifer with infinite domain, the Theis solution for the residual drawdown at time t'($t' = 0$ at the moment the pump is turned off) is given as follows (Theis 1935):

$$s'(t') = s_1(t' + t_p) + s_2(t') \tag{2}$$

with

$$s_1(t' + t_p) = \frac{Q}{4\pi T}\mathrm{W}(u_1) \tag{3}$$

$$s_2(t') = -\frac{Q}{4\pi T}\mathrm{W}(u_2) \tag{4}$$

and the arguments u_1 and u_2 are defined as

$$u_1 = \frac{r^2 S}{4T(t' + t_p)} \tag{5}$$

$$u_2 = \frac{r^2 S}{4Tt'} \tag{6}$$

and the Theis well function W(u) is given as

$$W(u) = \int_u^{\infty} \frac{e^{-u}}{u}du \tag{7}$$

Approximating the Theis Solution

The well function W(u) is an exponential integral and can be expanded into a converging series as follows:

$$W(u) = -0.577216 - \ln u + u - \sum_{n=2}^{\infty}(-1)^n \frac{u^n}{n \times n!} \tag{8}$$

Cooper and Jacob (1946) proposed that when u is very small (e.g., $u \leq 0.01$), the sum of the terms after the second term in the series becomes negligible. That is, when the pumping time is sufficiently long and/or the distance r is small, the series (Equation 8) can be reduced to the Cooper-Jacob approximation:

$$\mathrm{W}(u) = -0.577216 - \ln u \tag{9}$$

During the recovery phase, it is reasonable to expect $t' + t_p$ to be sufficiently large so that u_1 as defined in Equation 5 satisfies the condition of $u_1 \leq 0.01$, and W(u_1) in Equation 3 can be approximated as in Equation 9:

$$s_1 = \frac{Q}{4\pi T}\ln\frac{2.25T(t' + t_p)}{r^2 S} \tag{10}$$

With respect to s_2, since t' starts from the moment of pumping cessation, for many recovering water level records, u_2 as given in Equation 6 may not satisfy the condition of being no more than 0.01, especially for the early-time data. Similar to the approach used in Banton and Bangoy (1996), we truncate the W(u_2) expansion to its first three terms and reduce the full expression of s_2 to the following form:

$$s_2 = -\frac{Q}{4\pi T}\left[\ln\frac{2.25Tt'}{r^2 S} + u_2\right] \tag{11}$$

For the same level of truncation errors as introduced by Cooper-Jacob's approximation, the approximation used in Equation 11 will only require $u_2 \leq 0.2$. In analyzing recovery data, a requirement of $u_2 \leq 0.2$ will be easier to satisfy than the condition $u_2 \leq 0.01$ required for the Cooper-Jacob approximation. In the following section, we use Equations 10 and 11 to construct a linear relation and enable a simple straight-line fitting for estimating T and S from recovery data.

Formulating the Linear Relation for Parameter Estimation

Substituting Equations 10 and 11 into Equation 2, we obtain

$$s' = \frac{Q}{4\pi T}\left[\ln\frac{(t' + t_p)}{t'} - \frac{r^2 S}{4Tt'}\right] \tag{12}$$

Multiplying both sides of Equation 12 by t' gives

$$t's' = \frac{Q}{4\pi T}\left[t'\ln\left(\frac{t' + t_p}{t'}\right) - \frac{r^2 S}{4T}\right] \tag{13}$$

For recovery data from a single observation well, the distance between observation well and pumping well r is a known constant. We can then perform the following transformations for Equation 13. Let

$$Y = t's' \tag{14}$$

$$X = t'\ln\left(\frac{t' + t_p}{t'}\right) \tag{15}$$

$$A = \frac{Q}{4\pi T} \tag{16}$$

$$B = -A\frac{r^2 S}{4T} = -\frac{Qr^2 S}{16\pi T^2} \tag{17}$$

Equation 13 can then be rewritten as:

$$Y = AX + B \tag{18}$$

with A and B being constants. Using observed data pairs s' and t', we can compute Y and X data pairs based on Equations 14 and 15. We can then plot Y against X and fit a straight line through these data pairs. The slope of this line is A and the line intercept of the Y axis is at B (where $X = 0$). Substituting A into Equation 16, we will obtain an estimate for the value of T as

$$T = \frac{Q}{4\pi A} \tag{19}$$

Substituting B and T into Equation 17, we obtain an estimate of S as

$$S = -\frac{4TB}{Ar^2} \tag{20}$$

Application

To test the proposed method, we applied it to a set of residual drawdown data from the U.S. Department of the Interior (USDI) (1977, p. 120) to estimate T and S. This set of data has been analyzed by others, including Goode (1997), Chenaf and Chapuis (2002), and Samani and Pasandi (2003).

The residual drawdown data were recorded in an observation well located at 100 feet from the pumping well. The well was pumped 800 min with a discharge rate of 162.9 ft^3/min, and the recovery period was also recorded for 800 min after the pump was turned off. Applying Equations 15 and 16 to the observed residual drawdown s' and the corresponding t', we obtained Y and X data series (Figure 1). Almost all the Y and X data pairs fall onto a straight line, except for a few late time points. Examining the original data (table 5-4, USDI 1977, p. 120), we found that the observation data started to wiggle a bit around $t' = 540$ min, which might be due to some late-time measurement errors. Wiggling was also visible in the straight-line plots in Chenaf and Chapuis (Figures 2 to 5, 2002), even though the amount of fluctuation was somewhat suppressed by the use of semilog plots in their figures. We then performed linear regression with and without the late time points. In both cases, the correlation coefficients of fitting R were >0.999, and the impacts of the late time points on the fitting results were minimal. The slope and intercept determined from linear regression with these late time points were $A = 0.422$ and $B = -2.389$. Substituting into Equations 16 and 17 yields $T = 30.73$ ft^2/min and $S = 0.0696$.

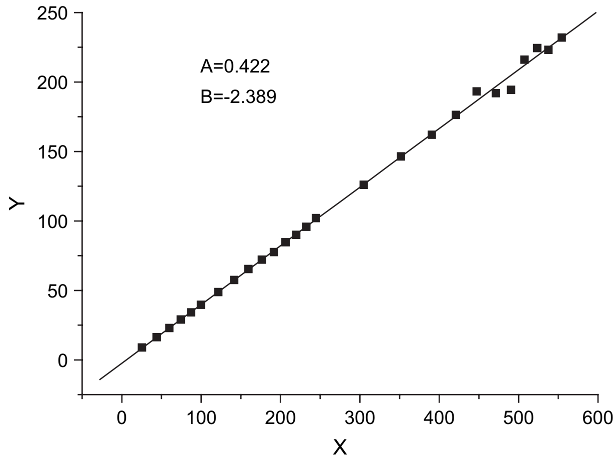

Figure 1. Straight-line fit of transformed recovery data (X and Y) and the estimated values of slope A and intercept B.

When compared to previous studies (Table 1), our method gave comparable results while requiring much less strict limits on the length of the recovery phase.

Discussion and Conclusions

All methods listed in Table 1 gave comparable results, which does not enable us to differentiate the performance of these methods. However, the unique features of our approach become obvious when we compare the straight-line plot in Figure 1 with straight-line fitting reported by others, e.g., Chenaf and Chapuis (2002, Figures 2 to 5) and Samani and Pasandi (2003, Figure 5b).

Table 1
Comparison With Other Methods

This paper	$T = 30.73$ ft^2/min $S = 0.0696$
USDI—original results	
Theis recovery method:	
extrapolation of pumping period drawdown	$T = 32.1$ ft^2/min $S = 0.07$
Samani and Pasandi (2003)	
Cooper-Jacob approximation	$T = 32.42$ ft^2/min $S = 0.06$
Chenaf and Chapuis (2002)	
Pumping phase	
Cooper-Jacob method	$T = 31.99$ ft^2/min $S = 0.0604$
Recovery phase	
Cooper-Jacob method	$T = 31.95$ ft^2/min $S = 0.0599$
Normalized residual drawdown method	$T = 32.16$ ft^2/min $S = 0.0599$
Water recovery height method	$T = 32.16$ ft^2/min $S = 0.0588$

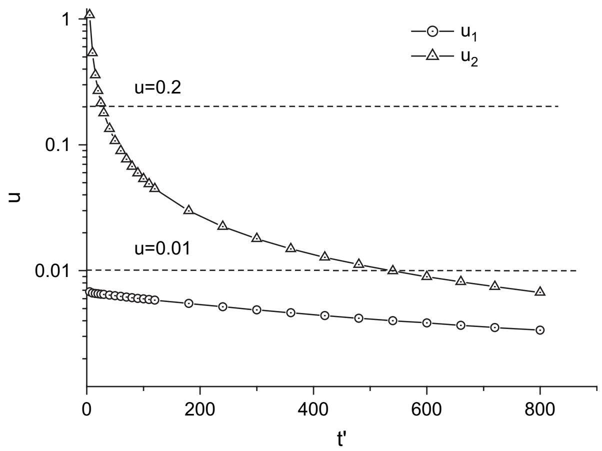

Figure 2. Decreasing values of u_1 (circle) and u_2 (triangle) as recovery proceeds (t' is the recovery time).

In our Figure 1, all the points transformed from the recovery data fall onto a straight line nicely except for a couple of late time points, which may be due to measurement errors. All other plots reported in Table 1 showed deviation from a straight-line fit for early-time data. In fact, both Chenaf and Chapuis (2002) and Samani and Pasandi (2003) adopted the Cooper-Jacob approximation to arrive at linear relations that require both u_1 and u_2 to be no more than 0.01. For the application described in the previous section, we plotted the values of u_1 and u_2 vs. the corresponding t' in a semilog graph (Figure 2). As shown, u_1 is well below 0.01 for all t', while u_2 reaches the threshold value of 0.01 well beyond 540 min. While the USDI test example lasted long enough to enable the application of all the methods reported in Table 1, in many field cases, the requirement of $u_2 \leq 0.01$ might be difficult to satisfy. For example, in the pumping test described by Xue (1986), among all the recovery data collected, there were only three sets with $u_2 < 0.01$. The approximation adopted in our method (Equation 10), however, requires $u_2 \leq 0.2$ for its application. From Figure 2, it is obvious that most of the early-time recovery data can be used in the application of our method.

In addition to the relaxation of the restriction on u_2, in our method linear regression is performed only once using recovery data from a single observation well, and no knowledge of the drawdown at the moment of pumping cessation is required. Multiple sets of recovery data from multiple wells can be analyzed separately and results compared to test the reliability of each independent estimate. Furthermore, we should note that Chenaf and Chapuis' (2002) approach made no assumption regarding the storage coefficient during the pumping period being equal to the storage coefficient during the recovery period, which may be important if pumping has caused consolidation of the confined aquifer.

In addition, for the case when many residual drawdowns are available from multiple observation wells at a single moment (t' is a constant), it is also easy to see that Equation 12 gives another linear relation between s' and r^2. Following similar steps, drawdown data from multiple wells may be used to derive estimates of T and S.

In conclusion, we have proposed a simple and effective method for determining the aquifer transmissivity and storage coefficient based solely on pumping test recovery data. The application of this method in the field may be hindered by conditions that violate the assumptions in the Theis solution (e.g., boundary effects and wellbore storage). How these common field conditions will impact the performance of the proposed method is unclear. More in-depth analyses are needed to resolve this issue when applying the proposed method to nonideal field conditions.

Acknowledgments

This research was supported by Zheng's "Young Talents Award" from the Chinese Academy of Sciences and the Innovation Project (award No. KZCX3-SW-428) of the Chinese Academy of Sciences. We highly appreciate the constructive comments and suggestions from four anonymous reviewers and an associate editor, as well as editing for English usage by Editor in Chief Mary Anderson, which helped us to improve the quality of this manuscript. The assistance of Dr. X. Zhan in locating the USDI pumping test recovery data set is also highly appreciated.

References

Agarwal, R.G. 1980. A new method to account for producing time effects when drawdown type curves are used to analyze pressure buildup and other test data. SPE Paper 9289 presented at the 55th SPE Annual Technical Conference and Exhibition, 21–24 September, Dallas, Texas.

Banton, O., and L.M. Bangoy. 1996. A new method to determine storage coefficient from pumping test recovery data. *Ground Water* 34, no. 5: 772–777.

Chapuis, R.P. 1992. Discussion of "Estimation of storativity from recovery data," by P.N. Ballukraya and K.K. Sharma, 1991. *Ground Water* 30, no. 2: 269–272.

Chenaf, D., and R.P. Chapuis. 2002. Methods to determine storativity of infinite confined aquifer from a recovery test. *Ground Water* 40, no. 4: 385–389.

Cooper, H.H., and C.E. Jacob. 1946. A generalized graphical method for evaluating formation constants and summarizing well field history. *Transactions of American Geophysical Union* 27, no. 4: 526–534.

de Marsily, G. 1986. *Quantitative Hydrogeology: Groundwater Hydrology for Engineering*. London: Academic Press.

Goode, D.J. 1997. Composite recovery type curves in normalized time from Theis' exact solution. *Ground Water* 35, no. 4: 672–678.

Samani, N., and M. Pasandi. 2003. A single recovery type curve from Theis' exact solution. *Ground Water* 41, no. 5: 602–607 [Notice of Plagiarism with regard to this paper appeared in *Ground Water* 42, no. 1].

Singh, S.K. 2003. Storage coefficient and transmissivity from residual drawdowns. *Journal of Hydraulic Engineering* 129, no. 8: 637–644.

Theis, C.V. 1935. The relation between the lowering of the piezomertric surface and the rate and duration of discharge of a well using ground-water storage. *Transactions of American Geophysical Union* 16, 519–524.

U.S. Department of the Interior (USDI). 1977. *Ground Water Manual*. Washington, DC: Bureau of Reclamation, U.S. Government Printing Office.

Xue, Y.Q. 1986. *Ground Water Dynamics* [in Chinese]. Beijing, China: Geology Publish House.