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A NEW METHOD TO ACCOUNT FOR PRODUCING TIME EFFECTS WHEN DRAWDOWN TYPE CURVES ARE USED TO ANALYZE PRESSURE BUILDUP AND OTHER TEST DATA

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ABSTRACT

Currently, type curve analysis methods are being commonly used in conjunction with the conventional methods to obtain better interpretation of well test data. Although the majority of published type curves are based on pressure drawdown solutions, they are often applied indiscriminately to analyze both pressure drawdown and buildup data. Moreover, the limitations of drawdown type curves, to analyze pressure buildup data collected after short producing times, are not well understood by the practicing engineers. This may often result in an erroneous interpretation of such buildup tests. While analyzing buildup data by the conventional semi-log method, the Horner method takes into account the effect of producing time. On the other hand, for type curve analysis of the same set of buildup data, it is customary to ignore producing time effects and utilize the existing drawdown type curves. This causes discrepancies in results obtained by the Horner method and type curve methods. Although a few buildup type curves which account for the effect of producing times have appeared in the petroleum literature, they are either limited in scope or somewhat difficult to use.

In view of the preceding, a novel but simple method has been developed which eliminates the dependence on producing time effects and allows the user to utilize the existing drawdown type curves for analyzing pressure buildup data. This method may also be used to analyze two-rate, multiple-rate and other kinds of tests by type curve methods as well as the conventional methods. The method appears to work for both unfractured and fractured wells. Wellbore effects such as storage and/or damage may be taken into account except in certain cases.

The purpose of this paper is to present the new method and demonstrate its utility and application by means of example problems.

References and illustrations at end of paper.

INTRODUCTION

Type curves have appeared in the petroleum literature since 1970 to analyze pressure transient (pressure drawdown and pressure buildup) tests taken on both unfractured and fractured wells. The majority of type curves[1-8] which have been developed and published to date were generated using data obtained from pressure drawdown solutions and obviously are most suited to analyze pressure drawdown tests. These drawdown type curves are also commonly used to analyze pressure buildup data. The application of drawdown type curves in analyzing pressure buildup data is not as bad as it may first appear. As long as the producing time, t_p, prior to shut-in is sufficiently long compared to the shut-in time, Δt [that is $(t_p+\Delta t)/t_p \simeq 1$], for liquid systems, it is reasonable to analyze pressure buildup data using drawdown type curves. However, for cases where producing times prior to pressure buildup tests are of the same magnitude or only slightly larger than the shut-in times [that is, $(t_p+\Delta t)/t_p >> 1$], the drawdown type curves may not be used to analyze data from pressure buildup tests. The above requirement on the duration of producing times is the same for the conventional semi-log analysis. If pressure buildup data obtained after short producing time are to be analyzed, the Horner method[10] is recommended over the MDH (Miller-Dyes-Hutchinson) method.[9] The MDH method is generally used to analyze buildup data collected after long producing times, whereas the Horner method is used for those obtained after relatively short producing times. Although pressure buildup tests with short producing times may occur often under any situation, they are rather more common in the case of drill stem tests and pre-fracturing tests on low permeability gas wells.

Thus, there is a need for generating buildup type curves, which account for the effects of producing time. Some limited work has been done in this regard. McKinley[11] has published type curves for analyzing buildup data for a radial flow system. However, his buildup type curves were generated on the assumption of long producing times; and these type curves are therefore very similar to drawdown type curves and are obviously unsuitable for cases where producing times prior to shut-in are relatively short. Crawford, et al.,[12] pointed out the above limitations for McKinley type curves in analyzing pressure buildup data from the DST tests. They also presented buildup curves for short producing times. Since their curves deal with specific values of real producing times prior to shut-in, they are limited in scope and utility. Recently, the effect of producing time on analysis of pressure buildup data using drawdown type curves has been discussed by Raghavan.[13] His study clearly points out the limitations of drawdown type curves for analyzing buildup data collected after small producing times. A family of buildup type curves is presented both for unfractured and fractured wells with producing time as a parameter. Although these type curves offer a definite advantage over the existing drawdown type curves, they are difficult to use because of the multiplicity of type curves. In a recent paper, Agarwal[14] also discussed the limitations of using drawdown type curves for analyzing buildup data obtained after small producing times but no details were given. These limitations are discussed here in this paper. Recently Gringarten, et al.[15], presented drawdown type curves, plotted in a slightly different form, and suggested some guidelines regarding the portions of buildup data which may be analyzed by drawdown type curves. Although these guidelines may be useful in certain cases, the basic problem still remains.

To overcome the above-mentioned difficulties and to eliminate dependence on producing time, a new method has been developed. This method should provide a significant improvement over the current methods because (1) this permits us to account for the effects of producing time, and (2) data are normalized in such a fashion that instead of utilizing a family of type curves with producing time as a parameter, the existing drawdown type curves may be used. This concept appears to work for both unfractured and fractured wells. Wellbore storage effects with or without damage may also be taken into account provided that producing time prior to shut-in is long enough to be out of such wellbore effects.

This method has been extended to include analysis of data from two-rate tests[8,16,17] and multiple rate tests[8,17,18] by type curve methods. Although not shown, it appears to have a potential for applying type curve methods to other kinds of testing.

This new method, although originally conceived for type curve analysis of buildup data, is quite suitable for the conventional semi-log analysis. It is similar to the Horner method because it includes the effects of producing time, and may be used to determine formation flow capacity, skin factor and the initial reservoir pressure. However, it has an added advantage. It allows the plotting of pressure buildup data, with and without producing time effects, on the same time scale as the graph paper. This enables a better comparison of data using the MDH and Horner type graphs.

Although the new method will be developed using the solutions for liquid systems, its applicability to gas wells will also be indicated.

BASIS OF DRAWDOWN AND BUILDUP TYPE CURVES

A type curve is a graphical representation of a mathematical solution (obtained analytically or numerically) for a specific flow type. The solution is normally plotted, in terms of dimensionless variables, on log-log graph paper. The graph thus prepared becomes the type curve for the specific flow problem with given inner and outer boundary conditions. Depending on the type of solution (drawdown or buildup), drawdown and buildup type curves are generated.

Drawdown Type Curves

As the name implies, these type curves are based on the drawdown solutions. The pressure drawdown solution for a well producing at a constant rate as a function of flowing time, t may be written as

$$\frac{kh[p_i - p_{wf}(t)]}{141.2\ qB\mu} = p_{wD}(t_D) \qquad (1)$$

where,

$$t_D = \frac{2.634 \times 10^{-4} kt}{\phi(\mu c_t)_i\ r_w^2} \qquad (2)$$

Eq. (1) is a general solution and is not meant to be restricted to any particular drainage shape or well location. The majority of the published type curves[1-8] for both unfractured and fractured wells are based on pressure drawdown solutions for liquid systems. Examples of pressure drawdown type curves for unfractured wells are those presented by Agarwal, et al.,[1] Earlougher and Kersch[4] and Gringarten, et al.[15] In another publication Gringarten, et al.,[5] presented type curves for vertically fractured wells with infinite flow capacity and uniform flux fractures. Type curves for finite flow capacity fractures were provided by Cinco et al.,[6] and Agarwal, et al.[7] More regarding the use of above type curves for analyzing buildup data will be said later.

Buildup Type Curves

To obtain pressure buildup solutions, superposition may be applied in the normal manner to pressure drawdown solutions. This provides buildup pressures at shut-in times, Δt after a producing time, t_p. Fig. 1 shows a schematic of pressure buildup behavior obtained following a constant rate drawdown for a production period, t_p. Flowing pressures $p_{wf}(t)$ are shown as a function of flowing time, t up to a production period, t_p, when a buildup test is initiated. Buildup pressures, $p_{ws}(t_p+\Delta t)$, are shown as a function of shut-in time, Δt. Instead of taking a buildup test, if the well was allowed to produce beyond time, t_p flowing pressures as shown by $p_{wf}(t_p+\Delta t)$ would have been obtained. Note that the flowing pressure at the end of the production period which is denoted by $p_{wf}(t_p)$ is same as the buildup pressure at the instant of shut-in which is shown as $p_{ws}(\Delta t=0)$. Superposition when applied to drawdown solutions provides the following.

$$\frac{kh[p_i-p_{ws}(t_p+\Delta t)]}{141.2\ qB\mu}$$

$$= p_{wD}[(t_p+\Delta t)_D]-p_{wD}[(\Delta t)_D] \quad (3)$$

The flowing pressure, $p_{wf}(t_p)$ at the end of producing period, t_p is given by

$$\frac{kh[p_i-p_{wf}(t_p)]}{141.2\ qB\mu} = p_{wD}[(t_p)_D] \quad (4)$$

Subtracting Eq. (3) from Eq. (4) and substituting $p_{ws}(\Delta t=0)$ for $p_{wf}(t_p)$, we obtain

$$\frac{kh[p_{ws}(t_p+\Delta t)-p_{ws}(\Delta t=0)]}{141.2\ qB\mu}$$

$$= p_{wD}[(t_p)_D]-p_{wD}[(t_p+\Delta t)_D]+p_{wD}[(\Delta t)_D] \quad (5)$$

Eq. (5) provides a basis for buildup type curves and has been utilized in this paper.

Before discussing the new method, let us review the simplified version of Eq. (5) which has been commonly used in the past and has provided the basis of utilizing drawdown type curves for analyzing pressure buildup data. If producing time, t_p, is significantly larger than the shut-in time, Δt, it is reasonable to assume that $[(t_p+\Delta t)/t_p] \simeq 1$. Although approximate, this also implies that $(t_p+\Delta t) \simeq t_p$, or $p_{wD}[(t_p+\Delta t)_D] \simeq p_{wD}[(t_p)_D]$. Thus, Eq. (5) can be simplified as a pressure buildup equation as shown below:

$$\frac{kh[p_{ws}(t_p+\Delta t)-p_{ws}(\Delta t=0)]}{141.2\ qB\mu} = p_{wD}[(\Delta t)_D] \quad (6)$$

A comparison of pressure buildup equation (6) and the pressure drawdown equation (1) indicates that they are similar at least for cases where producing period, t_p is significantly longer than the shut-in time, Δt. It also implies that $(\Delta p)_{drawdown}$ vs. flowing time, t is equivalent to $(\Delta p)_{buildup}$ vs. shut in time, Δt, where

$$(\Delta p)_{drawdown} = p_i-p_{wf}(t) \quad (7)$$

$$(\Delta p)_{buildup} = p_{ws}(t_p+\Delta t) - p_{ws}(\Delta t=0) \quad (8)$$

Since Eq. (6) has been derived from Eq. (5) based on the assumption of long producing period, t_p, the difference

$$p_{wD}[(t_p)_D]-p_{wD}[(t_p+\Delta t)_D] \simeq 0 \quad (9)$$

On Fig. 1, the above difference has been shown as the cross-hatched area and may be defined as

$$(\Delta p)_{difference} = p_{wf}(t_p)-p_{wf}(t_p+\Delta t) \quad (10)$$

or $$(\Delta p)_{difference} = p_{ws}(\Delta t=0)-p_{wf}(t_p+\Delta t) \quad (11)$$

As producing period t_p gets smaller or Δt gets larger, the difference shown by Eqs. (9) through (11) can no longer be ignored and the use of drawdown type curves to analyze pressure buildup data becomes invalid. The impact of the assumption shown by Eq. (9) will be discussed first in a generalized fashion followed by its impact on type curves for specific flow regimes. Finally, the new method will be discussed which accounts for producing time effects for analyzing pressure buildup data.

Fig. 2 schematically shows pressure buildup behavior obtained following a constant rate drawdown but at the end of three successively increasing producing periods, t_p such that $t_{p_3} > t_{p_2} > t_{p_1}$. The cross-hatched area shown at the end of each production period denotes the difference between $(\Delta p)_{drawdown}$ and $(\Delta p)_{buildup}$ represented by Eq. (10) or (11). Note that $(\Delta p)_{difference}$ gets smaller as the length of the producing period increases.

This can be better shown by means of Fig. 3 where $(\Delta p)_{drawdown}$ vs. flowing time, t has been compared with $(\Delta p)_{buildup}$ vs. shut-in time, Δt with producing period t_p as a parameter. Although schematic, Fig. 3 clearly indicates that there is a significant difference between $(\Delta p)_{drawdown}$ and $(\Delta p)_{buildup}$ for small producing periods. However, this difference gets smaller as the length of the producing period increases. Also note that for a given producing period, the difference between the two (Δp)s is small at early shut-in times but it gets bigger as shut-in time, Δt, increases. Fig. 3 clearly indicates the limitations of using drawdown type curves for analyzing pressure buildup data where producing period, t_p, prior to shut-in is relatively small.

Next we will examine the impact of this difference on type curve analysis for the specific flow regimes (radial flow, linear flow, etc.) and discuss the new method which accounts for producing time effects.

UNFRACTURED WELL

Infinite Radial System ($s=0$; $C_D=0$)

Let us first consider the pressure drawdown solution for a well producing at a constant rate in a radial system.

$$p_{wD}(t_D) = \frac{1}{2}\,[\ln(t_D) + 0.80907] \qquad (12)$$

Eq. (12) is based on the assumption that wellbore effects (storage and skin) are negligible and the dimensionless time, $t_D \geq 100$ such that the log approximation applies to the ε_i-solution. Substitution of Eq. (12) in Eq. (1) provides

$$\frac{kh[p_i - p_{wf}(t)]}{141.2\ qB\mu} = \frac{1}{2}\,[\ln(t_D) + 0.80907] \qquad (13)$$

Eq. (13) is a pressure drawdown solution for a radial system which also forms the basis for semi-log straight line on a semi-log graph paper. If Eq. (12) is substituted in Eq. (3), the well known Horner pressure buildup equation is obtained.

$$\frac{kh[p_i - p_{ws}(t_p + \Delta t)]}{141.2\ qB\mu} = \frac{1}{2}\,[\ln \frac{(t_p + \Delta t)_D}{\Delta t_D}] \qquad (14)$$

In Eq. (14) the subscript D may be dropped if desired. The above equation also takes into account producing time effects. Unfortunately $\Delta p = [p_i - p_{ws}(t_p + \Delta t)]$ on the left hand side of Eq. (14) requires a knowledge of initial reservoir pressure, p_i which is generally not known. Consequently, Eq. (14) is not suitable for the purposes of type curve matching. However, $(\Delta p)_{buildup}$ defined by Eq. (8) is generally known and is normally used for type curve analysis. If Eq. (12) is substituted in Eq. (6), the simplified pressure buildup equation is obtained.

$$\frac{kh[p_{ws}(t_p + \Delta t) - p_{ws}(\Delta t = 0)]}{141.2\ qB\mu}$$

$$= \frac{1}{2}\,[\ln(\Delta t_D) + 0.80907] \qquad (15)$$

Eq. (15) is the familiar MDH (Miller-Dyes-Hutchinson) equation for pressure buildup and assumes that the producing period prior to shut-in is sufficiently long such that transients during the flow period do not affect the subsequent pressure buildup data. Thus, it should be obvious that Eq. (15) is not suitable for pressure buildup analysis (conventional or type curve) when producing times prior to shut-in are small. However, Eq. (5) may be used, as was done by Raghavan, to generate a family of pressure buildup curves with dimensionless producing time, t_{pD}, as a parameter. Fig. 4 presents such results for an infinite radial system. Dimensionless pressure change, $\tilde{p}_{wDs}$ during buildup has been plotted as a function of dimensionless shut-in time, Δt_D, with dimensionless producing time, t_{pD}, as a parameter. Note that $\tilde{p}_{wDs}$ will be defined later as Eq. (19) and is the same as the left hand side of Eq. (5). The buildup curve with $t_{pD}=\infty$ corresponds to the pressure drawdown solution given by Eq. (12). Since data are plotted on semi-log graph paper (for $t_D \geq 100$), the pressure drawdown solution on Fig. 4 is a straight line. This figure clearly points out the limitations of using pressure drawdown solution in a conventional or type curve analysis mode to analyze pressure buildup data obtained after short producing times. However, buildup curves may be utilized for type curve matching purposes as discussed by Raghavan.[13] The obvious disadvantage is that this requires the use of a family of type curves.

To overcome the above difficulty, a new method has been developed which should allow us to analyze pressure buildup data by means of pressure drawdown type curves. This new method may also be used to perform the conventional analysis.

NEW METHOD

Eq. (5), presented earlier as a pressure buildup solution, forms the basis for this new method. Substitution of Eq. (12) in Eq. (5)

$$\frac{kh[p_{ws}(t_p + \Delta t) - p_{ws}(\Delta t = 0)]}{141.2\ qB\mu}$$

$$= \frac{1}{2}\,[\ln \frac{(t_{pD} \times \Delta t_D)}{(t_p + \Delta t)_D} + 0.80907] \qquad (16)$$

Eq. (16) thus becomes a pressure buildup solution for the infinite radial system. A comparison of pressure drawdown, Eq. (13) with Eq. (16), indicates that pressure drawdown curves generated by Eq. (13) should be the same as the pressure buildup curves obtained by Eq. (16). To demonstrate this, the family of buildup curves shown in Fig. 4 and plotted as a function of dimensionless shut-in time, Δt_D, were replotted as a function of the time group, $(t_{pD} \times \Delta t_D)/(t_p + \Delta t)_D$. Results are shown in Fig. 5.

Note that it is possible to normalize a family of buildup curves into a single curve. Moreover, this single curve is the same as the pressure drawdown curve. The preceding suggests that for analyzing pressure buildup data by drawdown type curves, $(\Delta p)_{buildup}$ data should be plotted as a function of $(t_p \times \Delta t)/(t_p + \Delta t)$ rather than just the shut-in time, Δt.

For the sake of simplicity and brevity let us define the new time group as an equivalent drawdown time, Δt_{edt} or further abbreviated as Δt_e, where

$$\Delta t_e = \frac{t_p \times \Delta t}{t_p + \Delta t} \qquad (17)$$

In a dimensionless form, Eq. (17) may be expressed as

$$\Delta t_{eD} = \frac{(t_{pD} \times \Delta t_D)}{(t_p + \Delta t)_D} \quad (18)$$

The dimensionless pressure change during buildup or a rate change may be defined as[13]

$$\tilde{p}_{wDs} = \frac{kh[p_{ws}(t_p + \Delta t) - p_{ws}(\Delta t=0)]}{141.2\ qB\mu} \quad (19)$$

In establishing the new method, it was previously assumed that wellbore effects (such as storage and skin) are negligible. It appears that skin effect, s, may be considered in the development of this method.

Infinite Radial System ($s \neq 0$; $C_D=0$)

If skin effect, s, is introduced in the pressure drawdown solution given by Eq. (12), we obtain

$$p_{wD}(t_D) = \frac{1}{2}\,[\ln(t_D) + 0.80907] + s \quad (20)$$

If we go through the same steps as we did for an infinite radial system ($s=0$; $C_D=0$) and instead of utilizing Eq. (12) we use Eq. (20), it will be obvious that the new method is equally valid when $s \neq 0$. Pressure drawdown Eq. (13) and the new pressure buildup Eq. (16) will respectively become Eqs. (21) and (22) as shown below:

$$\frac{kh[p_i - p_{wf}(t)]}{141.2\ qB\mu}$$

$$= \frac{1}{2}\,[\ln(t_D) + 0.80907] + s \quad (21)$$

$$\frac{kh[p_{ws}(t_p+\Delta t) - p_{ws}(\Delta t=0)]}{141.2\ qB\mu}$$

$$= \frac{1}{2}\,[\ln\frac{(t_{pD} \times \Delta t_D)}{(t_p + \Delta t)_D} + 0.80907] + s \quad (22)$$

The above equations establish the validity of using pressure drawdown type curves for pressure buildup analysis even when skin is present.

Based on the encouraging results obtained thus far, we wanted to apply this concept of the equivalent drawdown time, Δt_e to other wellbore effects and also to other flow regimes. The attempt was made to establish the validity of this concept for the above situations by graphical means rather than the mathematical solutions. Let us first consider the infinite radial system with wellbore storage effects.

Infinite Radial System ($s=0$; $C_D \neq 0$)

To study the effect of storage on buildup type curves, data presented by Agarwal, et al.,[1] were utilized. Pressure drawdown data p_{wD} vs. t_D data for $s=0$ and $C_D=1000$ were taken from Table 3 of the above paper. Eq. (5) was used to generate the pressure buildup data for a number of producing times as was done by Raghavan.[13] Both pressure drawdown data and pressure buildup data are plotted on Fig. 6 (semi-log graph paper) as a function of t_D and Δt_D respectively. Note that a family of buildup curves is obtained with producing time, t_{pD} (10^3 to 10^6) as a parameter. These data are also plotted on log-log graph paper as shown by Fig. 7. These figures further emphasize the limitations of using pressure drawdown curves to analyze pressure buildup data obtained after short producing periods. Fig. 7 shows that unit slope lines for buildup data are shifted to the right of the drawdown curves. If dimensionless storage, C_D, is computed using the buildup data, the computed value of C_D will be erroneously high. Moreover, if pressure buildup data are forced to match the pressure drawdown type curve, the computed value of formation flow capacity (kh) will be erroneously optimistic. The magnitude of error will increase with decreasing producing period.

Figs. 8 and 9 are the replots of pressure buildup solution on semi-log and log-log graph papers utilizing the new time group. Fig. 8 indicates that almost all pressure buildup curves are normalized except two which correspond to dimensionless producing period, t_{pD} equal to 10^3 and 10^4. Although these curves do not seem to appear bad on the semi-log paper, they look rather poor on the log-log graph in Fig. 9. The reason for this may be obvious if we inspect the following equation[19] which provides the time for storage effects to become negligible.

For $s=0$

$$t_D > 60C_D \quad (23)$$

If Eq. (23) is used for the subject problem, $C_D=1000$, the minimum producing time required for the storage effects to become negligible will be equal to 6×10^4. Since the producing periods in the two cases were only 10^3 and 10^4 respectively, pressure buildup data could not be normalized. Based on a number of cases studied, it appears that it is possible to normalize the pressure buildup curves provided that the producing time, t_{pD}, is at least equal to or greater than that given by Eq. (23).

Infinite Radial System ($s \neq 0$; $C_D \neq 0$)

Agarwal, et al.,[1] data were taken for a number of cases for non-zero values of C_D and s. Although not shown in this paper, results indicate that pressure buildup curves are normalized when the new method is used. The lower limit of the producing time for $s \neq 0$ is determined by the following equation.[20]

$$t_{pD} \geq (60 + 3.5\ s)C_D \qquad (24)$$

The preceding discussion establishes the validity of using Agarwal, et al's[1] pressure drawdown type curves (radial flow with storage and skin effect) for analyzing pressure buildup data provided that the new method is used.

APPLICABILITY OF NEW METHOD TO OTHER TYPE CURVES

It appears that it is possible to extend this method to other drawdown type curves which have appeared in the petroleum literature. Two sets of type curves will be considered: (1) Earlougher and Kersch,[4] and (2) Gringarten et al.[15]

Earlougher and Kersch Type Curves[4]

These type curves are based on pressure drawdown solution and are applicable to an infinite radial system with wellbore storage and skin. They are basically the same type curves as that of Agarwal et al.[1] because both use the same solution. However, they are distinctly different in appearance because data are plotted differently. A schematic of their drawdown type curves is shown in Fig. 10, where $(p_{wD}C_D)/t_D$ has been plotted as a function of t_D/C_D with C_De^{1} as a parameter. where C_D is the dimensionless storage coefficient, and is defined as

$$C_D = \frac{5.615\ C}{2\pi\phi hc_t r_w^2} \qquad (25)$$

Although no proof is demonstrated, their type curves may be converted for analyzing pressure buildup data if dimensionless time, t_D, appearing both in y-axis and x-axis is replaced by Δt_{eD}, and p_{wD} on the y-axis is replaced by $\tilde{p}_{wDs}$ as shown in Fig. 11. In performing type curve analysis, the basic steps as outlined by Earlougher and Kersch[4] remain the same except for some minor changes in the preparation of the data plot $(\Delta p_{buildup})/\Delta t_e$ vs. Δt_e should be plotted instead of plotting $\Delta p/t$ vs t. Limitations on the lower limits of producing time, t_{pD}, given by Eqs. (23) and (24) should also apply in this case.

Gringarten et al. Type Curves[15]

Recently, Gringarten et al.[15] presented another set of type curves in a form different than that of Agarwal et al.[1] and Earlougher and Kersch[4]. Pressure drawdown data have been plotted as p_{wD} vs. (t_D/C_D) with C_De^{1} as a parameter. Their drawdown type curves are schematically shown in Fig. 12. To convert their type curves for pressure buildup data, p_{wD} is changed to $\tilde{p}_{wDs}$ and the parameter t_D/C_D on the abscissa should be replaced by $\Delta t_{eD}/C_D$. In using their type curves for buildup, $(\Delta p)_{buildup}$ vs. Δt_e should be plotted on the data plot. Steps for type curve matching remain the same. The limitations on the lower limit of producing time as discussed earlier should also apply in this case.

CONVENTIONAL ANALYSIS USING THE NEW METHOD

Although the new method was originally conceived as to be used for type curve analysis purposes, it also appears useful for analyzing pressure buildup data by the conventional semi-log analysis methods. This can be seen by rewriting Eq. (22) in the following familiar form:

$$[p_{ws}(t_p+\Delta t)-p_{ws}(\Delta t=0)] = \frac{162.6\ qB\mu}{kh}$$

$$[\log \frac{(t_p \times \Delta t)}{(t_p + \Delta t)} + \log \frac{k}{\phi\mu c_t r_w^2} -$$

$$3.23 + 0.87\ s] \qquad (26)$$

or

$$[\Delta p_{buildup}] = m[\log(\Delta t_e) +$$

$$\log \frac{k}{\phi\mu c_t r_w^2} - 3.23 + 0.87s] \qquad (27)$$

where, m is the slope per log cycle, s is the usual skin effect and Δt_e was defined earlier by Eq. (17).

$$m = \frac{162.6\ q\mu B}{kh} \qquad (28)$$

$(\Delta p)_{buildup}$ is the left hand side of Eq. (22) and was defined earlier by Eq. (8). The form of Eq. (26) or Eq. (27) suggests that a graph of buildup pressure, p_{ws} or $(\Delta p)_{buildup}$ vs. Δt_e, should be linear on a semi-log graph paper. This will be shown later by means of Fig. 16. The slope of the line should provide the value of formation flow capacity, kh. Note that the graph utilizing Δt_e is similar to the Horner graph because it also takes into account the effect of producing time, t_p. Moreover, this graph appears more general than the Horner graph because the value of Δt_e increases with the increasing value of shut-in time, Δt as opposed to the Horner time group $(t_p + \Delta t)/\Delta t$ where it decreases as Δt increases. This permits plotting of buildup data on the same time scale using Δt and Δt_e so that the effect of including or excluding the producing time can be compared. Eq. (17) also indicates that for long producing times, when $(t_p + \Delta t)/t_p \simeq 1$. Eq. (17) reverts back as

$$\Delta t_e \simeq \Delta t \qquad (29)$$

Eq. (29) also provides the basis for making a MDH plot for long producing times. Eq. (26) may be solved for the skin effect, s as

$$s = 1.151\left[\frac{p_{ws}(\Delta t_e = 1\ \text{hr}) - p_{ws}(\Delta t = 0)}{m} - \log\frac{k}{\phi\mu c_t r_w^2} + 3.23\right] \quad (30)$$

Note that in Eq. (30), $p_{ws}(\Delta t_e = 1\ \text{hr})$ should be read on the semi-log straight line or its extension.

The initial reservoir pressure, p_i, or a false pressure, p^*, can be directly read from the straight line portion of the semi-log graph [p_{ws} vs. Δt_e] where Δt_e is equal to producing time, t_p. Inspection of Eq. (17) indicates that this corresponds to the Horner time ratio, $(t_p+\Delta t)/\Delta t$ equal to unity or shut-in time, Δ close to infinity. The estimation of initial reservoir pressure by this method will be illustrated later by means of a field example and will be shown on Fig. 16.

FIELD EXAMPLE

Pressure Buildup Analysis Using New Method

A field example taken from Gringarten et al.'s[15] paper will be utilized to illustrate the application of the new method to analyze pressure buildup data taken on an acidized well. Both conventional and type curve methods will be used to analyze the data. Results will be compared with those of Gringarten et al.[15] To maintain the continuity, part of the information appearing in their paper will be reproduced here.

Table 1 lists the pertinent reservoir and well data, along with pressure-time data both during the drawdown and buildup periods. Fig. 13 is a graph showing well pressures both during the constant rate drawdown [vs. flowing time, t] and during the subsequent buildup [vs. $(t_p + \Delta t)$]. Buildup data were replotted using the new time group, $(t_p \times \Delta t)/(t_p + \Delta t)$ or Δt_e. On Fig. 13, they are plotted as a function of $[t_p+\Delta t_e]$. Note that there is a significant difference between the two buildup curves. Data plotted simply as a function of shut-in time, Δt (shown by open circles) appear flatter compared to the second buildup curve plotted vs. Δt_e and shown by solid circles. This is to be expected because the first curve does not take into account the effect of producing time. More will be said about this later.

Although initial reservoir pressure was not known for this problem, it is possible to estimate it by means of an expanded plot (not shown here) of early time buildup and drawdown data and recognizing that a graph of $[(\Delta p)_{buildup}$ vs. $\Delta t_e]$ is equivalent to $[(\Delta p_{drawdown}$ vs. flowing time, t]. Using the known value of $(\Delta p)_{buildup}$ at a given Δt_e, the corresponding $(\Delta p)_{drawdown}$ at the same value of flowing time, t, was estimated. This provided p_i = 3251 psi, which was used to compute $(\Delta p)_{drawdown}$ vs. flowing time, t.

Fig. 14 shows a comparison of drawdown data [plotted as $(\Delta p)_{drawdown}$ vs. t and shown by triangles] with the pressure buildup data [$(\Delta p)_{buildup}$ vs. Δt_e] (shown by solid circles). The comparison between the two plots is excellent. $(\Delta p)_{buildup}$ data plotted as a function of conventional shut-in time, Δt, are also shown by a dotted line with open circles. Note that between 200 and 250 minutes there is a departure between the conventional buildup curve and the other two curves. Gringarten et al.[15] observed a similar departure between the conventional buildup curve and the drawdown curve at about 250 minutes and concluded that the buildup data beyond this time should not be analyzed by drawdown type curves. However, the modified buildup plot does not suffer from the above restriction. If the new method is used, the majority of data may be type curve matched. Fig. 14 also suggests that in this case, the real shut-in time, Δt, of 4281 minutes is only equal to (1347 x 4281)/(1347 + 4281) = 1025 minutes in terms of equivalent drawdown time, Δt_e.

Next $(\Delta p)_{buildup}$ vs. Δt_e data were type curve matched using Gringarten et al's[15] type curve as shown in Fig. 15. Note that a very satisfactory match has been obtained. Computations for type curve analysis are shown in Table 2 and results summarized in Table 3. It is also possible to read the initial pressure directly from the log-log plot. To accomplish this, read $(\Delta p)_{buildup}$ at $\Delta t_e = t_p$. This provides $p_i = p(\Delta t{=}0)+(\Delta p)_{buildup}$. In this case p_i = 3251 psi, as shown on Fig. 15.

To demonstrate applicability of the new method to conventional semi-log analysis, buildup pressures, p_{ws} were plotted on a semi-log graph paper both as a function of conventional shut-in time, Δt (shown by open circles) and the equivalent time, Δt_e (shown by solid circles). This is shown in Fig. 16. As expected, there is a significant difference between the plots. In a way it is similar to comparing a MDH plot with a Horner plot. However, the new method is better because data can be compared on an equivalent time scale. It also provides a reasonable straight line, whose slope was used to compute formation flow capacity, kh, as given by Eq. (28). Eq. (30) was used to compute the skin effect, s. It is also possible to directly read the initial pressure from the semi-log straight line or its extension where $\Delta t_e = t_p$. Results of both conventional semi-log and type curve analyses are listed in Table 3. For comparison purposes, analysis results obtained by Gringarten et al.[15] are also shown in Table 3.

Note that excellent agreement has been obtained between the conventional semi-log and type curve methods when the new method is used. Moreover, results also agree very well with that of Gringarten et al.[15] when they used the Horner method for the semi-log analysis and the desuperposed data for type curve matching purposes. Obviously, their MDH type results shown in Table 3 obtained by ignoring the effect of producing time (using semi-log or type curve method) will be wrong, as expected. This was also pointed out by Gringarten et al.[15] In regard to the desuperposition principle, it should be pointed out that it is not always possible to desuperpose the buildup data because it requires a knowledge of pressure vs. time data from the preceding flow period. If the new method is used, desuperposition of data may not be necessary. Although not shown, the new method may be used to desuperpose pressure buildup data.

EXTENSION OF NEW METHOD TO OTHER KINDS OF TESTING

It appears that the new method may be extended to analyze other types of testing such as two-rate and multiple rate tests in an infinite radial system. Once this method is used, data may be analyzed by both the conventional semilog method and type curve matching techniques.

Two Rate Testing[8,16,17]

A schematic of two rate testing with rate and pressure history is shown in Fig. 17. This type of testing consists of flowing a well at a constant rate, q_1, for time, t_1, when the rate is changed to q_2 during the incremental time, Δt. The flowing pressure, $p_{wf1}(t_1)$ at the end of the first flow rate, q_1, can be obtained from Eq. (21) as

$$\frac{kh[p_i - p_{wf1}(t_1)]}{141.2\ \mu B} = \frac{1}{2} q_1[\ln(t_1)_D + 0.80907 + 2s] \qquad (31)$$

Note that the above equation assumes the log approximation. By applying the superposition principle, we obtain the following equation for the flowing pressure, $p_{wf2}(t_1 + \Delta t)$ during the second flow rate, q_2.

$$\frac{kh[p_i - p_{wf2}(t_1 + \Delta t)]}{141.2\ \mu B} = \frac{1}{2} q_1[\ln(t_1 + \Delta t)_D + 0.80907 + 2s] - \frac{1}{2}(q_1 - q_2)[\ln(\Delta t_D) + 0.80907 + 2s] \qquad (32)$$

Subtracting Eq. (32) from Eq. 31, we obtain

$$\frac{kh[p_{wf2}(t_1 + \Delta t) - p_{wf1}(t_1)]}{141.2(q_1 - q_2)\mu B} = \frac{1}{2}[\ln\left(\frac{t_1}{t_1 + \Delta t}\right)^{\left(\frac{q_1}{q_1 - q_2}\right)} \cdot (\Delta t)_D + 0.80907 + 2s] \qquad (33)$$

Comparison of the above equation with Eq. (21) indicates that a plot of $[p_i - p_{wf}(t)]$ vs. t during a pressure drawdown test is equivalent to plotting of data as

$$[p_{wf2}(t_1 + \Delta t) - p_{wf1}(t_1)] \text{ vs. } \left[\left(\frac{t_1}{t_1 + \Delta t}\right)^{\left(\frac{q_1}{q_1 - q_2}\right)} \cdot \Delta t\right]$$

obtained during the second rate testing. Also note that Eq. (33) reverts back to the pressure buildup Eq. (22) when q_2 is set equal to zero. This is to be expected in view of the fact that a pressure buildup test is a special case of two rate tests with $q_2 = 0$.

For a two rate testing, let us define equivalent drawdown time, Δt_{e2} as

$$\Delta t_{e2} = \left(\frac{t_1}{t_1 + \Delta t}\right)^{\left(\frac{q_1}{q_1 - q_2}\right)} \cdot \Delta t \qquad (34)$$

Substituting Eq. (34) in Eq. (33) and expressing it for conventional semilog analysis, we obtain

$$[p_{wf2}(t_1 + \Delta t) - p_{wf1}(t_1)] = \frac{162.6\ (q_1 - q_2)B\mu}{kh}[\log(\Delta t_{e2}) + \log\frac{k}{\phi\mu c_t r_w^2} - 3.23 + 0.87s] \qquad (35)$$

Eq. (35) suggests that a graph of $[p_{wf2}(t_1 + \Delta t) - p_{wf1}(t_1)]$ or $p_{wf2}(t_1 + \Delta t)$ vs. Δt_{e2} should be linear with slope, m. Formation flow capacity is computed as

$$kh = \frac{162.6(q_1 - q_2)B\mu}{m} \text{ md-ft} \qquad (36)$$

and skin effect, s, is computed using

$$s = 1.151[\frac{p_{wf2}(\Delta t_{e2} = 1\text{ hr}) - p_{wf1}(t_1)}{m} - \log\frac{k}{\phi\mu c_t r_w^2} + 3.23] \qquad (37)$$

An example will be shown later for multiple rate testing.

Multiple Rate Testing[8,17,18]

A schematic of multiple rate testing is shown in Fig. 18. This type of testing consists of flowing a well at a constant rate q_1 for time, t_1, at rate q_2 for time t_1 to t_2 and so on. Say the final rate is q_n for time t_{n-1} to any incremental time, Δt. Although not shown, pressures are denoted as $p_{wf1}(t_1)$, $p_{wf2}(t_2)$....and $p_{wfn-1}(t_{n-1})$ at the end of first, second and t_{n-2} time periods. $p_{wfn}(\Delta t)$ are the pressures during the final (nth) period. If the steps similar to those shown for two rate testing are followed for multiple rate testing, the following equation is obtained.

$$\frac{kh[p_{wfn}(\Delta t) - p_{wfn-1}(t_{n-1})]}{141.2\ (q_{n-1} - q_n)B\mu}$$

$$= \frac{1}{2}\{\ln\left[\prod_{j=1}^{n-1}\left(\frac{t_{n-1} - t_{j-1}}{\Delta t + t_{n-1} - t_{j-1}}\right)^{\left(\frac{q_j - q_{j-1}}{q_{n-1} - q_n}\right)}(\Delta t)_D\right]$$

$$+ 0.80907 + 2s\} \qquad (38)$$

where $t_o = 0$; $q_o = 0$ and $n \geq 2$. Eq. (38) is very general and should apply to any number of flow and buildup periods, provided that the system is behaving like an infinite radial system and log approximation is valid. Eq. (38) also suggests that multiple rate test data during any flow or buildup period may be analyzed using drawdown type curves. For multiple rate testing, the equivalent drawdown time may be defined as

$$\Delta t_{en} = \left[\prod_{j=1}^{n-1}\left(\frac{t_{n-1} - t_{j-1}}{\Delta t + t_{n-1} - t_{j-1}}\right)^{\left(\frac{q_j - q_{j-1}}{q_{n-1} - q_n}\right)}\right]\cdot\Delta t \qquad (39)$$

For conventional semi-log analysis, Eq. (38) may be written as

$$[p_{wfn}(\Delta t) - p_{wfn-1}(t_{n-1})] = \frac{162.6(q_{n-1} - q_n)B\mu}{kh}[\log(\Delta t_{en}) +$$

$$\log\frac{k}{\phi\mu c_t r_w^2} - 3.23 + 0.87s] \qquad (40)$$

Eq. (40) also suggests a linear relationship on a semi-log paper with slope, m where

$$kh = \frac{162.6(q_{n-1} - q_n)B\mu}{m}\ \text{md-ft} \qquad (41)$$

and Eq. (40) is expressed for the skin effect, s, as follows:

$$s = 1.151[\frac{p_{wfn}(\Delta t_{en}=1\ \text{hr}) - p_{wfn-1}(t_{n-1})}{m}$$

$$- \log\frac{k}{\phi\mu c_t r_w^2} + 3.23] \qquad (42)$$

Next a computer simulated example will be considered to illustrate the application to multiple rate data.

SIMULATED EXAMPLE

Multiple Rate Analysis Using New Method

To demonstrate the application, a computer generated example will be utilized. Table 4 lists the reservoir and well data for a gas well, where (μc_g) product is kept constant to eliminate the effects of (μc_g) variations as a function of pressure. Rate and pressure histories are shown on Fig. 19. Note that the gas well was produced at three different flow rates (15, 10 and 5 MMCF/D) with intermediate buildup periods. Each flow period was simulated to be 1/2 day long whereas each of the first two buildup periods was 1 day long. The third and the final buildup period was 2 1/2 days long. Pressure vs. time data for each flow period are shown on Fig. 19 by means of open symbols, whereas for each subsequent buildup period by means of corresponding solid symbols. Multiple rate test Eq. (40) was expressed in terms of real gas pseudo pressure[19], m(p) and rearranged slightly for the purpose. In practical gas units, the following equation is obtained:

$$\frac{m[p_{wfn}(\Delta t)] - m[p_{wfn-1}(t_{n-1})]}{(q_{n-1} - q_n)}$$

$$= \frac{1637T}{kh}[\log(\Delta t_{en}) + \log\frac{k}{\phi(\mu c_t)_i r_w^2}$$

$$- 3.23 + 0.87s] \qquad (43)$$

Eq. (43) was used to process the pressure vs. time data obtained for each test period. Results are shown on Fig. 20 using the same symbols as shown on Fig. 19. The left hand side of Eq. (43) denoted by $[\Delta m(p)/\Delta q]$ vs. the equivalent drawdown time, Δt_{en} have been plotted on a semilog graph paper for each test. Note that it was possible to normalize all test data on the semi-log straight line obtained using single rate drawdown data. Although details are not shown, the slope of the semi-log straight line provided the value of formation flow capacity (kh) which was consistent with the kh value entered into the program, as expected. The preceding example was used to demonstrate the application and establish the validity of the new method for multiple rate testing data.

Other Kinds of Testing

Although not shown in this paper, it appears that the new method may be applied to other kinds of testing methods such as interference, constant pressure testing, etc.

VERTICALLY FRACTURED WELL

The new method was next applied to vertically fractured wells with both infinite and finite flow capacity fractures. Results are discussed below. Eq. (5) again forms the basis of this study. Dimensionless time, t_{Dx_f} for a fractured well is defined as follows:

$$t_{Dx_f} = \frac{2.634 \times 10^{-4}\, kt}{\phi(\mu c_t)_i\, x_f^2} \quad (44)$$

Where x_f is the fracture half-length in feet. Definitions of real and dimensionless drawdown and buildup pressures were kept the same.

Infinite Flow Capacity Fracture

Gringarten et al.'s[5] pressure drawdown data (p_{wD} vs. t_{Dx_f} for the infinite reservoir case were taken from their Table 1. Eq. (5) was used to generate a family of pressure buildup curves with producing times, t_{pDx_f}, as a parameter. Results similar to those of Raghavan are presented in Fig. 21. Since Raghavan[13] adequately discussed the limitations of using pressure drawdown curves for analyzing pressure buildup data for fractured wells, only certain key points will be re-emphasized.

(i) Computed formation flow capacity will be optimistic.

(ii) Computed value of fracture length will be pessimistic.

(iii) The characteristic half slope line may not appear on the log-log paper.

Fig. 22 shows the replot of pressure buildup data utilizing the new method. Data are plotted as $\tilde{p}_{wds}$ vs. Δt_{eDx_f} or $(t_{pDx_f} \times \Delta t_{Dx_f})/(t_{pDx_f} + \Delta t_{Dx_f})$. Note that Δt_{eDx_f} is the equivalent drawdown time, expressed in the dimensionless form, for a vertically fractured well. The majority of buildup data have been normalized on the drawdown curve. It was rather a surprising observation in view of the fact that a time group developed for the radial system should also be applicable for a fractured well which is normally associated with linear, elliptical and radial flow regimes.

Although not included in this paper, the pressure drawdown data of Gringarten, et al.,[5] for the uniform flux fracture case were also considered. Pressure buildup data were generated and plotted using the new method. Once again it was possible to normalize the majority of buildup data on the drawdown type curve. The plot was very similar to that shown in Fig. 22.

Finite Flow Capacity Fracture

The new method was next applied to data for a vertically fractured with finite flow capacity fracture. Constant rate pressure drawdown data of Agarwal, et al.,[7] were used to generate a family of buildup type curves with producing time as a parameter. This had to be done for each value of dimensionless fracture flow capacity. These results were replotted using the new method. Once again, it was possible to normalize the majority of buildup data on drawdown type curves. For the sake of brevity, results are not presented here. However, it should be suffice to say that constant rate pressure drawdown type curves of Agarwal, et al.[7] and Cinco, et al.[6], may be utilized to analyze pressure buildup data. Requirement is that $(\Delta p)_{buildup}$ data are plotted as a function of Δt_e rather than the conventional shut-in time, Δt.

ANALYSIS OF GAS WELL BUILDUP DATA

The development of the new method, for analyzing pressure buildup data, has been discussed mainly utilizing solutions for liquid systems. However, it appears that the method may be extended to include the analysis of data from gas wells, if real gas pseudo-pressure m(p) of Al-Hussainy, et al.[19], is used and variations of (μc_g) vs. pressure are accounted for. The latter may be accomplished if real times in the new time group are replaced by real gas pseudo-time, $t_a(p)$ of Agarwal[14]. For example, if pressure buildup data collected after short producing time from an MHF gas well are to be analyzed by drawdown type curves, the following procedure is recommended.

Graph $\{m[p_{ws}(t_p + \Delta t)] - m[p_{ws}(\Delta t{=}0)]\}$ vs. $\dfrac{(t_{ap} \times \Delta ta)}{(t_{ap} + \Delta ta)}$ on

the data plot, utilizing the appropriate type curve. Steps outlined in Ref. 14 for type curve matching remain the same. In the above time group, t_{ap} and Δt_{ap} represent flowing time, t, and shut-in time, Δt, expressed in terms of real gas pseudo-time. If variations of (μc_g) vs. pressure during the test period appear to be small, instead of using pseudo-time, real times may be used.

CONCLUDING REMARKS

1. A new method has been developed to analyze pressure buildup data by pressure drawdown type curves. It provides a significant improvement over the current methods because

   (i) the effects of producing time are accounted for;

   (ii) data are normalized in such a way that instead of using a family of buildup curves, the existing drawdown type curves may be used;

   (iii) wellbore storage and damage effects may be considered except under certain conditions.

2. This method can also be used to perform conventional semi-log analysis to estimate formation flow capacity, kh, skin effect, s, and initial pressure, p_i. It appears similar to the Horner method because both methods take into account producing time effects. However, this method is more general and has the advantage that

   (i) both the MDH plot and the plot using the new method utilize the common time scale, which permits comparing the two plots and determining the effects of including or excluding the producing time;

   (ii) it provides a relationship between the flowing time, t, during a drawdown test to an equivalent time, Δt_e, during a buildup test.

3. For long producing periods, the new method reverts back to the MDH method.
4. A field example is included to demonstrate the application of the new method and point out the utility.
5. This method has been extended to include the analysis of two rate and multiple rate test data by both type curve and conventional methods. This was shown both theoretically and also by means of example problem.
6. Although originally developed for radial systems, this method appears to work well for vertically fractured wells with infinite and finite flow capacity fractures.
7. The method, although developed using liquid solutions, should be applicable to data from gas wells, as shown in the paper.
8. Finally, it appears that the new method may be applied to a variety of testing methods such as interference testing and constant pressure testing to name a few.

NOMENCLATURE

B	= formation volume factor, RB/STB (res m^3/stock tank m^3)
c_g	= gas compressibility, psi^{-1} (kPa^{-1})
c_t	= total system compressibility, psi^{-1} (kPa^{-1})
C	= storage coefficient, RB/psi (res. m^3/kPa)
C_D	= dimensionless storage coefficient [see Eq. (25)]
F_{CD}	= dimensionless fracture flow capacity (see Ref. 7)
h	= formation thickness, ft (m)
k	= formation permeability, md
log	= logarithm to base 10
ln	= natural logarithm
m	= slope of the semilog straight line, psi/log cycle (kPa/log cycle)
m(p)	= real gas pseudo pressure, psi^2/cp (kPa^2/Pa·s)
p_i	= initial pressure, psi (kPa)
p_{wD}	= dimensionless pressure drop [see Eq. (1)]
$\tilde{p}_{wDs}$	= dimensionless pressure rise or change [see Eq. (19)]
p_{wf}	= wellbore flowing pressure, psi (kPa)
p_{wfn-1}	= pressure at the end of test period, t_{n-1}, psi (kPa)
p_{wfn}	= pressures during nth test period, psi (kPa)
p_{ws}	= shut-in pressure, psi (kPa)
$(\Delta p)_{buildup}$	= pressure change during buildup, psi (kPa) [see Eq. (8)]
$(\Delta p)_{difference}$	= pressure difference, psi (kPa) [see Eq. (10) and (11(]
$(\Delta p)_{drawdown}$	= pressure change during drawdown, psi (kPa) [see Eq. (7)]
q	= flow rate, STB/D or MCF/D ("standard" m^3/D)
q_n	= flow rate during the final flow period, STB/D ("standard" m^3/D)
r_w	= wellbore radius, ft (m)
s	= skin effect
t	= flow time, hours
t_{n-1}	= total time up to (n-1) test period, hours
t_D	= dimensionless time based on wellbore radius (see Eq. 2)
$t_{a(p)}$	= real gas pseudo time (see Ref. 14)
t_{ap}	= real gas pseudo-producing time
t_{Dx_f}	= dimensionless time based on half fracture length (see Eq. 44)
t_p	= producing period, hours
t_{p1}	= producing period during the first test, hours.
t_{pD}	= dimensionless producing period
Δt	= shut-in time or incremental time during the final flow test period, hours
Δt_a	= real gas pseudo shut-in time

t_{ap} = real gas pseudo producing period

Δt_e = equivalent drawdown time, hours (see Eq. 17)

Δt_{Dx_f} = dimensionless shut-in time based on fracture half length

Δt_{e2} = equivalent drawdown time for two rate test, hours (see Eq. 34)

Δt_{en} = equivalent drawdown time for multiple rate test, hours (see Eq. 39)

T = reservoir temperature, °R

x_f = fracture half length, ft (m)

μ = viscosity, cp (Pa · s)

$(\mu c_t)_i$ = viscosity-compressibility product at initial condition, cp

ϕ = formation porosity, fraction

π = constant, 3.14159

Subscripts

a = adjusted or pseudo

CD = dimensionless flow capacity

D = dimensionless

Dx_f = dimensionless, based on x_f

e = equivalent

eD = equivalent, dimensionless

e2 = equivalent for two rate test

en = equivalent for multiple rate test

f = fracture

g = gas

i = initial

j = index

n = index

p = producing

t = total

w = wellbore

REFERENCES

1. Agarwal, Ram G., Al-Hussainy, Rafi, and Ramey, H. J., Jr.: "An Investigation of Wellbore Storage and Skin Effect in Unsteady Liquid Flow: I. Analytical Treatment," Soc. Pet. Eng. J. (Sept. 1970) 279-290; Trans., AIME, 249.

2. Wattenbarger, Robert A. and Ramey, H. J., Jr.: "An Investigation of Wellbore Storage and Skin Effect in Unsteady Liquid Flow: II. Finite Difference Treatment," Soc. Pet. Eng. J. (Sept. 1970) 291-297; Trans., AIME, 249.

3. Ramey, H. J., Jr.: "Short-Time Well Test Data Interpretation in the Presence of Skin Effect and Wellbore Storage," J. Pet. Tech. (Jan. 1970) 97-104; Trans., AIME, 249.

4. Earlougher, Robert C., Jr., and Kersch, Keith M.: "Analysis of Short-Time Transient Test Data by Type Curve Matching," J. Pet. Tech. (July 1974) 793-800; Trans., AIME, 257.

5. Gringarten, Alain C., Ramey, Henry J., Jr., and Raghavan, R.: "Pressure Analysis for Fractured Wells," paper SPE 4051 presented at the SPE-AIME 47th Annual Fall Meeting, San Antonio, Tex., Oct. 8-11, 1972.

6. Cinco-L, Hever, Samaniego-V., F. and Dominguez-A., N.: "Transient Pressure Behavior for a Well With a Finite-Conductivity Vertical Fracture," Soc. Pet. Eng. J. (Aug. 1978) 253-264.

7. Agarwal, R. G., Carter, R. D., and Pollock, C. B.: "Evaluation and Prediction of Performance of Low Permeability Gas Wells Stimulated by Massive Hydraulic Fracturing," J. Pet. Tech. (March 1979) 362-372; Trans., AIME, 267.

8. Earlougher, Robert C., Jr.: Advances in Well Test Analysis, Monograph Series, Society of Petroleum Engineers, Dallas (1977) 5.

9. Miller, C. C., Dyes, A. B., and Hutchinson, C. A., Jr.: "The Estimations of Permeability and Reservoir Pressure from Bottom-hole Pressure Build-Up Characteristics," Trans., AIME (1950) 189, 91-104.

10. Horner, D. R.: "Pressure Build-Up in Wells," Proc., Third World Pet. Cong., The Hague (1951) II, 503-521.

11. McKinley, R. M.: "Wellbore Transmiscibility From Afterflow-Dominated Pressure Buildup Data," J. Pet. Tech. (July 1971) 863-872; Trans., AIME, 251.

12. Crawford, G. E., Pierce, A. E., and McKinley, R. M.: "Type Curves for McKinley Analysis of Drill-Stem Test Data," paper SPE 6754, presented at the SPE 52nd Annual Fall Meeting, Denver, Colo., Oct. 9-12, 1977.

13. Raghavan, R.: "The Effect of Producing Time on Type Curve Analysis," J. Pet. Tech. (June, 1980) 1053-1064.

14. Agarwal, Ram G.: "Real Gas Pseudo-Time - A New Function for Pressure Buildup Analysis of MHF Gas Wells," paper SPE 8270 presented at SPE 54th Annual Fall Meeting, Las Vegas, Nev., Sept. 23-26, 1979.

15. Gringarten, A. C., Bourdet, D. P., Landel, P. A., and Kniazeff, V. J.: "A Comparison Between Different Skin and Wellbore Storage Type-Ruves to Early-Time Transient Analysis" paper SPE 8205 presented at SPE 54th Annual Fall Meeting, Las Vegas, Nev., Sept. 23-26, 1979.

16. Russell, D. G.: "Determination of Formation Characteristics From Two-Rate Flow Tests," J. Pet. Tech. (Dec. 1963) 1347-1355; Trans., AIME, 228.

17. Matthews, C. S. and Russell, D. G.: Pressure Buildup and Flow Tests in Well, Monograph Series, Society of Petroleum Engineers of AIME, Dallas (1967) 1, Chapter 5.

18. Odeh, A. S. and Jones, L. G.: "Pressure Drawdown Analysis, Variable-Rate Case," J. Pet. Tech. (Aug. 1965) 960-964; Trans., AIME, 234.

19. Al-Hussainy, R., Ramey, H. J., Jr., and Crawford, P. B.: "The Flow or Real Gases Through Porous Media," J. Pet. Tech. (May 1966) 624-636; Trans., AIME, 237.

TABLE 1

RESERVOIR AND WELL DATA
(Field Example - Ref. 15)

Formation thickness, h	30 ft
Formation porosity, ϕ	0.15 fraction PV
Wellbore radius, r_w	0.3 ft
Fluid viscosity, μ	1.0 cp
System compressibility, c_t	10 X 10-6 psi-1
Formation volume factor, B	1.25 RB/STB
Production rate, q	800 STB/D
Producing period, t_p	1347 minutes

Drawdown		Buildup			
Δt (min)	p_{wf} (psi)	Δt (min)	p_{ws} (psi)	$p_{ws}(t_p+\Delta t)-p_{ws}(\Delta t=0)$ (psi)	$\frac{(t_p \times \Delta_t)}{(t_p + \Delta_t)}$
45	3198	3	3105	8	2.99
85	3180	5	3108	11	4.48
192	3154	9	3115	18	8.64
297	3141	16	3125	28	15.61
417	3130	30	3139	42	29.74
654	3116	40	3146	49	38.6
983	3196	66	3159	62	62.6
1347	3097	100	3171	74	92.9
		138	3180	83	125.0
		252	3195	98	210.5
		334	3203	106	269.
		423	3208	111	321.
		574	3216	119	396.
		779	3222	125	499.
		1092	3228	131	612.
		1674	3234	137	748.
		2186	3238	141	842.
		2683	3242	145	898.
		3615	3246	149	962.
		4281	3246	149	1036.

TABLE 2

TYPE CURVE ANALYSIS
(Field Example - Ref. 15)

Match Point

$[\Delta p]_M = 10$ psi $\qquad [p_{wD}]_M = 0.17$

$[\Delta t_e]_M = 10$ min $\qquad [\frac{t_D}{c_D}]_M = 0.64$

$[c_D e^{2s}]_M = 1.0$

(i) Formation flow capacity, kh

$$kh = \frac{[p_D]_M\ (141.2)\ qB\mu}{[\Delta p]_M} \text{ md-ft}$$

$$kh = \frac{(0.17)(141.2)(800)(1.25)(1)}{10}$$

$$kh = 2400 \text{ md-ft}$$

(ii) Skin effect, s

$$C = (0.000295)\ \frac{kh}{\mu}\ \frac{[\Delta t_e]_M}{[t_D/c_D]_M} \text{ Res Bbl/psi}$$

$$= \frac{(0.000295)(2400)(10/60)}{(1)(0.64)} \text{ Res Bbl/psi}$$

$$C = 0.184 \text{ Res Bbl/psi}$$

Eq. (25) is used to compute

$$c_D = \frac{(C)(5.615)}{2\pi\phi h c_t r_w^2} = \frac{(0.184)(5.615)}{2\pi(.15)(30)(1X10^{-5})(.3)^2} = 40600$$

$$s = \frac{1}{2}\ \ell n\ [\frac{(c_D e^{2s})_M}{c_D}] = \frac{1}{2}\ \ell n\ [\frac{1}{40821}]$$

$$= -5.31$$

TABLE 3

SUMMARY OF RESULTS
(Field Example - Ref. 15)

Analysis Using This Method

	Semilog	Type Curve
kh (md-ft)	2323	2400
s	-5.15	-5.31
C (Res Bbl/psi)	--	0.184
p_i	3253	3251

Gringarten et al.[15] Analysis

	Semilog		Type Curve	
	Horner	MDH	Desuperposition	MDH
kh (md-ft)	2274	4279	2259	4095
s	-5.0	-4.0	-5.1	-4.0
C (Res Bbl/psi)	--	--	0.19	0.25
p_i	3253	3230	--	--

TABLE 4

RESERVOIR AND WELL DATA
(Simulated Example)

Initial reservoir pressure, p_i	500 psi
Reservoir temperature, T	720°R
Formation permeability, k	5 md
Formation thickness, h	40 feet
Hydrocarbon porosity, ϕ	5%
Viscosity-compressibility product, (μc_t)	3.93 X 10^{-6} cp/psi
Wellbore radius, r_w	0.25 feet

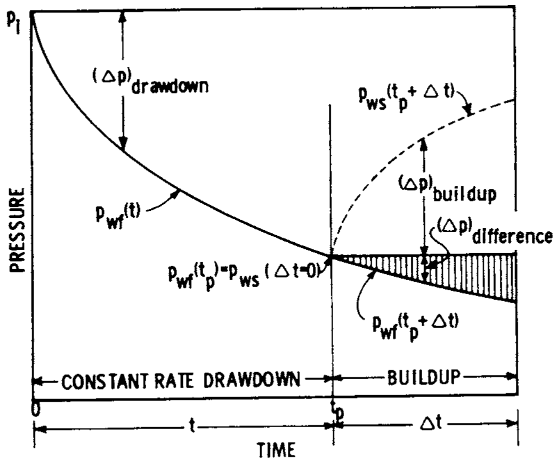

FIGURE 1: SCHEMATIC OF PRESSURE BUILDUP BEHAVIOR FOLLOWING A CONSTANT RATE DRAWDOWN FOR A PRODUCTION PERIOD, t_p

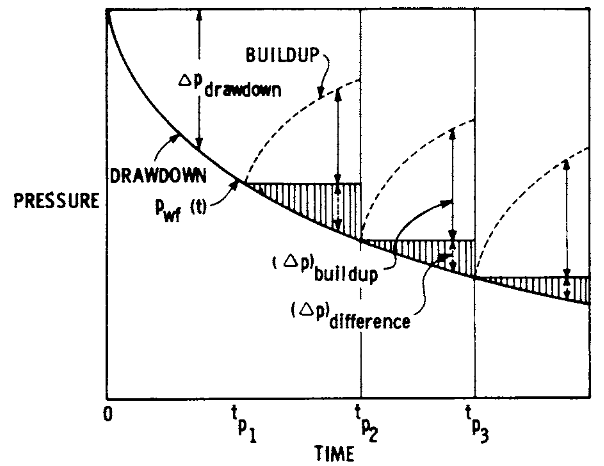

FIGURE 2: SCHEMATIC OF PRESSURE BUILDUP BEHAVIOR FOLLOWING CONSTANT RATE DRAWDOWN OF SUCCESSIVELY INCREASING FLOW PERIODS, t_p

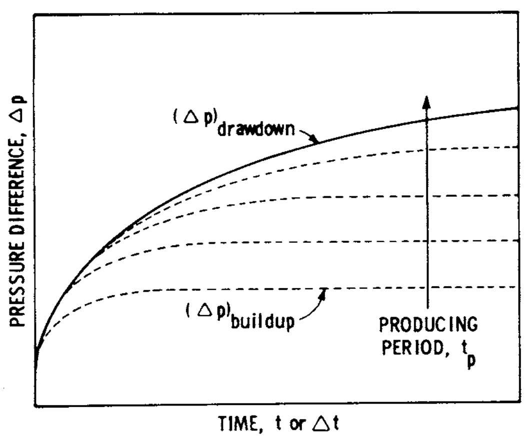

FIGURE 3: SCHEMATIC SHOWING COMPARISON BETWEEN $[(\Delta p)_{drawdown}$ vs $t]$ AND $[(\Delta p)_{buildup}$ vs $\Delta t]$

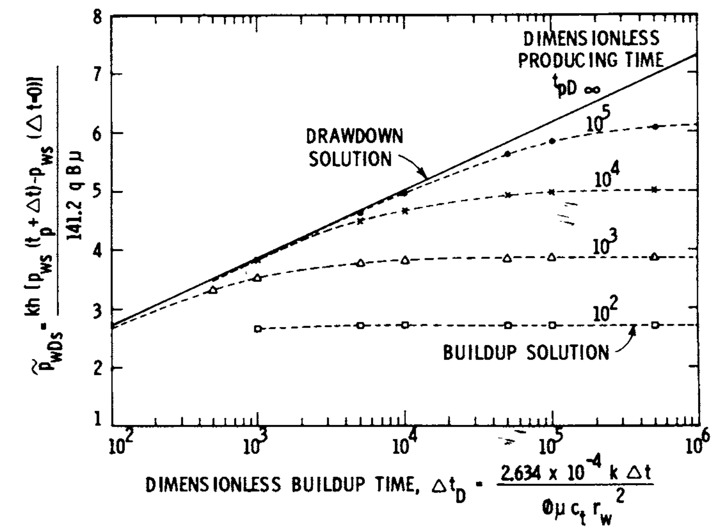

FIGURE 4: BUILDUP TYPE CURVES FOR VARIOUS PRODUCING TIMES - SEMI-LOG GRAPH (INFINITE RADIAL SYSTEM, $C_D = 0$ & $S = 0$)

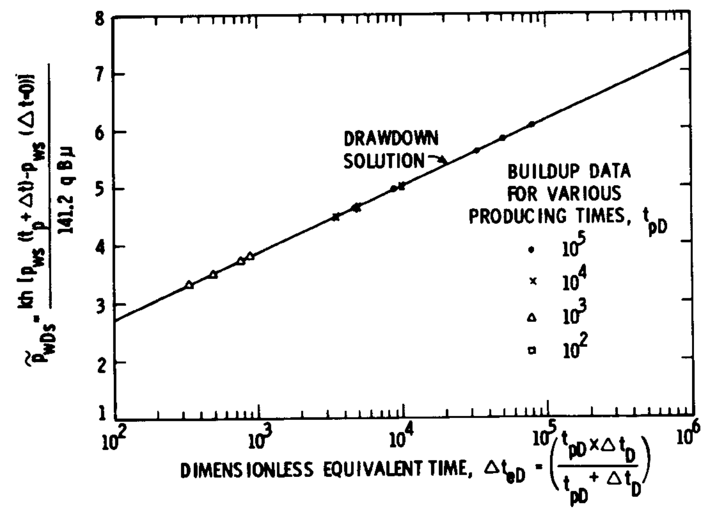

FIGURE 5: NORMALIZED BUILDUP TYPE CURVE-SEMI-LOG GRAPH (INFINITE RADIAL SYSTEM, $C_D=0$; $s = 0$)

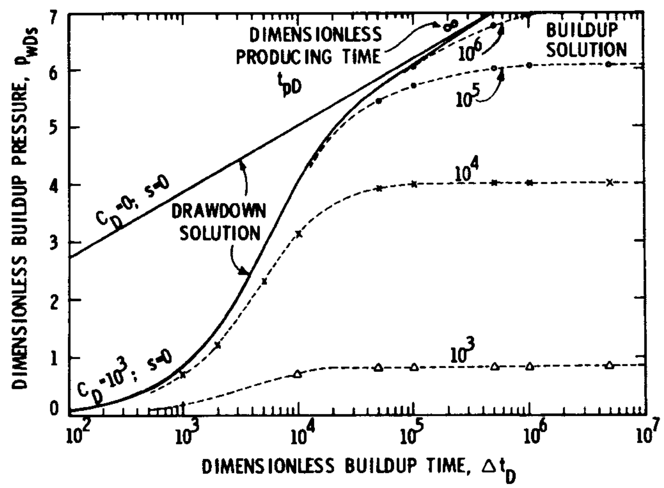

FIGURE 6: BUILDUP TYPE CURVES FOR VARIOUS PRODUCING TIMES-SEMI-LOG GRAPH (INFINITE RADIAL SYSTEM, $C_D=1000$; $s=0$)

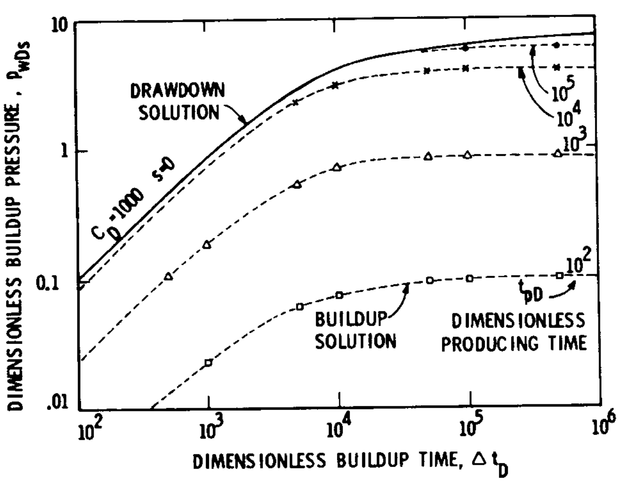

FIGURE 7: LOG-LOG BUILDUP TYPE CURVES FOR VARIOUS PRODUCING TIMES (INFINITE RADIAL SYSTEM, $C_D=1000$; $S=0$)

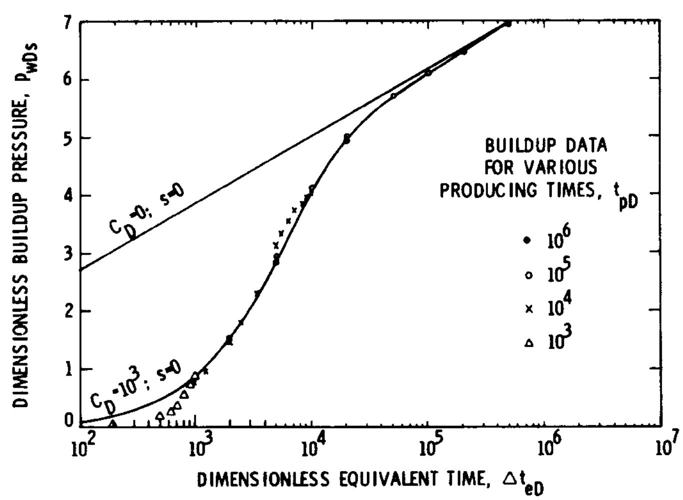

FIGURE 8: NORMALIZED BUILDUP TYPE CURVE (SEMI-LOG GRAPH) (INFINITE RADIAL SYSTEM, $C_D=1000$; $S=0$)

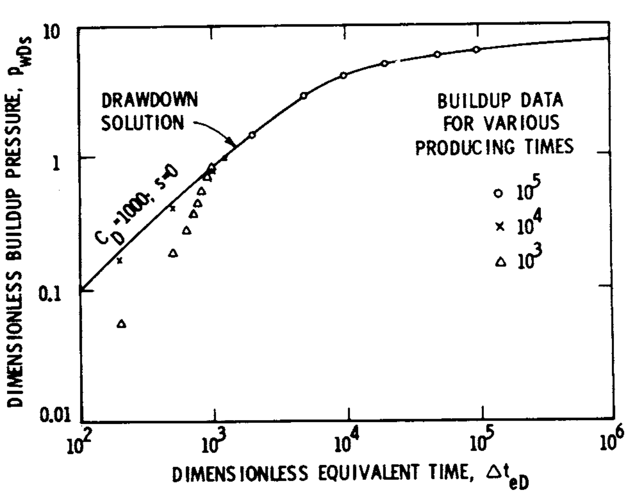

FIGURE 9: NORMALIZED BUILDUP TYPE CURVES - LOG-LOG GRAPH (INFINITE RADIAL SYSTEM, C_D=1000, S=0)

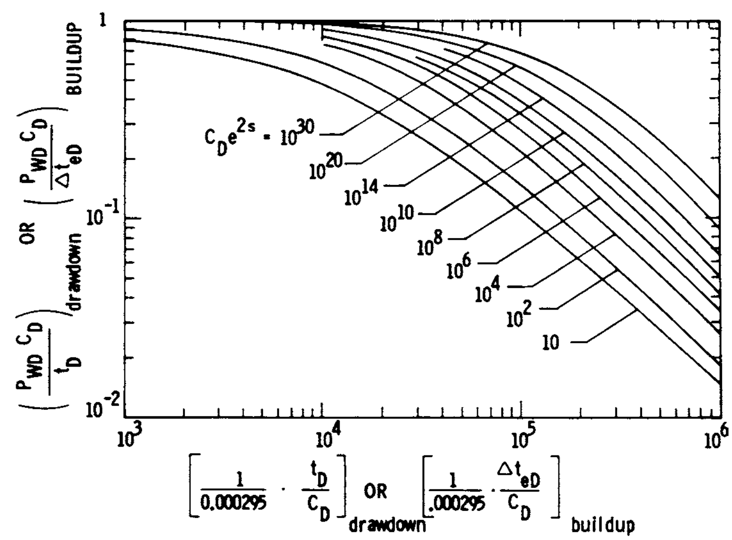

FIGURE 10: A GENERALIZED SCHEMATIC OF EARLOUGHER AND KERSCH DRAWDOWN TYPE CURVES[4]

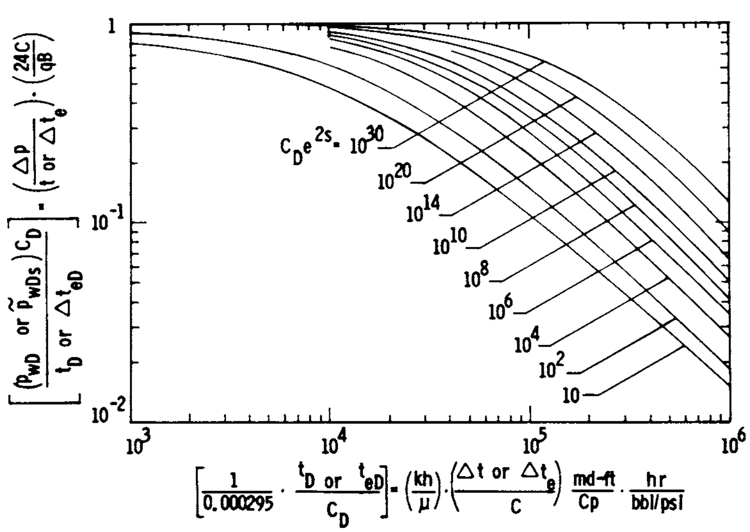

FIGURE 11: A SCHEMATIC OF EARLOUGHER AND KERSCH[4] DRAWDOWN TYPE CURVES FOR A WELL WITH STORAGE AND SKIN (INFINITE RADIAL SYSTEM)-BY PERMISSION OF MARATHON OIL COMPANY

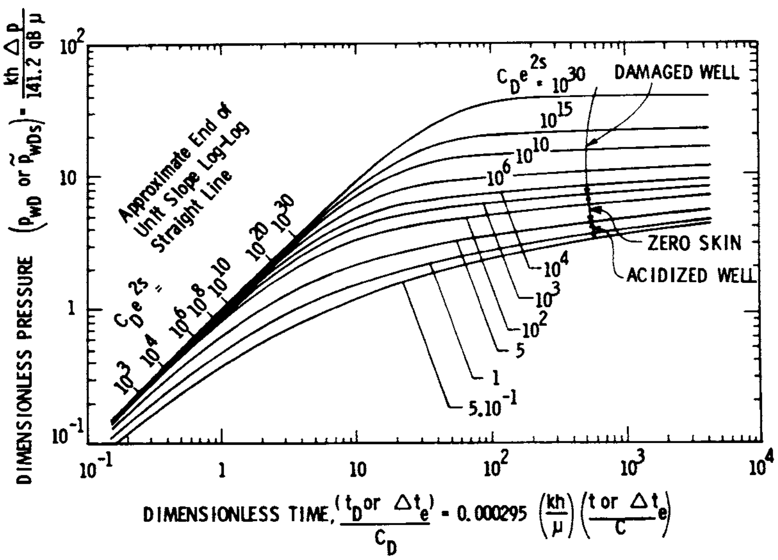

FIGURE 12: A SCHEMATIC OF GRINGARTEN Et Al[15] DRAWDOWN TYPE CURVES FOR A WELL WITH STORAGE AND SKIN (BY PERMISSION OF FLOPETROL)

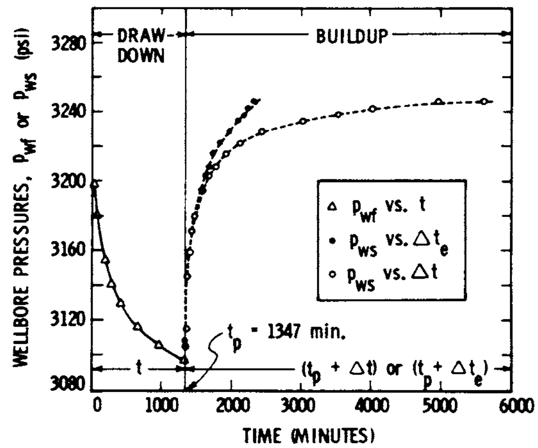

FIGURE 13: WELL PRESSURES Vs TIME DURING PRESSURE DRAWDOWN AND BUILDUP PERIODS (FIELD EXAMPLE - REF. 15)

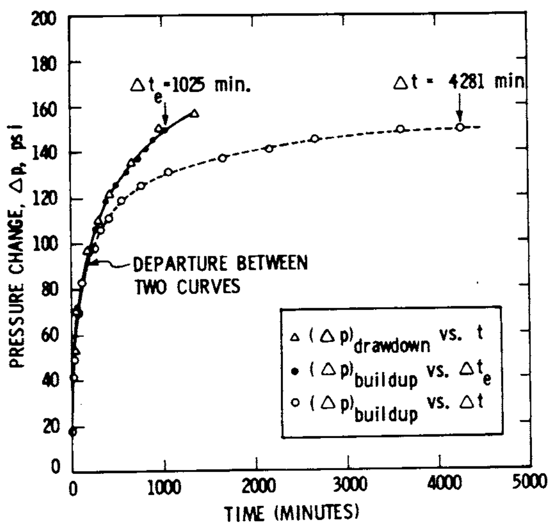

FIGURE 14: COMPARISON BETWEEN DRAWDOWN AND BUILDUP DATA USING NEW METHOD (FIELD EXAMPLE - REF. 15)

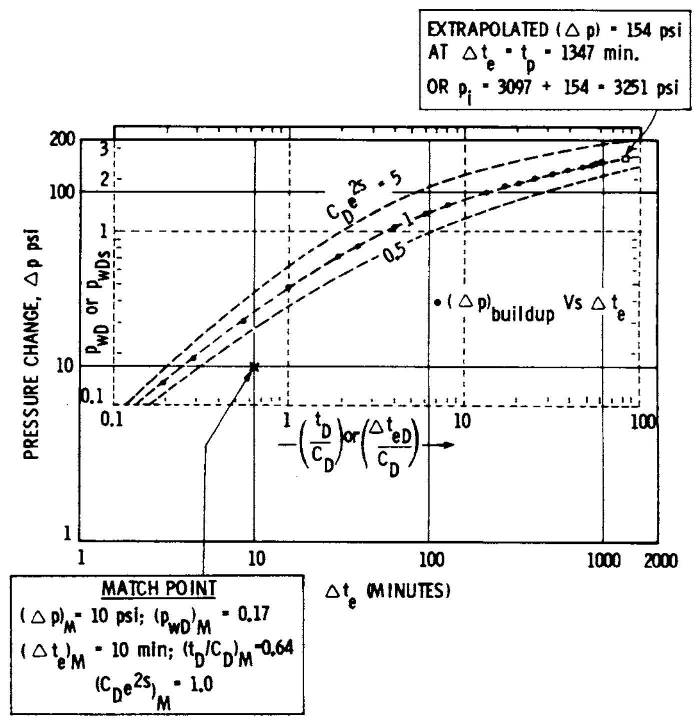

FIGURE 15: APPLICATION OF NEW METHOD USING GRINGARTEN Et Al TYPE CURVES (FIELD EXAMPLE - REF. 15)

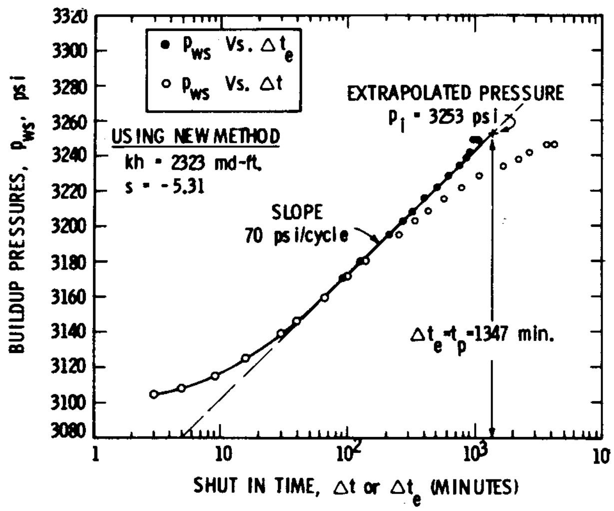

FIGURE 16: APPLICATION OF NEW METHOD TO CONVENTIONAL SEMILOG-ANALYSIS (FIELD EXAMPLE - REF. 15)

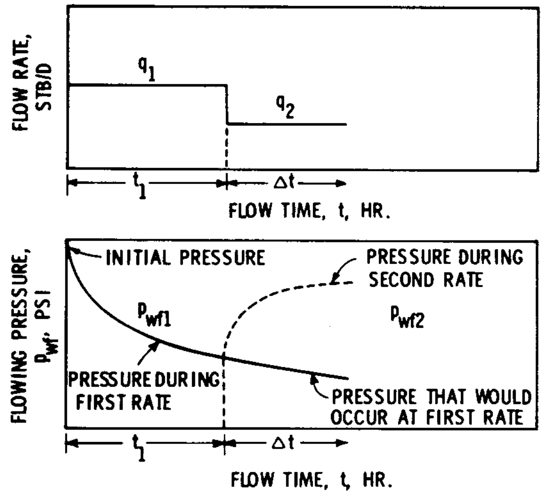

FIGURE 17: SCHEMATIC RATE AND PRESSURE HISTORY FOR A TWO-RATE TESTING, q > ı

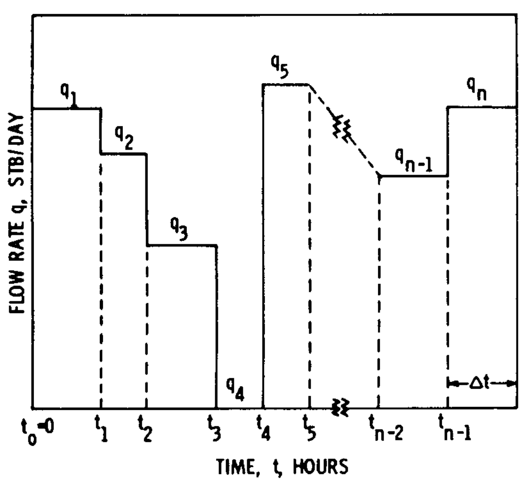

FIGURE 18: SCHEMATIC OF MULTIPLE RATE TESTING

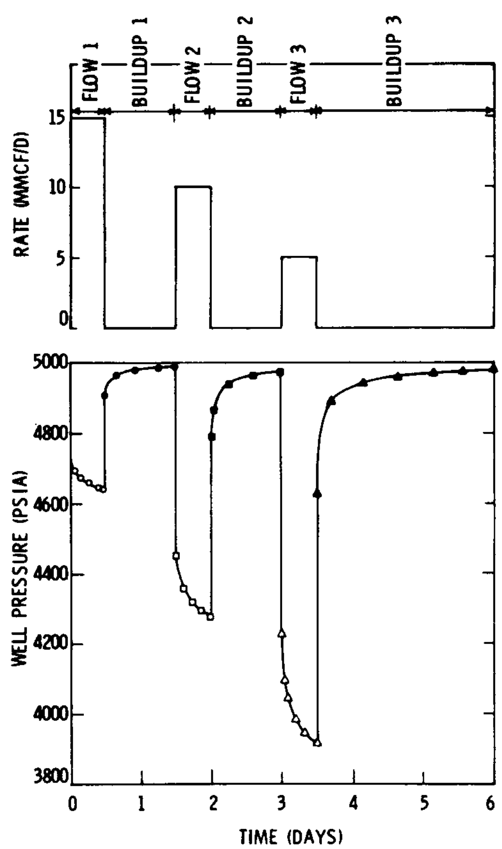

FIGURE 19: RATE AND PRESSURE HISTORY FOR SIMULATED EXAMPLE

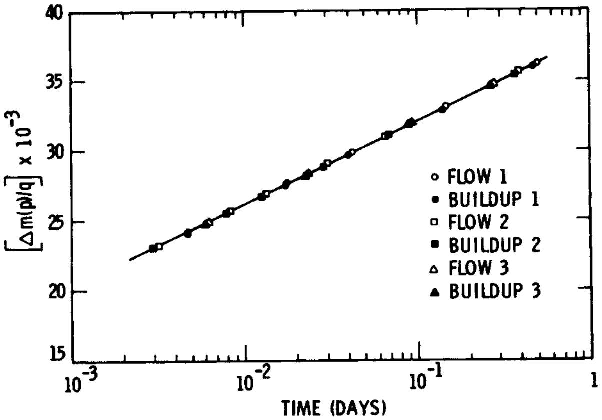

FIGURE 20: NORMALIZED MULTIPLE RATE TEST DATA (SIMULATED EXAMPLE)

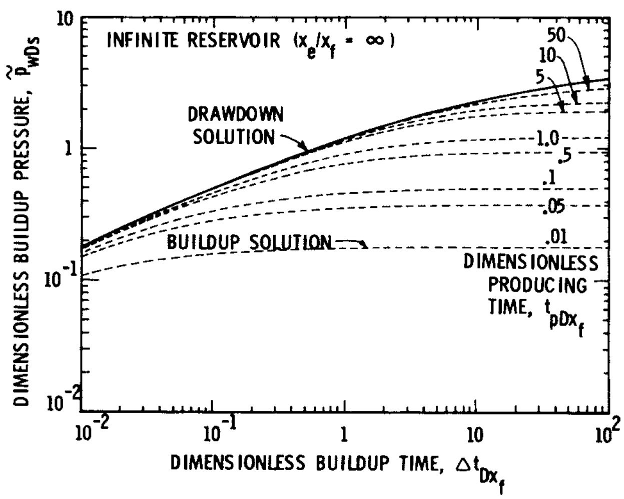

FIGURE 21: BUILDUP TYPE CURVES FOR VARIOUS PRODUCING TIMES-LOG-LOG GRAPH (VERTICALLY FRACTURED WELL WITH INFINITE FLOW CAPACITY FRACTURE)

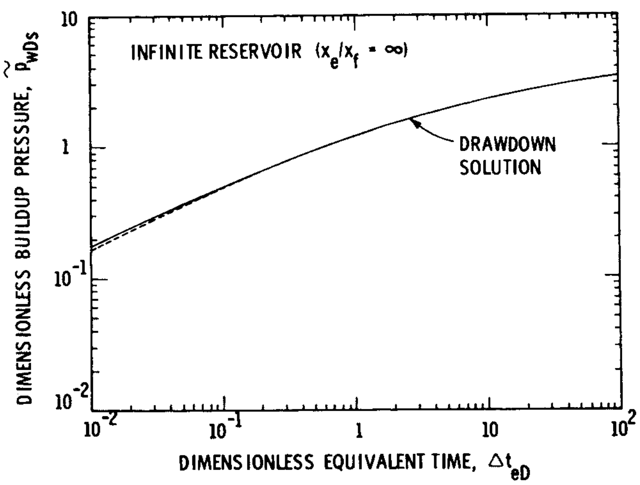

FIGURE 22: NORMALIZED BUILDUP TYPE CURVES (LOG-LOG GRAPH) (VERTICALLY FRACTURED WELL WITH INFINITE FLOW CAPACITY FRACTURE)